Certifying Confluence of Almost Orthogonal CTRSs via Exact Tree Automata Completion*

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1st FSCD

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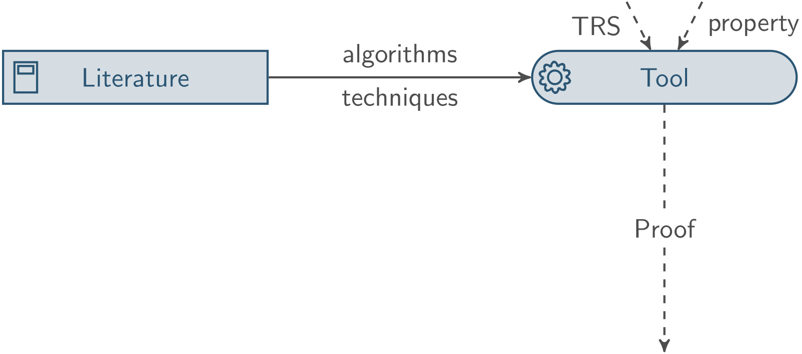
Outline

- The Big Picture
- Conditional Term Rewriting & Confluence
- Infeasibility & Tree Automata
- Certification
- Conclusion

The Big Picture

The State of the Art



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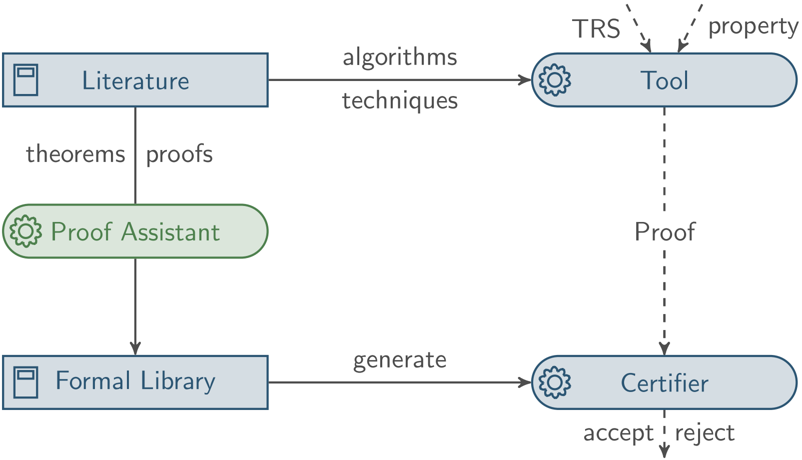

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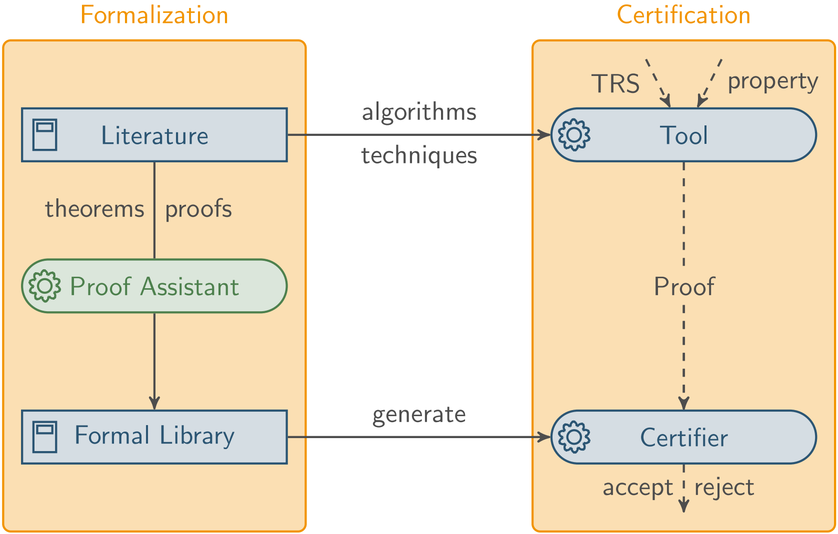

