Reliable Analysis of Conditional Term Rewrite Systems[1]

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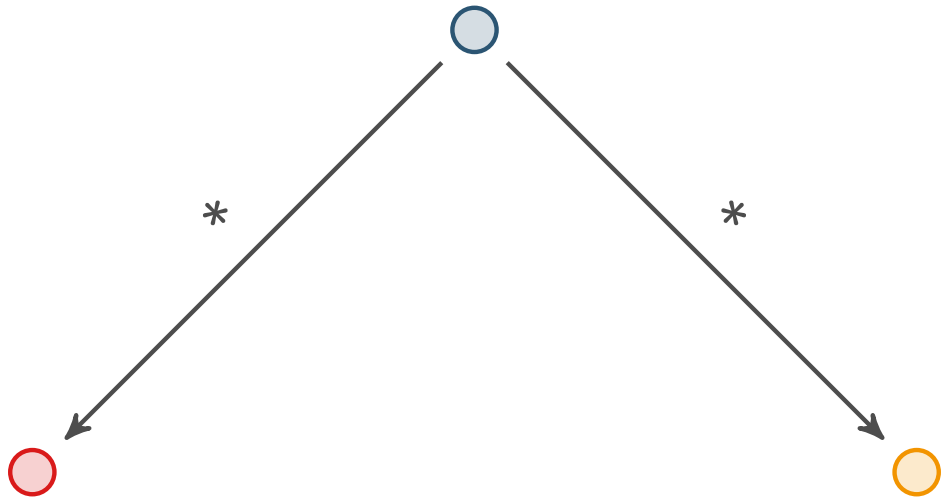
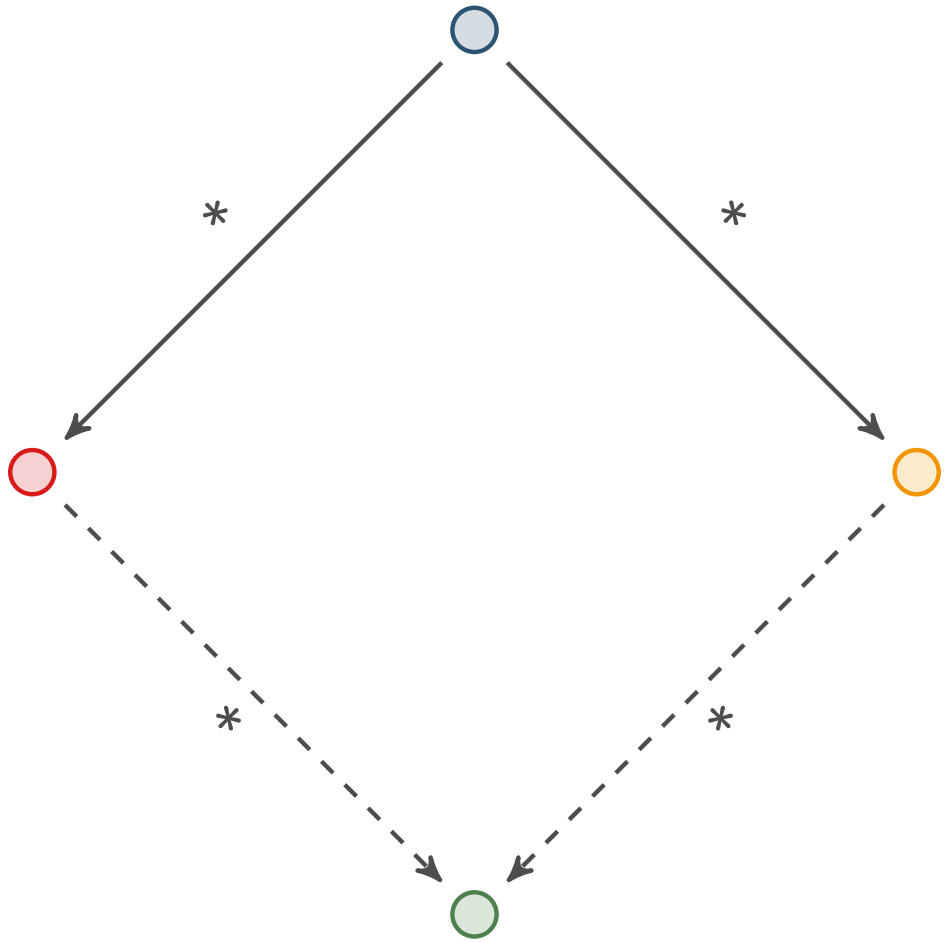
Computational Logic Group, University of Innsbruck

