Certifying Confluence of Quasi-Decreasing Strongly Deterministic Conditional Term Rewrite Systems*

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August 10, 2017

CADE-26

* Supported by the Austrian Science Fund (FWF): P27502

Motivation

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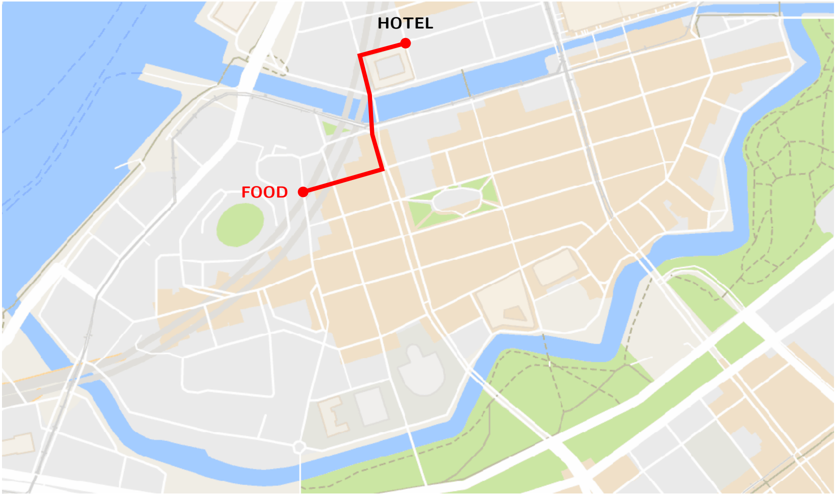

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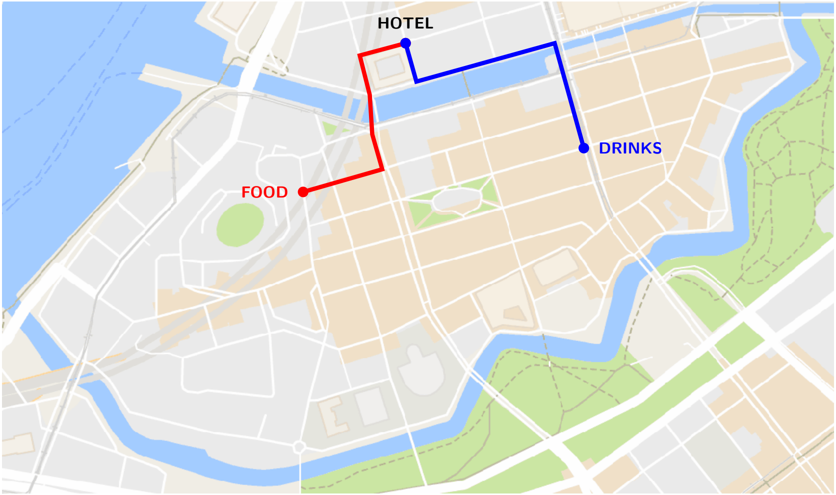

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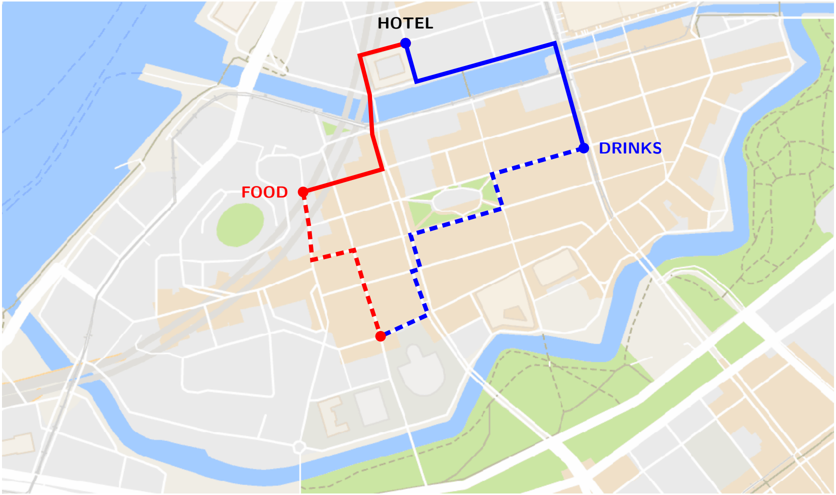
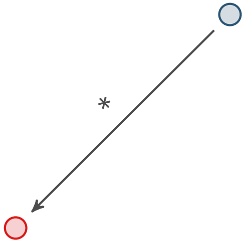


Confluence



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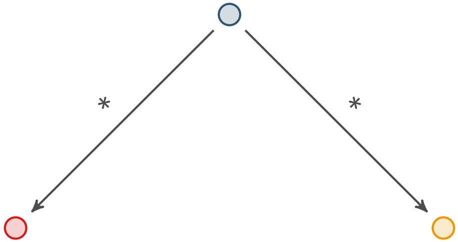