The Fruits of Formalization & Certification

- Understand, clarify, correct *literature*

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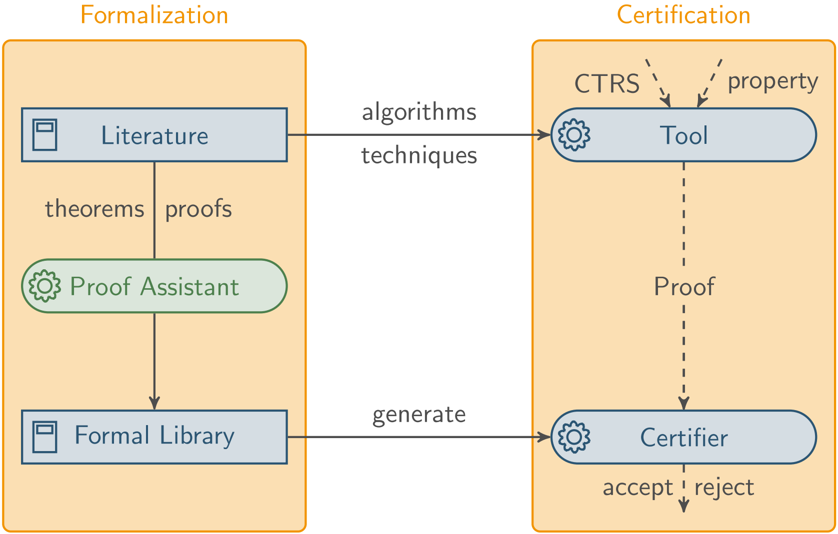
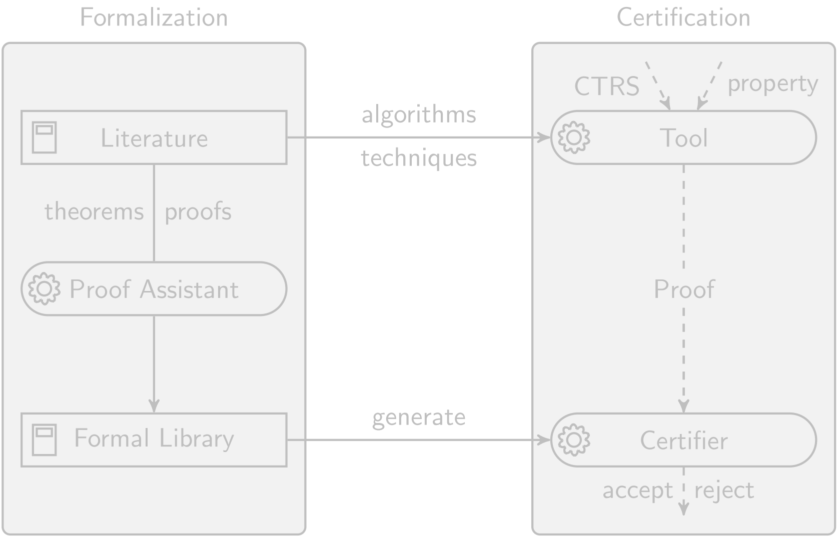
- Expose errors and increase *reliability* of tools

The Vision


Formalization
Certification
Literature
theorems
proofs
Proof Assistant
Formal Library
algorithms
techniques
generate
CTRS
property
Tool
Proof
Certifier
accept
reject

Our Contribution



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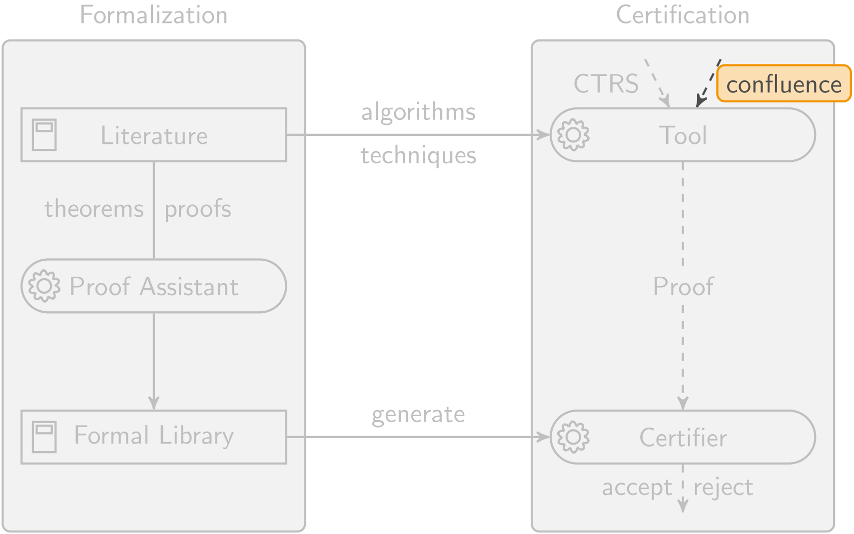