Valencia March 8, 2017

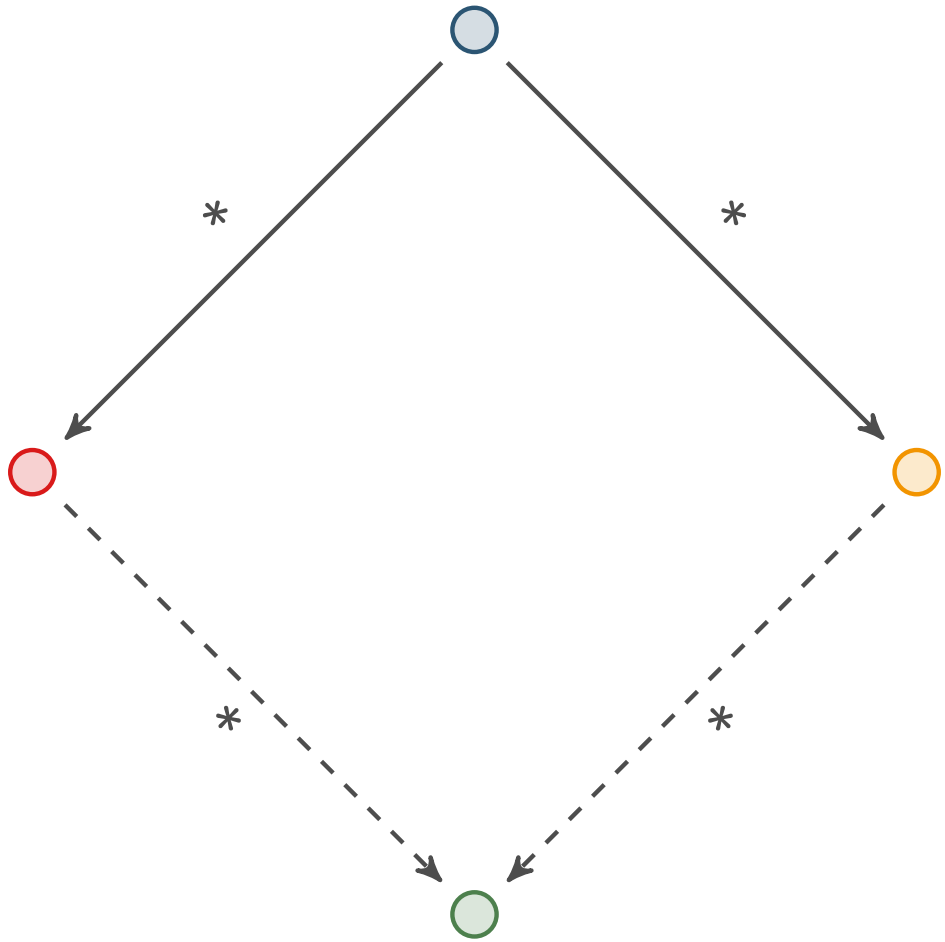
[1]Supported by the Austrian Science Fund (FWF): P27502