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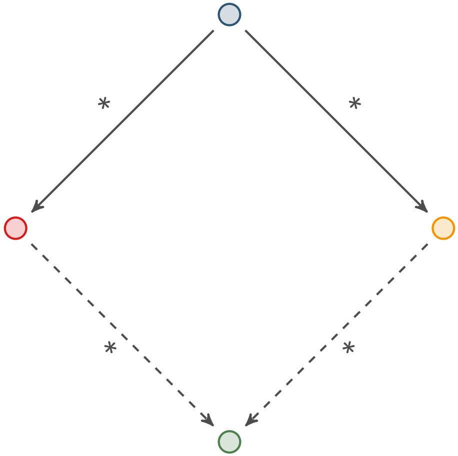

Conditional Term Rewriting


```
min (x:[])                 = x
min (x:xs) | x < y         = x
           | otherwise     = y
           where y = min xs
```

Conditional Term Rewriting

$$\begin{aligned}
\mathsf{min}(x : [\,]) &\to x \\
\mathsf{min}(x : xs) &\to x \Leftarrow \mathsf{min}(xs) \to^* y,\ x < y \to^* \mathsf{true} \\
\mathsf{min}(x : xs) &\to y \Leftarrow \mathsf{min}(xs) \to^* y,\ x < y \to^* \mathsf{false}
\end{aligned}$$

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\mathsf{s}(x) < \mathsf{s}(y) &\to x < y
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Conditional Term Rewriting

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\begin{aligned}
\mathsf{min}(\mathsf{x} : [\,]) &\to \mathsf{x} \\
\mathsf{min}(\mathsf{x} : \mathsf{xs}) &\to \mathsf{x} \Leftarrow \mathsf{min}(\mathsf{xs}) \to^* \mathsf{y},\ \mathsf{x} < \mathsf{y} \to^* \mathsf{true} \\
\mathsf{min}(\mathsf{x} : \mathsf{xs}) &\to \mathsf{y} \Leftarrow \mathsf{min}(\mathsf{xs}) \to^* \mathsf{y},\ \mathsf{x} < \mathsf{y} \to^* \mathsf{false} \\
\mathsf{x} < 0 &\to \mathsf{false} \\
0 < \mathsf{s}(\mathsf{y}) &\to \mathsf{true} \\
\mathsf{s}(\mathsf{x}) < \mathsf{s}(\mathsf{y}) &\to \mathsf{x} < \mathsf{y}
\end{aligned}
$$

$n = \mathsf{s}^n(0)$

$\mathsf{min}(2 : 1 : [\,]) \to$

Conditional Term Rewriting

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$\mathsf{min}(2 : 1 : [\,]) \to$

$\qquad \mathsf{min}(1 : [\,]) \to 1$

Conditional Term Rewriting

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$n = \mathsf{s}^n(0)$

$\mathsf{min}(2 : 1 : [\,]) \to$

$\mathsf{min}(1 : [\,]) \to 1$

$2 < 1 \to$

Conditional Term Rewriting

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\mathsf{s}(\mathsf{x}) < \mathsf{s}(\mathsf{y}) & \to & \mathsf{x} < \mathsf{y}
\end{array}
$$

$n = \mathsf{s}^n(0)$

$$
\begin{array}{l}
\mathsf{min}(2 : 1 : [\,]) \to \\
\qquad \mathsf{min}(1 : [\,]) \to 1 \\
\qquad 2 < 1 \to 1 < 0 \to \mathsf{false}
\end{array}
$$

Conditional Term Rewriting

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$$n = \mathsf{s}^n(0)$$

$$\mathsf{min}(2 : 1 : [\,]) \to 1$$

Conditional Critical Pairs

$$\begin{aligned}
\mathsf{min}(x : [\,]) &\to x \\
\mathsf{min}(x : xs) &\to x \Leftarrow \mathsf{min}(xs) \to^* y,\ x < y \to^* \mathsf{true} \\
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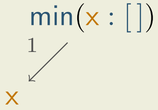

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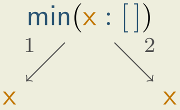

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\mathsf{s}(x) < \mathsf{s}(y) &\to x < y
\end{aligned}
$$

$$
x \;\overset{1}{\leftarrow}\; \mathsf{min}(x : [\,]) \;\overset{2}{\rightarrow}\; x
$$

$$
x \approx x \Leftarrow \mathsf{min}([\,]) \to^* y,\ x < y \to^* \mathsf{true}
$$

Conditional Critical Pairs

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\mathsf{min}(x : xs) &\to x \Leftarrow \mathsf{min}(xs) \to^* y,\ x < y \to^* \mathsf{true} \\
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0 < \mathsf{s}(y) &\to \mathsf{true} \\
\mathsf{s}(x) < \mathsf{s}(y) &\to x < y
\end{aligned}
$$

$$
\begin{aligned}
x \approx x &\Leftarrow \mathsf{min}([]) \to^* y,\ x < y \to^* \mathsf{true} \\
x \approx y &\Leftarrow \mathsf{min}([]) \to^* y,\ x < y \to^* \mathsf{false} \\
x \approx y &\Leftarrow \mathsf{min}(xs) \to^* z,\ x < z \to^* \mathsf{true}, \\
&\qquad \mathsf{min}(xs) \to^* y,\ x < y \to^* \mathsf{false}
\end{aligned}
$$