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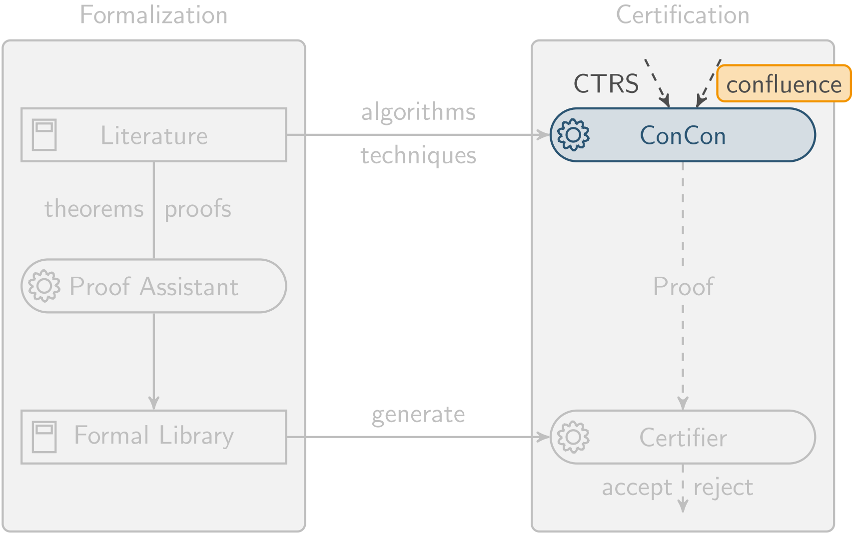

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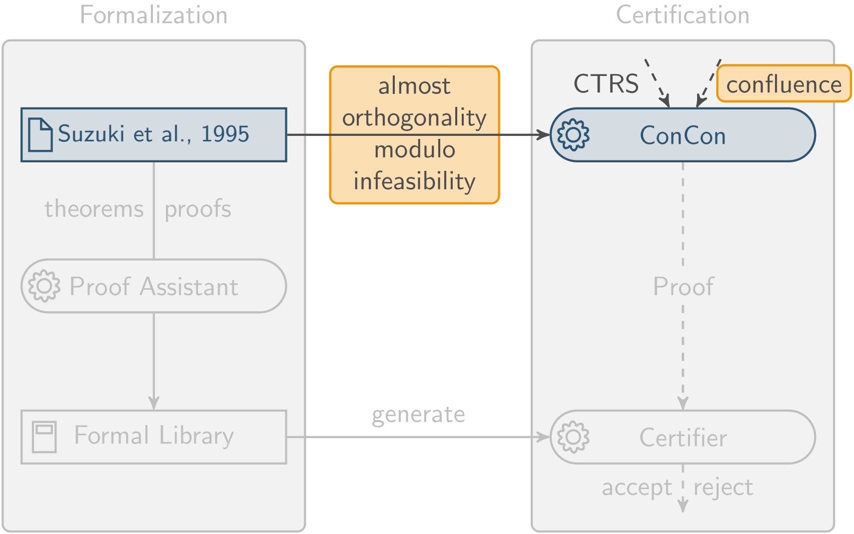

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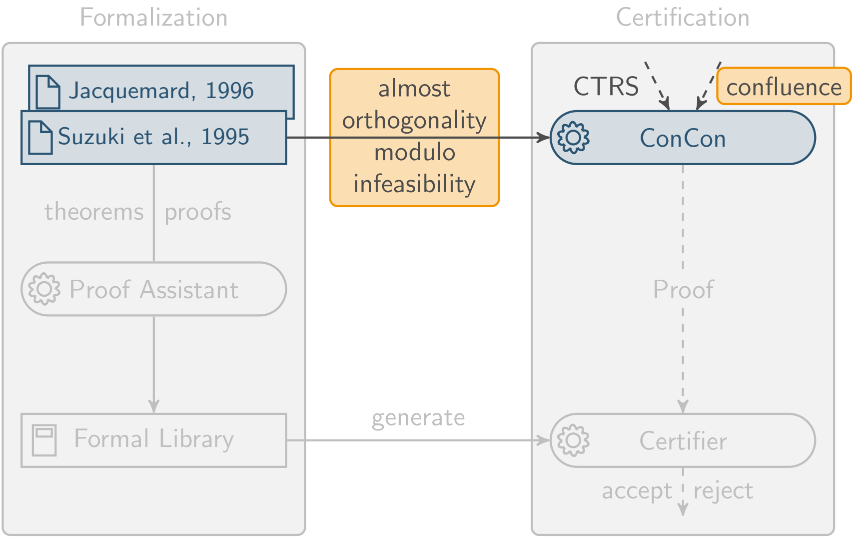