Termination





Confluence



Termination

result in finite time

Confluence

same result for same input

Undecidable Decision Problems

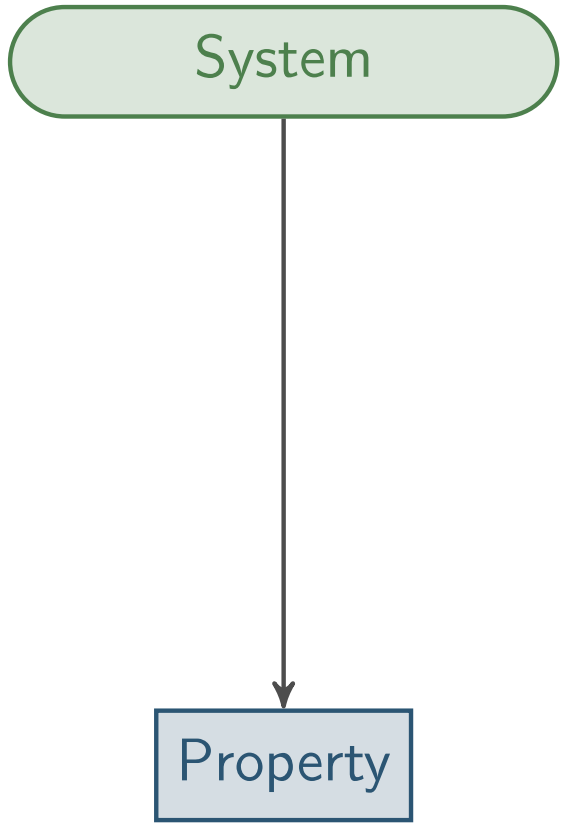
Termination

result in finite time

Confluence

same result for same input

Formal Verification



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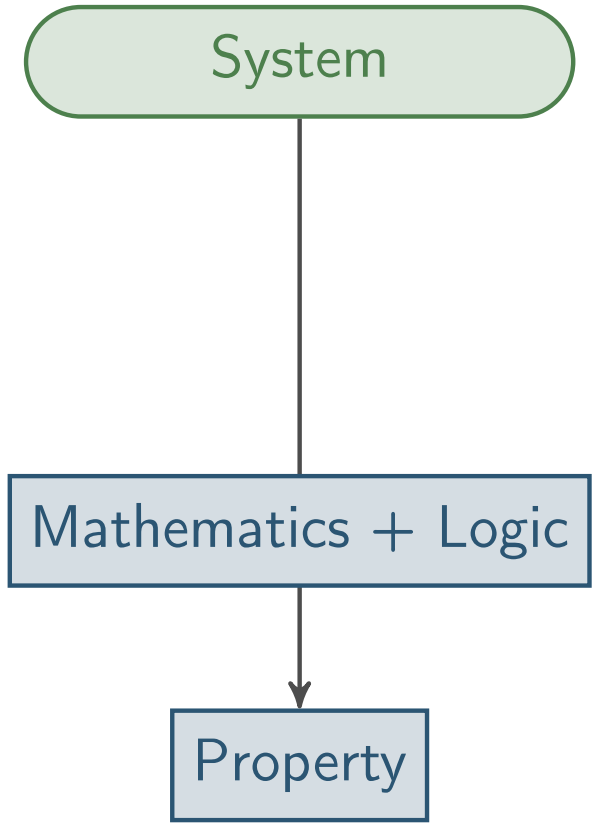

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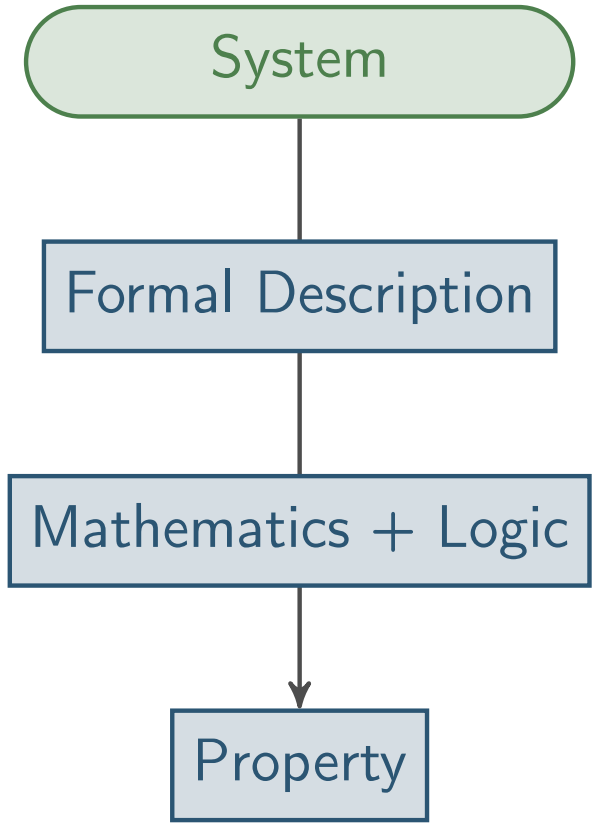