σ *satisfies* C if $u\sigma \rightarrow^* v\sigma$ for all $u \approx v \in C$

Joinability of CCPs

Some Definitions

σ *satisfies* C if $u\sigma \to^* v\sigma$ for all $u \approx v \in C$

$s \approx t \Leftarrow C$ is *joinable* if $s\sigma \to^* \cdot {}^*\!\leftarrow t\sigma$ for all σ that satisfy C

Joinability of CCPs

Some Definitions

σ *satisfies* C if $u\sigma \to^* v\sigma$ for all $u \approx v \in C$

$s \approx t \Leftarrow C$ is *joinable* if $s\sigma \to^* \cdot {}^*\!\!\leftarrow t\sigma$ for all σ that satisfy C

$s \approx t \Leftarrow C$ is *infeasible* if there is no σ that satisfies C

A Critical Pair Criterion

Theorem A

Quasi-decreasing strongly deterministic CTRSs where all CCPs are joinable are confluent.

A Critical Pair Criterion

Theorem A

Quasi-decreasing strongly deterministic CTRSs where all CCPs are joinable are confluent.

Critical Pair Lemma + Newman's Lemma

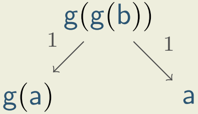
Terminating TRSs where all CPs are joinable are confluent.

Problem 1: Critical Pair Lemma doesn't hold

$$\begin{aligned} g(x) &\to a \Leftarrow x \to^* g(x) \\ b &\to g(b) \end{aligned}$$

Problem 1: Critical Pair Lemma doesn't hold

$$\begin{aligned} g(x) &\rightarrow a \Leftarrow x \rightarrow^* g(x) \\ b &\rightarrow g(b) \end{aligned}$$

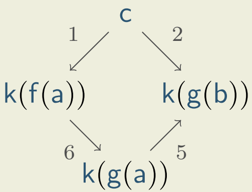


Problem 2: Termination is not enough

$$
\begin{aligned}
c &\to k(f(a)) & h(f(a)) &\to c \\
c &\to k(g(b)) & a &\to b \\
h(x) &\to k(x) & f(x) &\to g(x) \Leftarrow h(f(x)) \to^* k(g(b))
\end{aligned}
$$

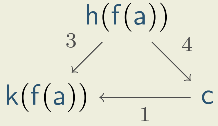
Problem 2: Termination is not enough

$$\begin{aligned} \mathsf{c} &\to \mathsf{k}(\mathsf{f}(\mathsf{a})) & \mathsf{h}(\mathsf{f}(\mathsf{a})) &\to \mathsf{c} \\ \mathsf{c} &\to \mathsf{k}(\mathsf{g}(\mathsf{b})) & \mathsf{a} &\to \mathsf{b} \\ \mathsf{h}(x) &\to \mathsf{k}(x) & \mathsf{f}(x) &\to \mathsf{g}(x) \Leftarrow \mathsf{h}(\mathsf{f}(x)) \to^{*} \mathsf{k}(\mathsf{g}(\mathsf{b})) \end{aligned}$$



Problem 2: Termination is not enough

$$\begin{aligned} c &\to k(f(a)) & h(f(a)) &\to c \\ c &\to k(g(b)) & a &\to b \\ h(x) &\to k(x) & f(x) &\to g(x) \Leftarrow h(f(x)) \to^* k(g(b)) \end{aligned}$$



Problem 2: Termination is not enough

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\begin{aligned}
\mathsf{c} &\to \mathsf{k}(\mathsf{f}(\mathsf{a})) & \mathsf{h}(\mathsf{f}(\mathsf{a})) &\to \mathsf{c} \\
\mathsf{c} &\to \mathsf{k}(\mathsf{g}(\mathsf{b})) & \mathsf{a} &\to \mathsf{b} \\
\mathsf{h}(x) &\to \mathsf{k}(x) & \mathsf{f}(x) &\to \mathsf{g}(x) \Leftarrow \mathsf{h}(\mathsf{f}(x)) \to^{*} \mathsf{k}(\mathsf{g}(\mathsf{b}))
\end{aligned}
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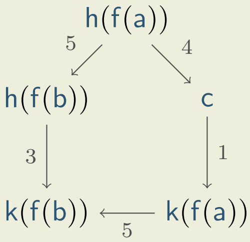
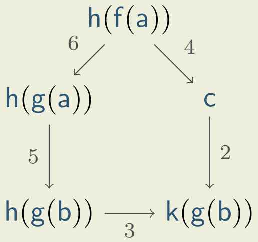


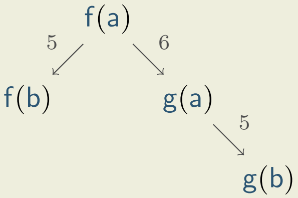
Problem 2: Termination is not enough

$$\begin{array}{rcl@{\qquad}rcl} \mathsf{c} & \to & \mathsf{k(f(a))} & \mathsf{h(f(a))} & \to & \mathsf{c} \\ \mathsf{c} & \to & \mathsf{k(g(b))} & \mathsf{a} & \to & \mathsf{b} \\ \mathsf{h}(x) & \to & \mathsf{k}(x) & \mathsf{f}(x) & \to & \mathsf{g}(x) \Leftarrow \mathsf{h(f}(x)) \to^* \mathsf{k(g(b))} \end{array}$$