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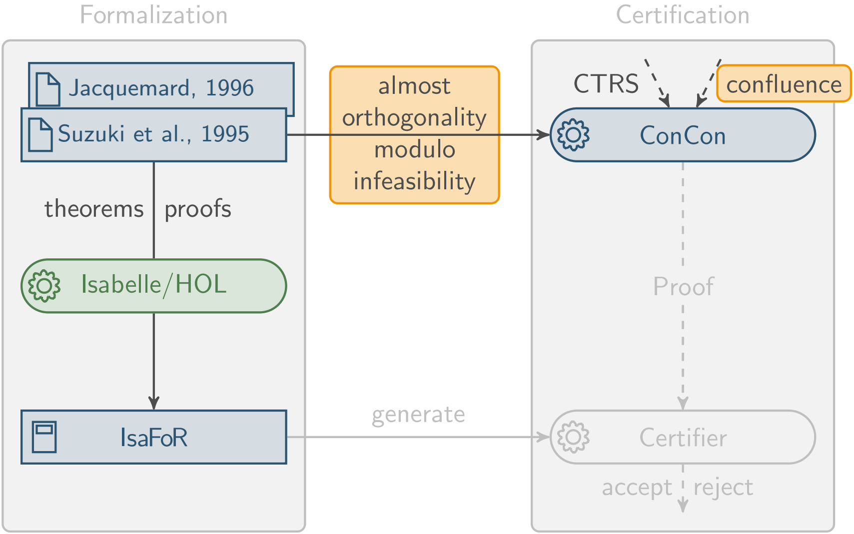

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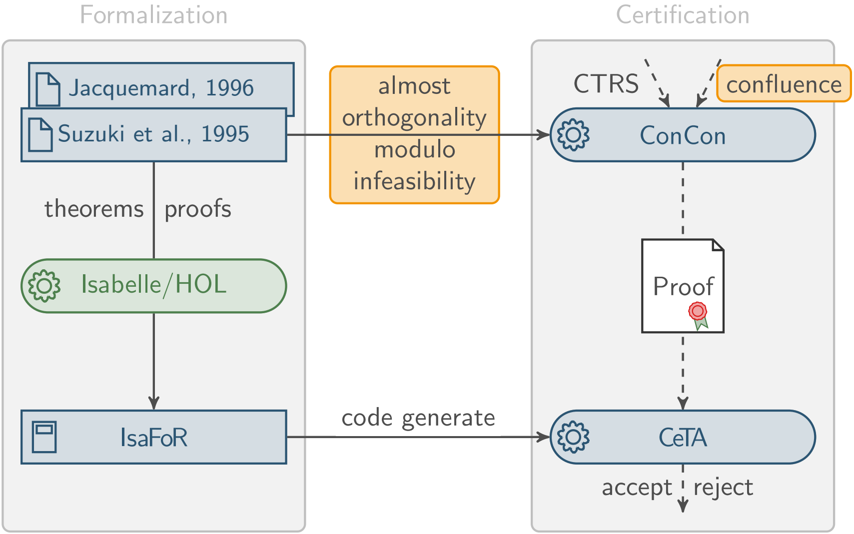

Conditional Term Rewriting & Confluence

Conditional Term Rewriting

Basic definitions

- conditional rewrite rule: $\ell \to r \Leftarrow s_1 \approx t_1, \ldots, s_n \approx t_n$

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- given CTRS $\mathcal{R}$, define TRS of level n, $\mathcal{R}_n$, inductively:

$$\mathcal{R}_0 = \varnothing$$

$$\mathcal{R}_{n+1} = \{\ell\sigma \to r\sigma \mid \ell \to r \Leftarrow c \in \mathcal{R} \land \forall s \approx t \in c.\, s\sigma \to^*_{\mathcal{R}_n} t\sigma\}$$

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Theorem [Suzuki, Middeldorp, Ida (RTA 1995)]
Oriented 3-CTRSs are level-confluent if they are

- Orthogonal,
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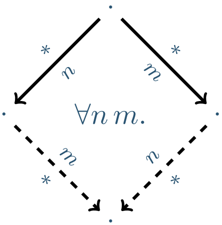

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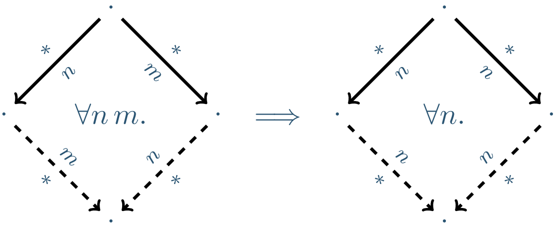
level-commutation level-confluence