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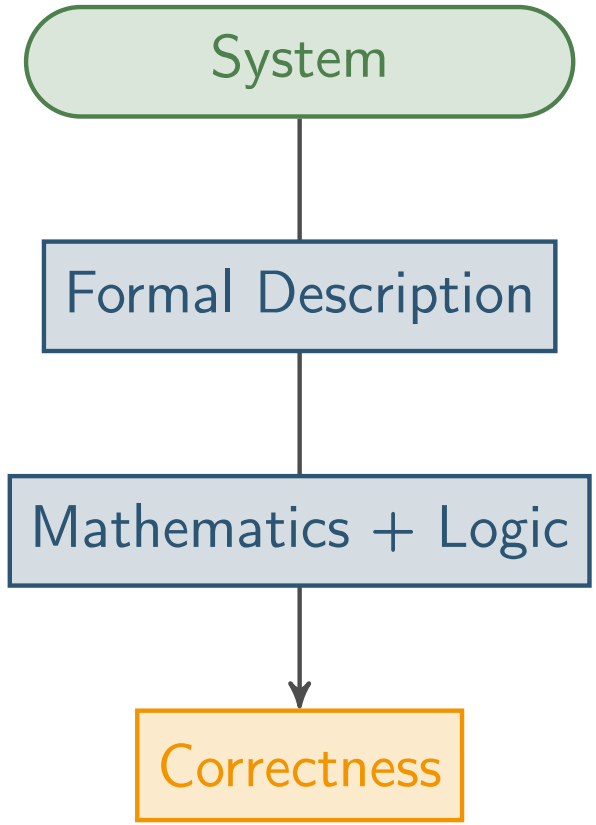

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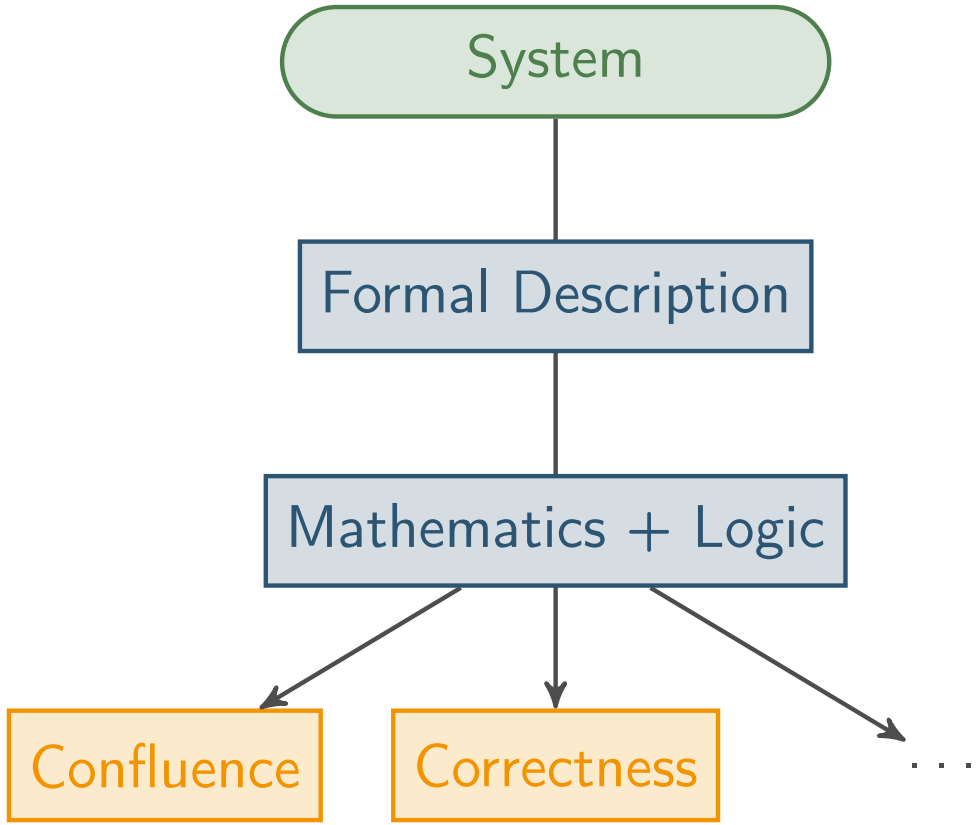
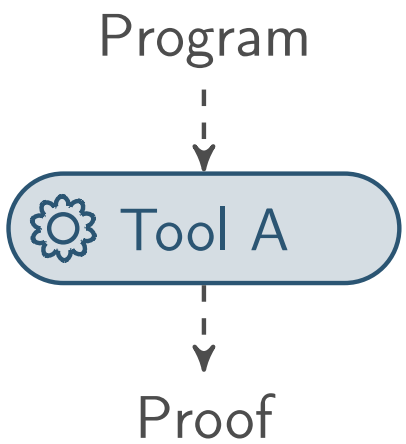