Problem 2: Termination is not enough

$$\begin{array}{rcl} \mathsf{c} & \to & \mathsf{k}(\mathsf{f}(\mathsf{a})) \\ \mathsf{c} & \to & \mathsf{k}(\mathsf{g}(\mathsf{b})) \\ \mathsf{h}(x) & \to & \mathsf{k}(x) \end{array} \qquad \begin{array}{rcl} \mathsf{h}(\mathsf{f}(\mathsf{a})) & \to & \mathsf{c} \\ \mathsf{a} & \to & \mathsf{b} \\ \mathsf{f}(x) & \to & \mathsf{g}(x) \Leftarrow \mathsf{h}(\mathsf{f}(x)) \to^* \mathsf{k}(\mathsf{g}(\mathsf{b})) \end{array}$$



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\begin{aligned}
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\end{aligned}
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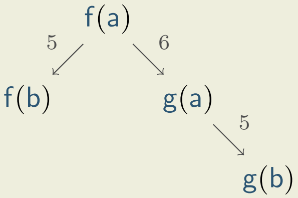

Solution:
Quasi-Decreasingness

Problem 3: Overlaps

Overlap of rule with itself at root position

$$
\begin{aligned}
0 + y &\to y \\
s(x) + y &\to x + s(y) \\
f(x, y) &\to z \Leftarrow x + y \to^* z + v
\end{aligned}
$$

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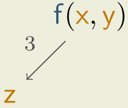

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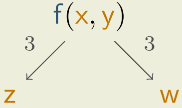

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0 + y &\to y \\
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\end{aligned}
$$

$f(x, y)$

$z \;\overset{3}{\leftarrow}\; f(x,y) \;\overset{3}{\rightarrow}\; w$

$$z \approx w \Leftarrow x + y \to^{*} z + v,\ x + y \to^{*} w + u$$

Problem 3: Overlaps

Overlap of rule with itself at root position

$$
\begin{aligned}
0 + y &\rightarrow y \\
s(x) + y &\rightarrow x + s(y) \\
f(x, y) &\rightarrow z \Leftarrow x + y \rightarrow^* z + v
\end{aligned}
$$

$$
\begin{array}{ccc}
 & f(x, y) & \\
{}^{3}\swarrow & & \searrow^{3} \\
z & \approx & w \Leftarrow x + y \rightarrow^* z + v,\ x + y \rightarrow^* w + u
\end{array}
$$

$$\sigma(x) = \sigma(z) = \sigma(u) = 1,\ \sigma(y) = \sigma(w) = \sigma(v) = 0$$

Problem 3: Overlaps

Overlap of rule with itself at root position

$$
\begin{aligned}
0 + y &\to y \\
s(x) + y &\to x + s(y) \\
f(x, y) &\to z \Leftarrow x + y \to^* z + v
\end{aligned}
$$

$$1 \quad \approx \quad 0 \Leftarrow 1 + 0 \to^* 1 + 0, 1 + 0 \to^* 0 + 1$$

$$\sigma(x) = \sigma(z) = \sigma(u) = 1, \ \sigma(y) = \sigma(w) = \sigma(v) = 0$$

Problem 3: Overlaps

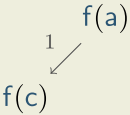
Variable overlap

$$
\begin{aligned}
\mathsf{a} &\to \mathsf{c} \\
\mathsf{g}(\mathsf{a}) &\to \mathsf{h}(\mathsf{b}) \\
\mathsf{h}(\mathsf{b}) &\to \mathsf{g}(\mathsf{c}) \\
\mathsf{f}(x) &\to y \Leftarrow \mathsf{g}(x) \to^* \mathsf{h}(y)
\end{aligned}
$$

Problem 3: Overlaps

Variable overlap

$$
\begin{aligned}
a &\to c \\
g(a) &\to h(b) \\
h(b) &\to g(c) \\
f(x) &\to y \Leftarrow g(x) \to^{*} h(y)
\end{aligned}
$$



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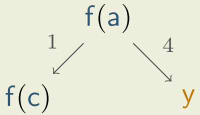

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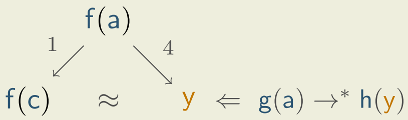

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\end{aligned}
$$

$$
\begin{array}{ccc}
 & f(a) & \\
{}^{1}\swarrow & & \searrow^{4} \\
f(c) & \approx & y \Leftarrow g(a) \to^* h(y)
\end{array}
$$