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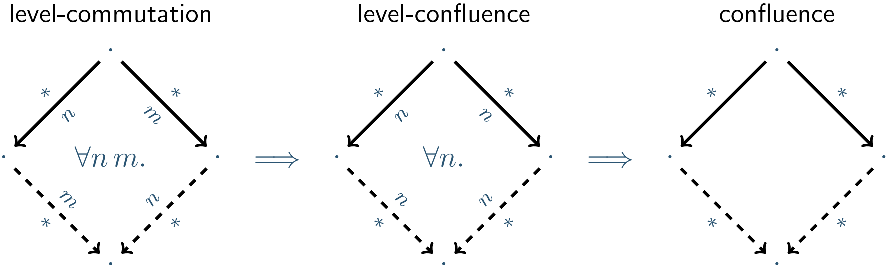
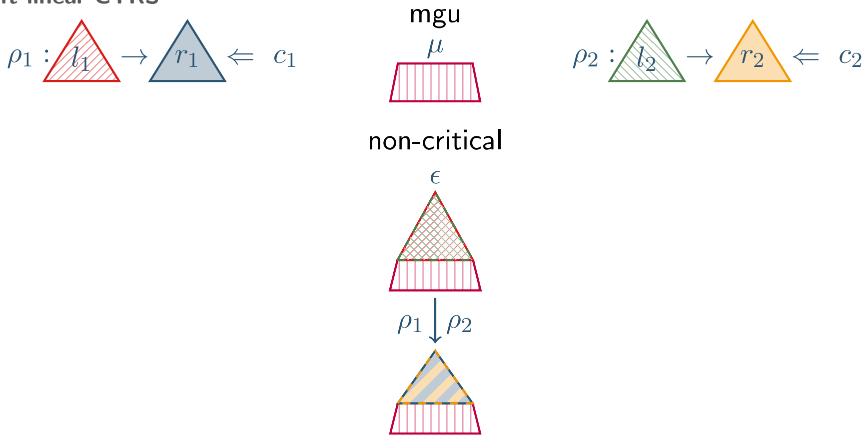


Orthogonality

Left-linear CTRS



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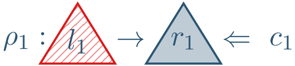
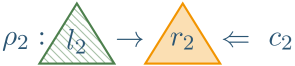


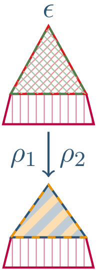
mgu



Example

$$\mathsf{f}(x) \to \mathsf{g}(y) \Leftarrow x \approx y$$

non-critical



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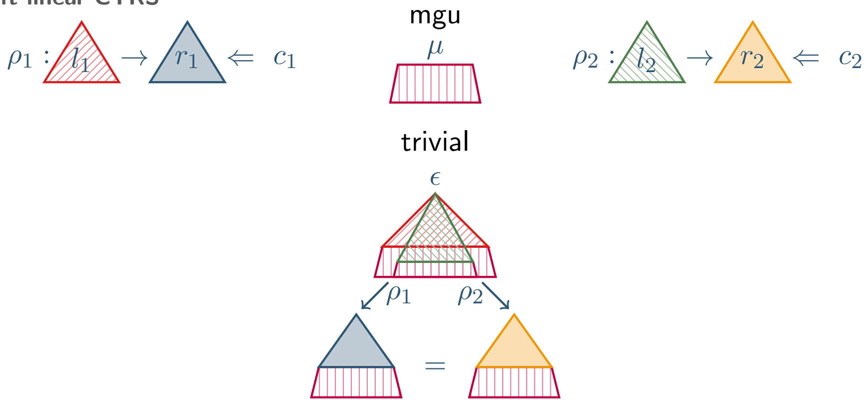

Almost Orthogonality

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Example

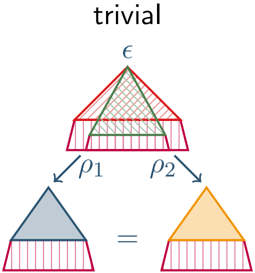
$$\mathsf{f}(\mathsf{a}, x) \to \mathsf{a}$$

$$\mathsf{f}(x, \mathsf{a}) \to \mathsf{a}$$

$$\mathsf{f}(\mathsf{a}, \mathsf{a})$$

$$\swarrow \qquad \searrow$$

$$\mathsf{a} \qquad\qquad \mathsf{a}$$



Almost Orthogonality modulo Infeasibility

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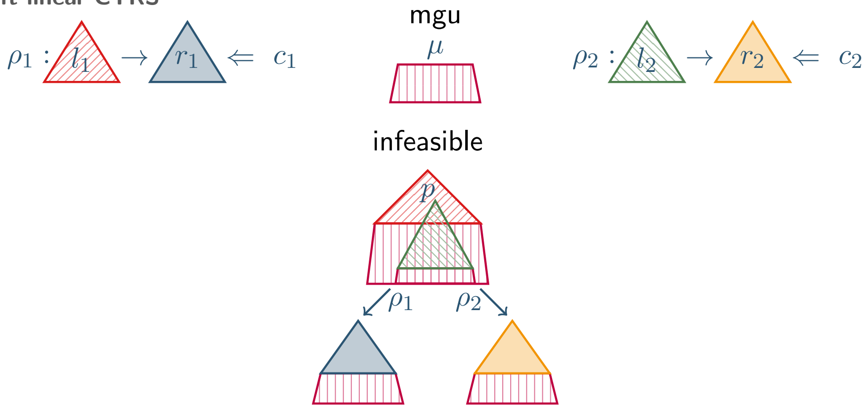

$$\nexists \sigma\, n.\, n, \sigma \vdash c_1\mu, c_2\mu$$

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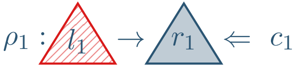