Automation



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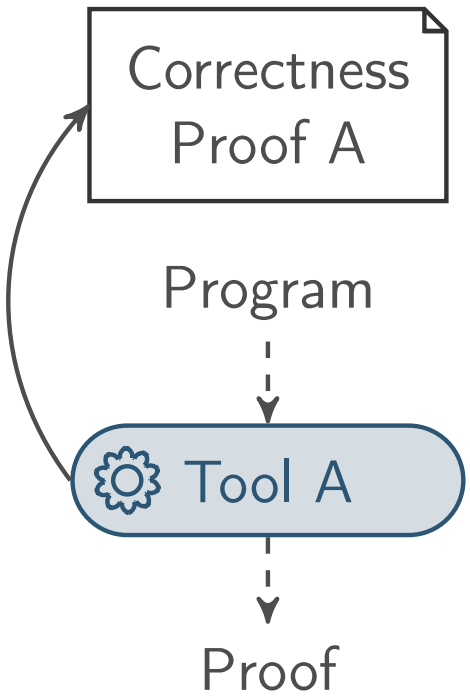
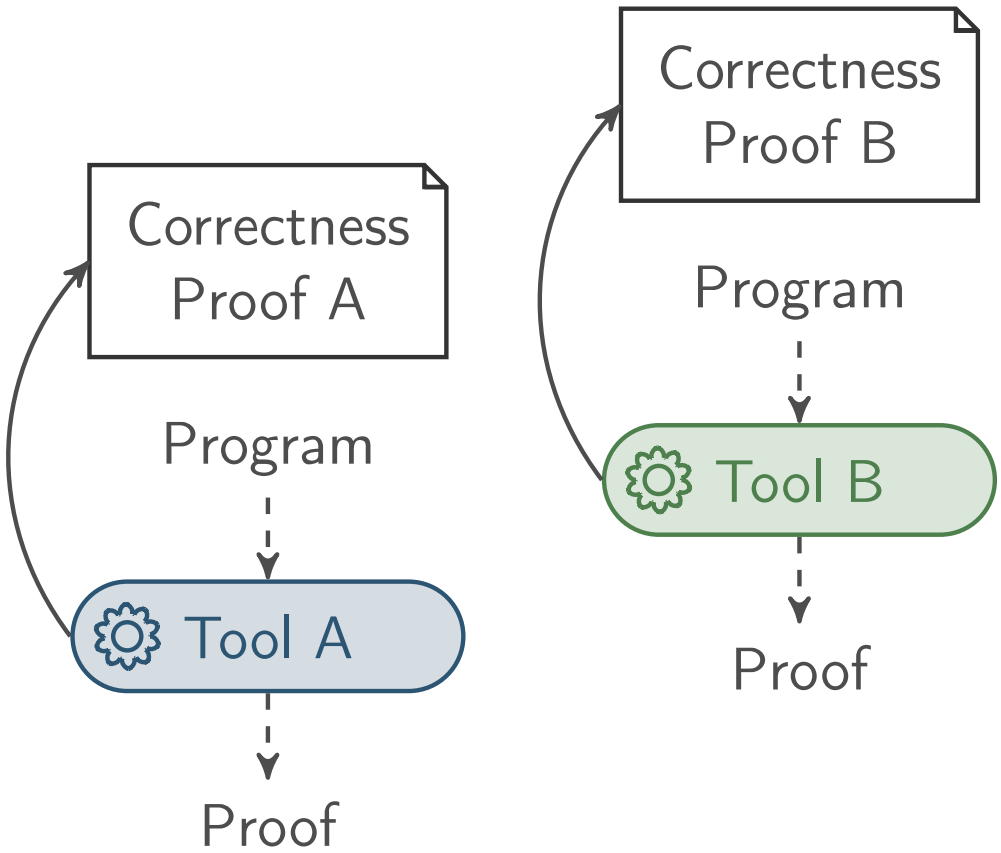
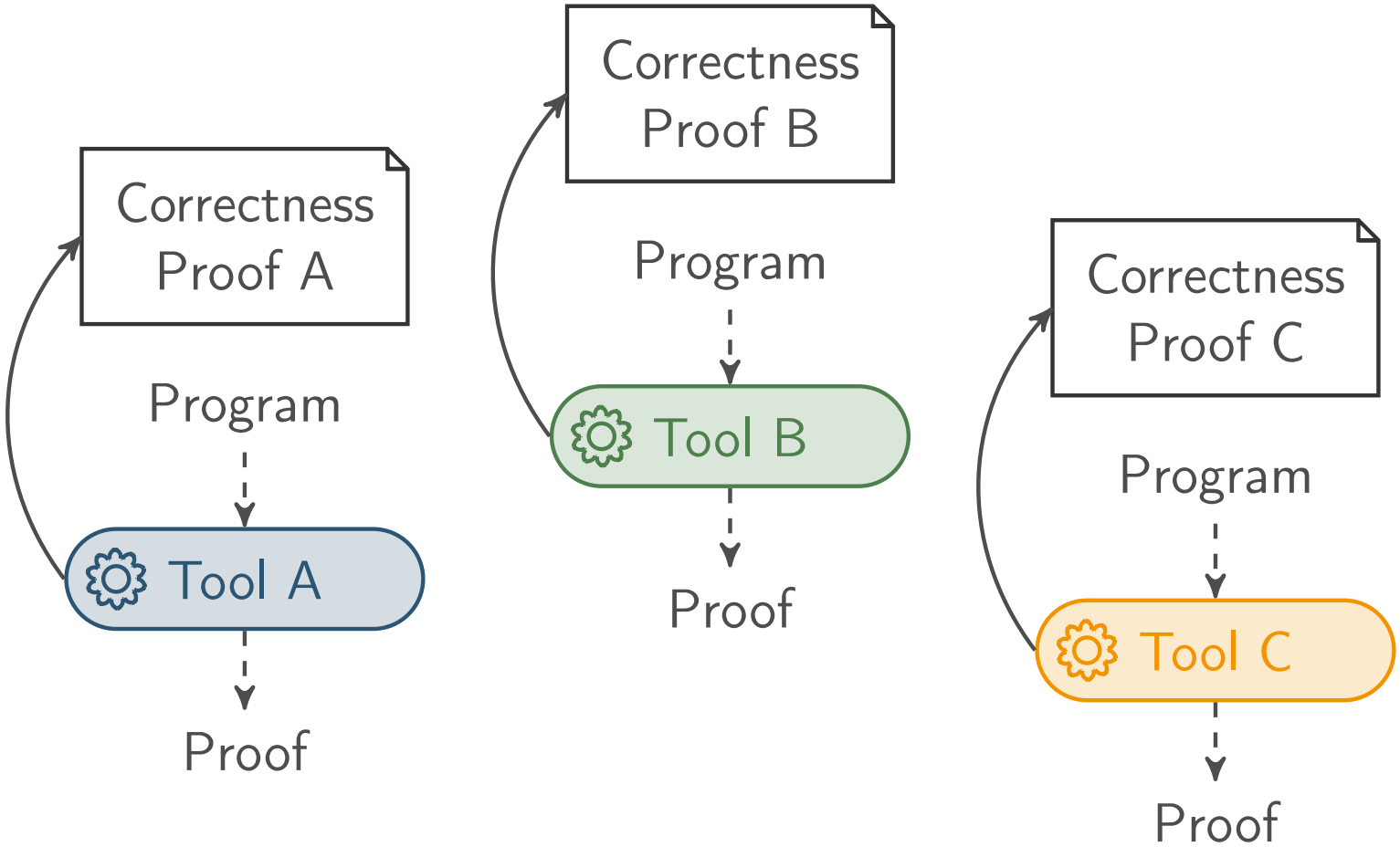