$\sigma(y) = b$

Problem 3: Overlaps

Variable overlap

$$\begin{aligned} a &\to c \\ g(a) &\to h(b) \\ h(b) &\to g(c) \\ f(x) &\to y \Leftarrow g(x) \to^* h(y) \end{aligned}$$

$$f(c) \quad \approx \quad b \Leftarrow g(a) \to^* h(b)$$

$$\sigma(y) = b$$

Problem 3: Overlaps

Variable overlap

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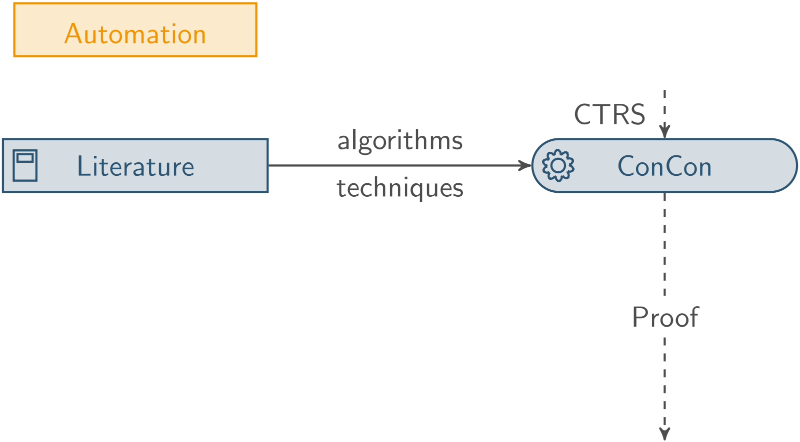
Solution:
Strong Determinism

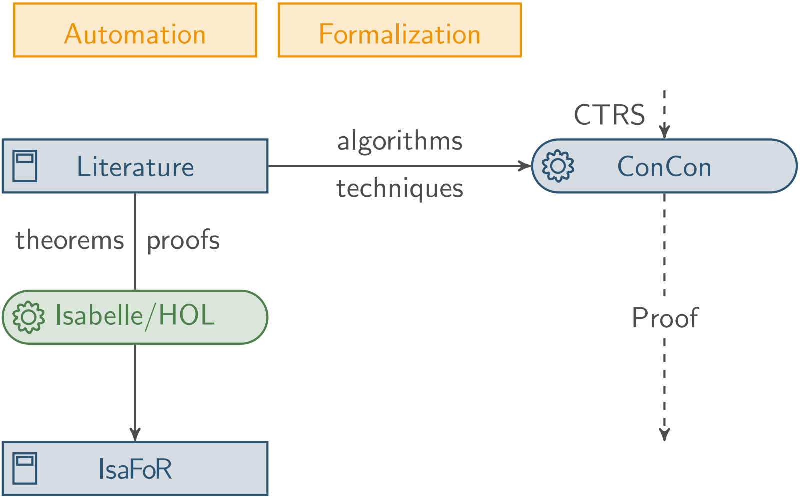
A Critical Pair Criterion

Theorem A
Quasi-decreasing strongly deterministic CTRSs where all CCPs are joinable are confluent.

Our Contributions



Our Contributions



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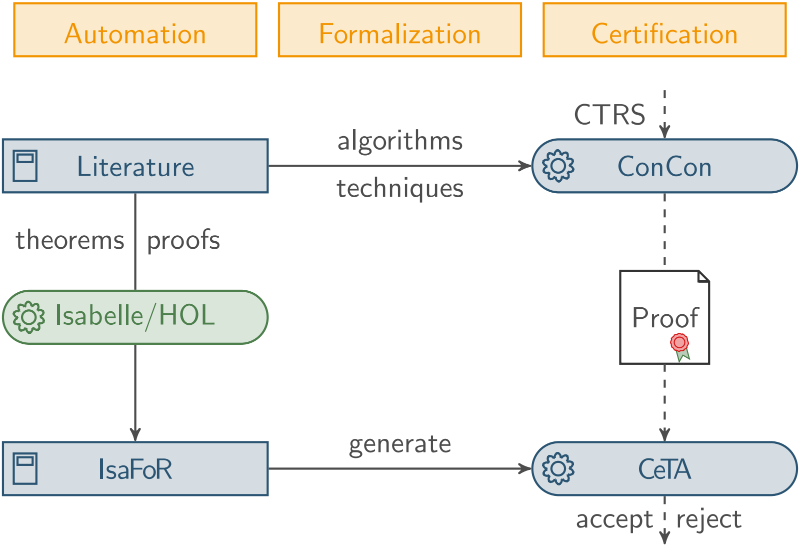

Quick Facts

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- 2 developers
- 10 contributors
- 182,000 LoI
- 2,317 definitions
- 8,525 lemmas
- 1,020 functions

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IsaFoR / CeTA

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→

CeTA

- Haskell
- code generation
- certifies proofs

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- 6 person-month
- 4.5 de Bruijn fact.
- 2,500 LoI
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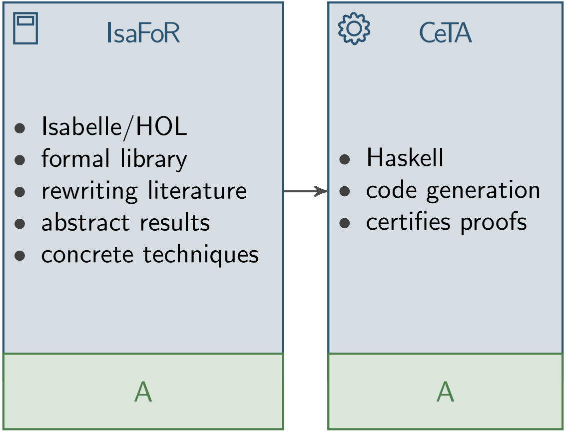
- 83 lemmas
- 14 functions