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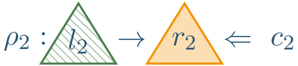

Example

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$\mathsf{f}(x) \to \mathsf{b} \Leftarrow x \approx \mathsf{b}$

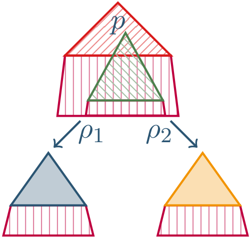
$\mathsf{f}(x)$

$\swarrow \quad \searrow$

$\mathsf{a} \qquad \mathsf{b}$

$x \approx \mathsf{a},\, x \approx \mathsf{b}$

infeasible



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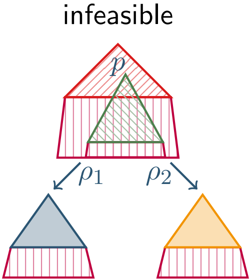

$$\forall m\, n.\, (\xleftarrow[m]{*} \cdot \xrightarrow[n]{*} \subseteq \xrightarrow[n]{*} \cdot \xleftarrow[m]{*} \implies \nexists \sigma.\, m, \sigma \vdash c_1\mu \wedge n, \sigma \vdash c_2\mu)$$

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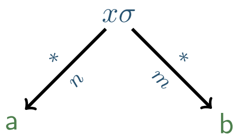

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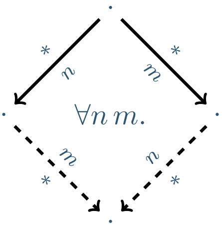

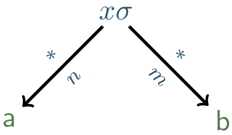

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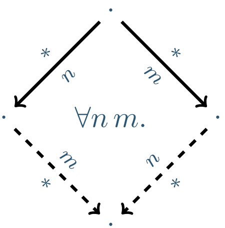

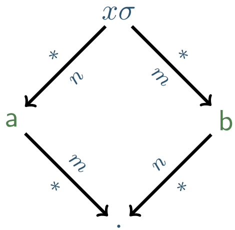

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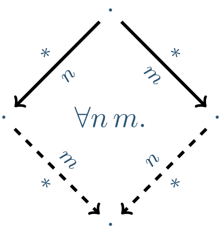

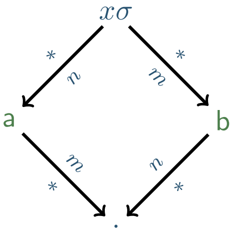

↯ $\mathsf{tcap}(\mathsf{a}) \not\sim \mathsf{tcap}(\mathsf{b})$

Formalized Theorem

Theorem

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Oriented 3-CTRSs are confluent if they are

- Almost Orthogonal modulo infeasibility,
- Extended properly oriented, and
- Right-stable.

Infeasibility & Tree Automata

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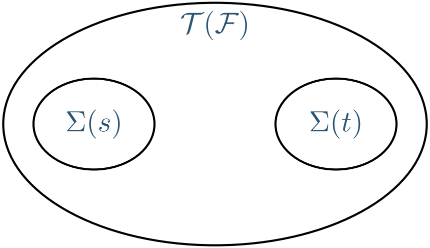

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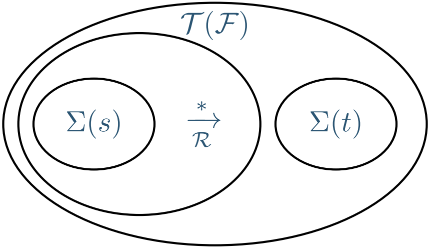

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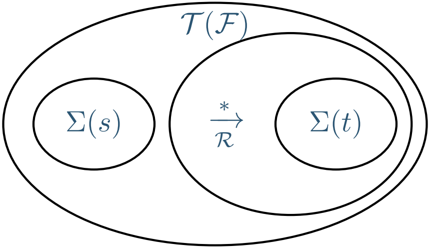