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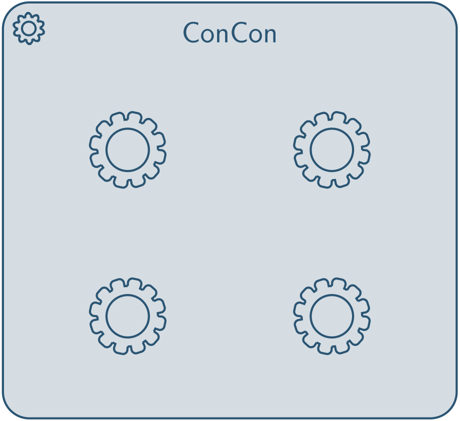

ConCon

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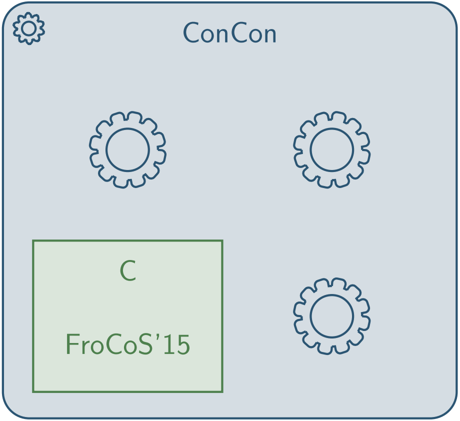

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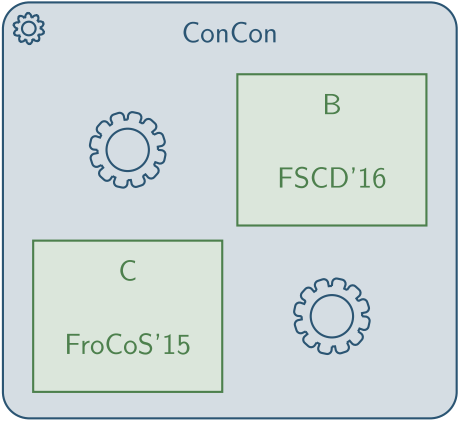

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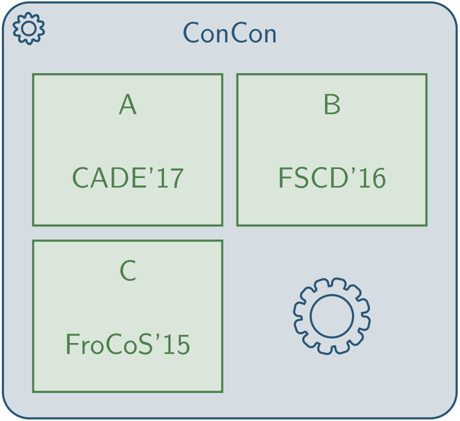

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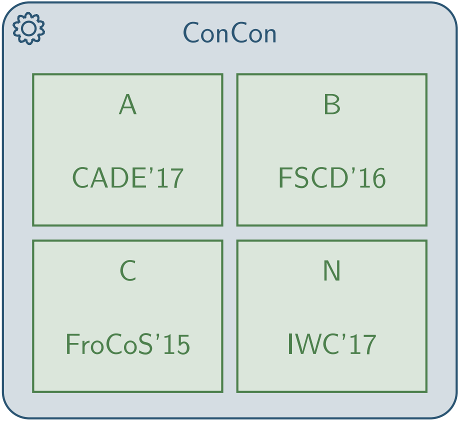
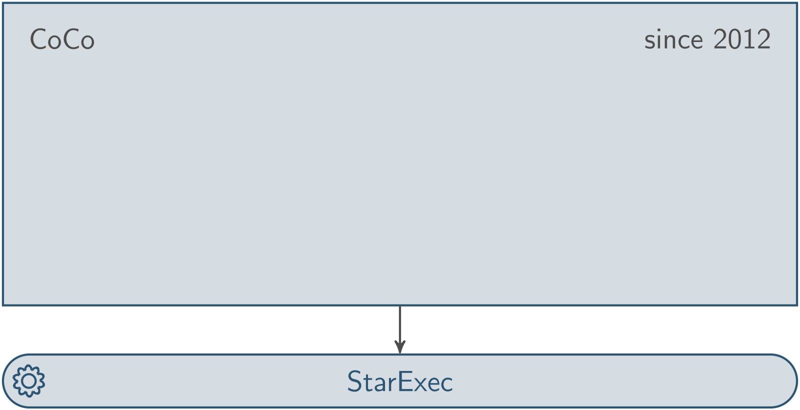


Confluence Competition

CoCo since 2012

Confluence Competition



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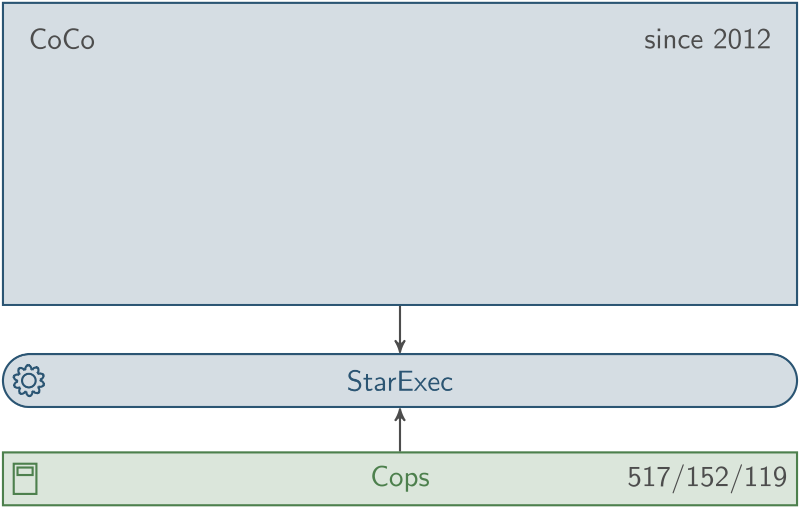

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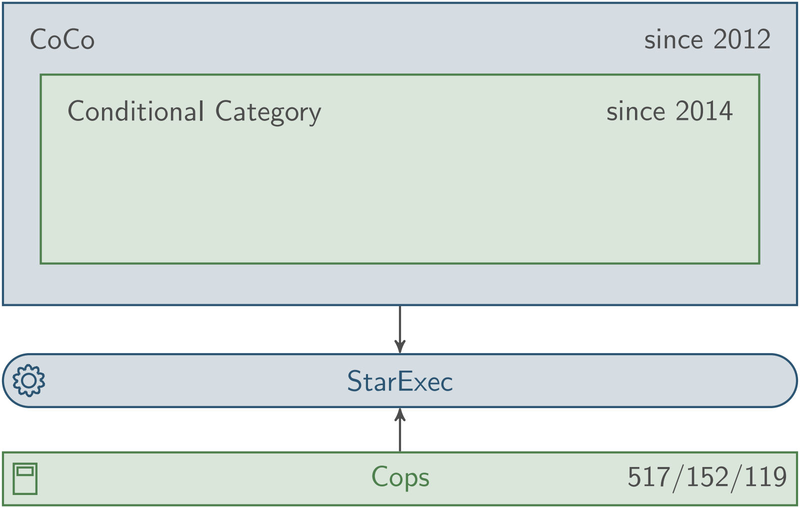

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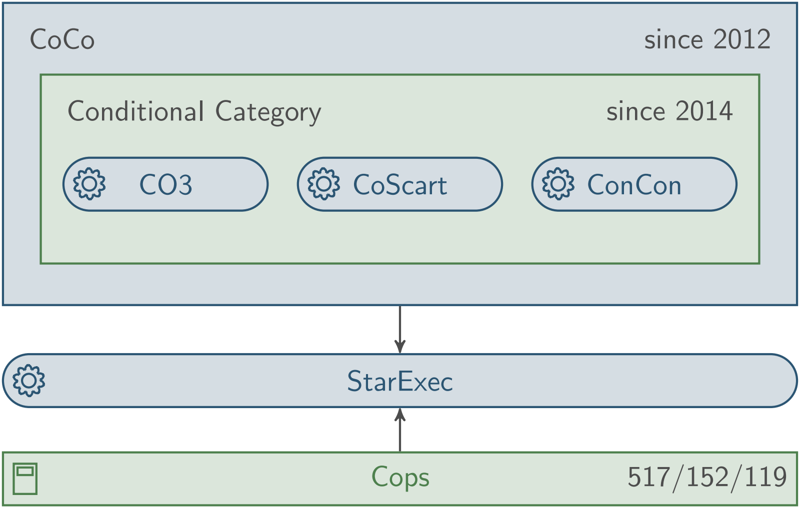

Can ConCon handle this example?

$$
\begin{array}{rcl}
\mathsf{min}(x : []) & \to & x \\
\mathsf{min}(x : xs) & \to & x \Leftarrow \mathsf{min}(xs) \to^* y,\ x < y \to^* \mathsf{true} \\
\mathsf{min}(x : xs) & \to & y \Leftarrow \mathsf{min}(xs) \to^* y,\ x < y \to^* \mathsf{false} \\
x < 0 & \to & \mathsf{false} \\
0 < \mathsf{s}(y) & \to & \mathsf{true} \\
\mathsf{s}(x) < \mathsf{s}(y) & \to & x < y
\end{array}
$$

$$
\begin{array}{l}
x \approx x \Leftarrow \mathsf{min}([]) \to^* y,\ x < y \to^* \mathsf{true} \\
x \approx y \Leftarrow \mathsf{min}([]) \to^* y,\ x < y \to^* \mathsf{false} \\
x \approx y \Leftarrow \mathsf{min}(xs) \to^* z,\ x < z \to^* \mathsf{true}, \\
\qquad\quad \mathsf{min}(xs) \to^* y,\ x < y \to^* \mathsf{false}
\end{array}
$$