Tree Automata

Basic definitions

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Example

$$\mathsf{f}(\alpha) \to \alpha$$
$$\mathsf{a} \to \alpha$$

$$L(\mathcal{A}) = \mathsf{f}^n(\mathsf{a})$$

Ancestor Automaton

ground-instance transitions

states: $[x] = \Box, \quad [f(t_1, \ldots, t_n)] = f([t_1], \ldots, [t_n])$

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$$\Delta \cup \bigcup_{\ell \to r \in \mathcal{R}} \Delta_\ell \qquad \frac{f(\ell_1, \ldots, \ell_n) \to r \in \mathcal{R} \quad r\theta \to^*_{\Delta_k} q}{f(q_1, \ldots, q_n) \to q \in \Delta_{k+1}} \quad (\dagger)$$

if $\ell_i \in \mathcal{V}(r)$ then $q_i = \ell_i\theta$ else $q_i = [\ell_i]$

Non-reachability via Ancestor Automaton

Theorem [cf. Jacquemard (RTA 1996)]
Given TA $\mathcal{A}$ and linear, growing TRS $\mathcal{R}$ the language of $\mathsf{anc}_{\mathcal{R}}(\mathcal{A})$ is exactly the set of $\mathcal{R}$-ancestors of $L(\mathcal{A})$.

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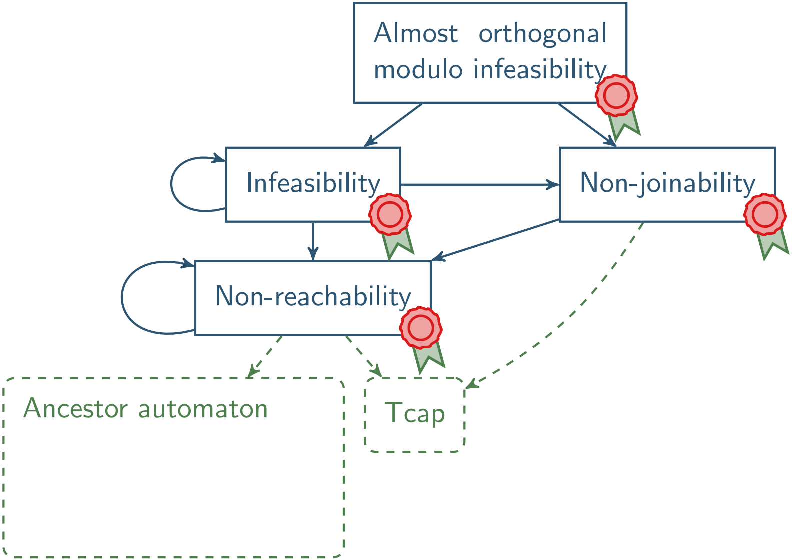
Lemma (Non-reachability via $\mathsf{anc}_{\mathcal{R}}(\mathcal{A})$)
For linear, growing TRS $\mathcal{R}$ if

$$L(\mathcal{A}_{\Sigma(s)} \cap \mathsf{anc}_{\mathcal{R}}(\mathcal{A}_{\Sigma(t)})) = \varnothing$$

then $t\tau$ is not reachable from $s\sigma$ for any σ, τ.

Certification

Certification Problem Format



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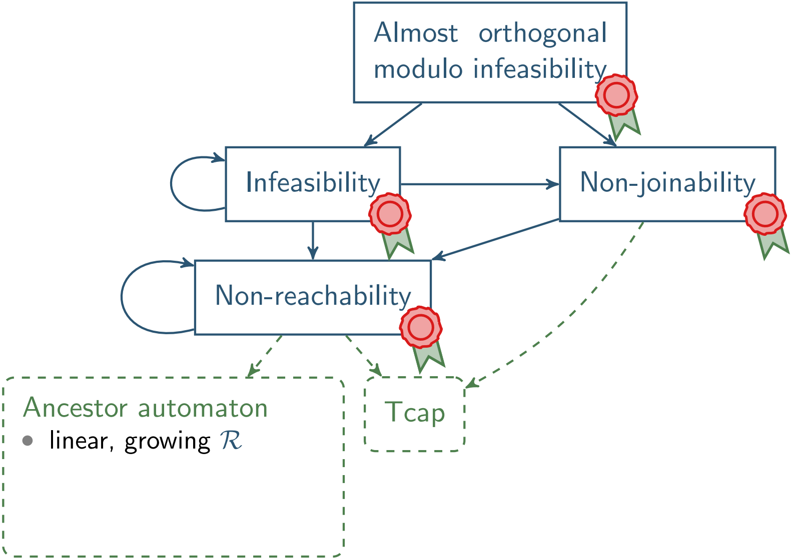