Yes! Using "inlining"

$$
\begin{array}{lcl}
\mathsf{min}(x : [\,]) & \to & x \\
\mathsf{min}(x : xs) & \to & x \Leftarrow \boxed{\mathsf{min}(xs) \to^* y,} \; x < y \to^* \mathsf{true} \\
\mathsf{min}(x : xs) & \to & y \Leftarrow \boxed{\mathsf{min}(xs) \to^* y,} \; x < y \to^* \mathsf{false} \\
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\mathsf{s}(x) < \mathsf{s}(y) & \to & x < y
\end{array}
$$

$$
\begin{array}{l}
x \approx x \Leftarrow \mathsf{min}([\,]) \to^* y, \; x < y \to^* \mathsf{true} \\
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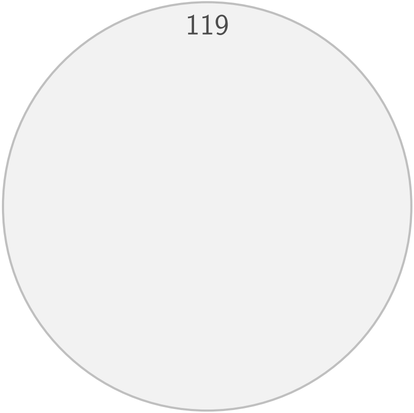
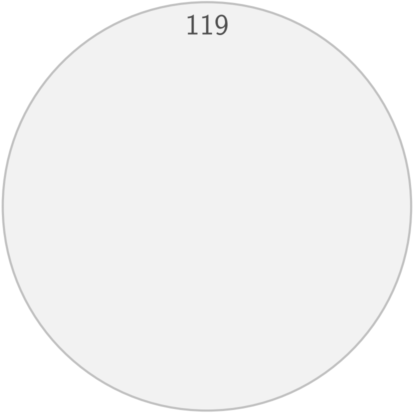
\qquad\qquad\qquad\quad x < \mathsf{min}(xs) \to^* \mathsf{false}
\end{array}
$$

Experimental Results

ConCon 1.3.2



ConCon 1.4.0



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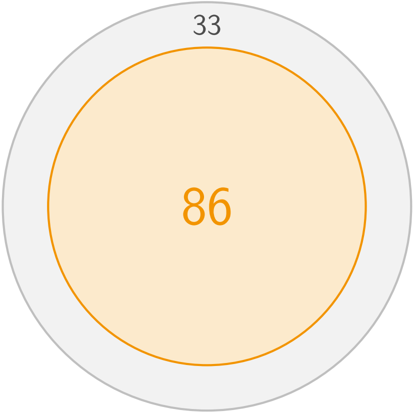
ConCon 1.4.0



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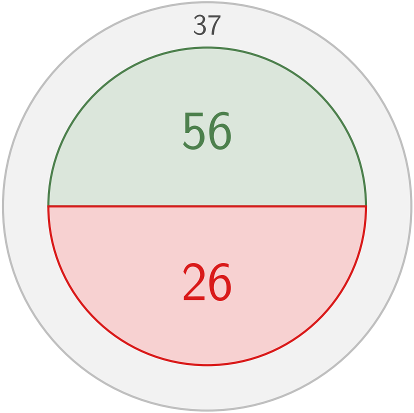

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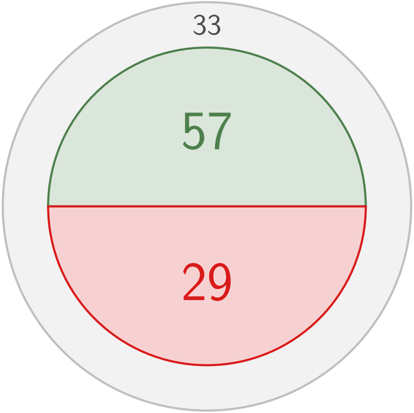

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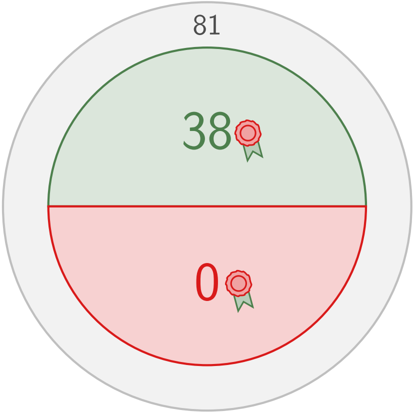

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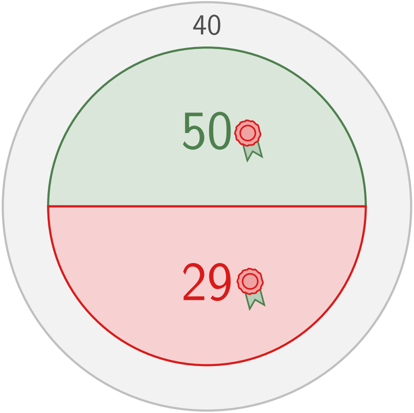

Conclusion

- Confluence of terminating TRS *decidable*
- BUT: joinability of CCPs *undecidable*
- Most parts of CTRS confluence literature formalized in *IsaFoR*
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Thank you for your attention!