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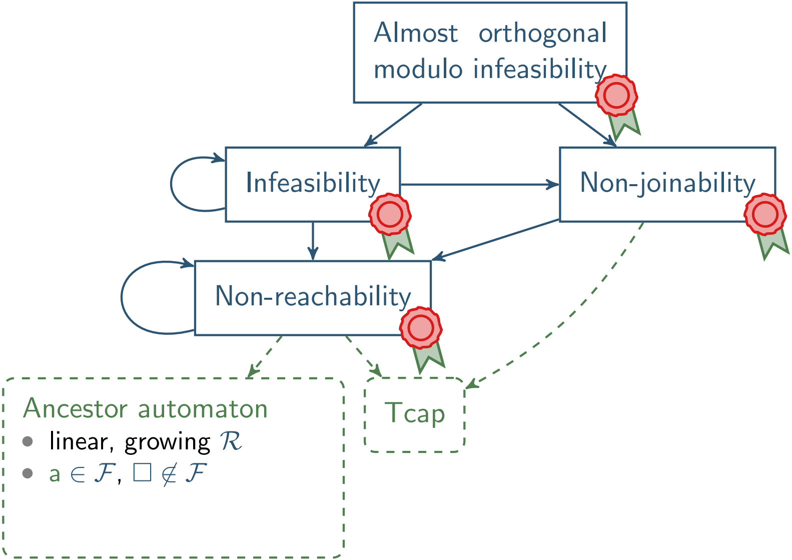

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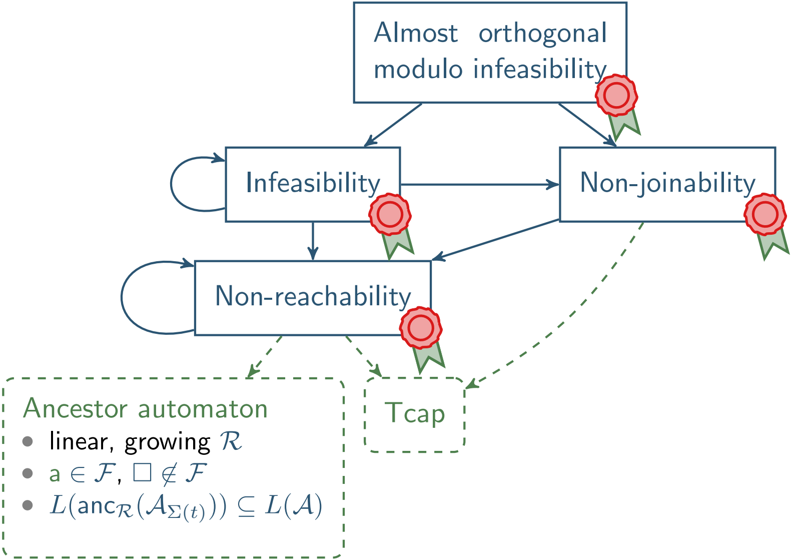

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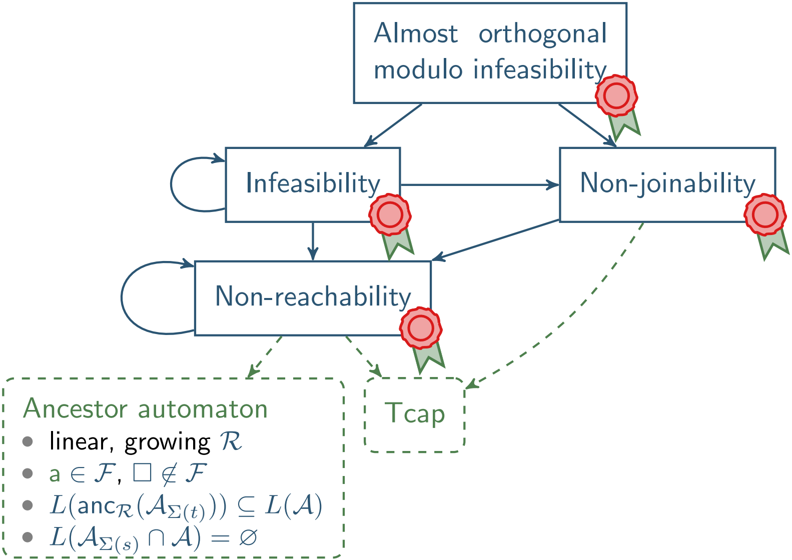

Conclusion

Summary & Future Work

- Confluence & non-reachability formalization in IsaFoR ($\sim 6,600$ LoI)
- Implemented techniques in ConCon
- Extended CPF format of CeTA

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Confluence *(82 CTRSs from Cops)*

	uncert	cert 2	cert 2+3	cert+
✓	47	23	32	35
✗	15	-	-	-
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Non-reachability *(412,829 potential dependency graph edges from TPDB)*

	1s	3s	10s
✓	10,217	24,291	43,364

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