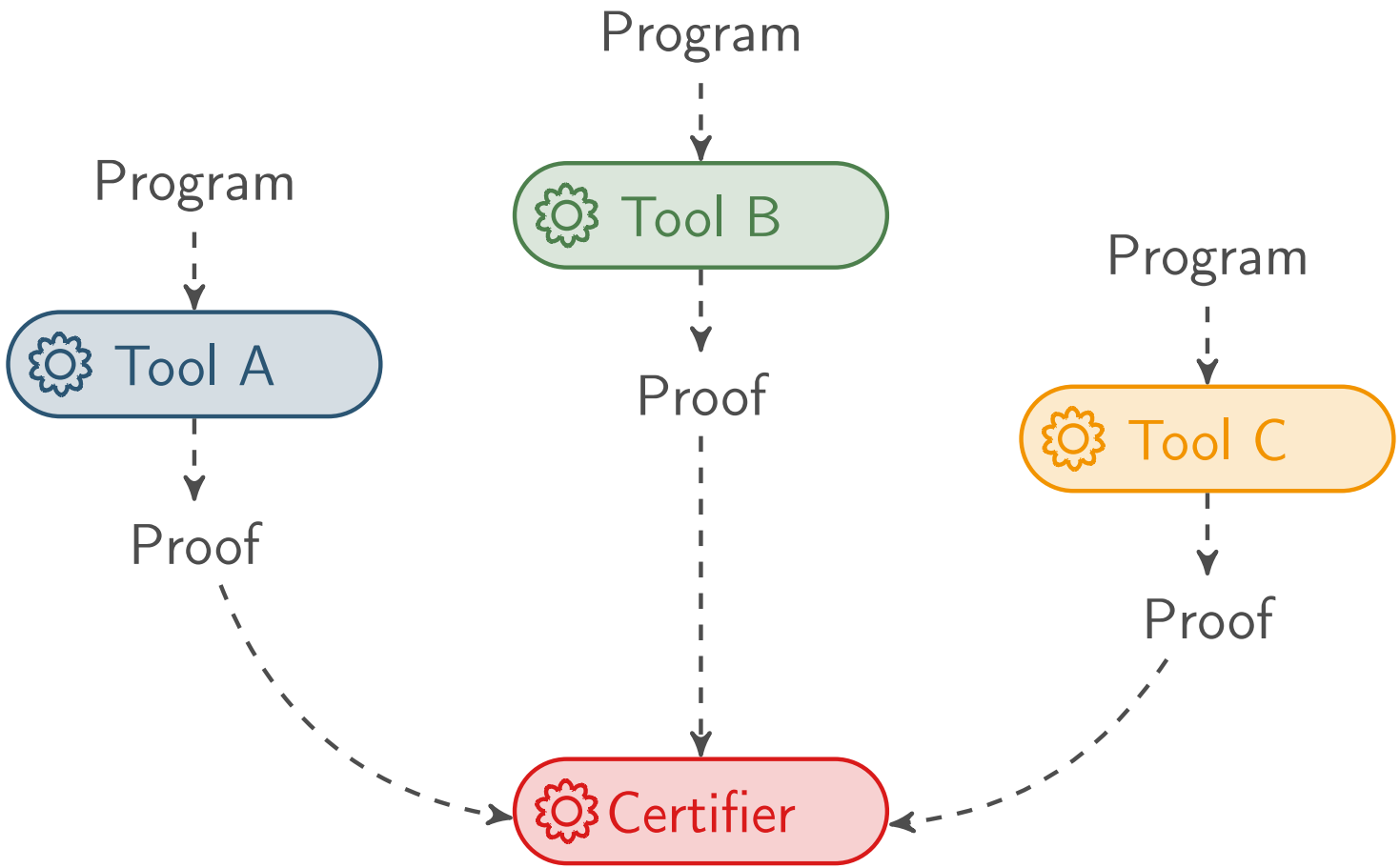
Automation


Correctness
Proof A
Program
Tool A
Proof
Correctness
Proof B
Program
Tool B
Proof

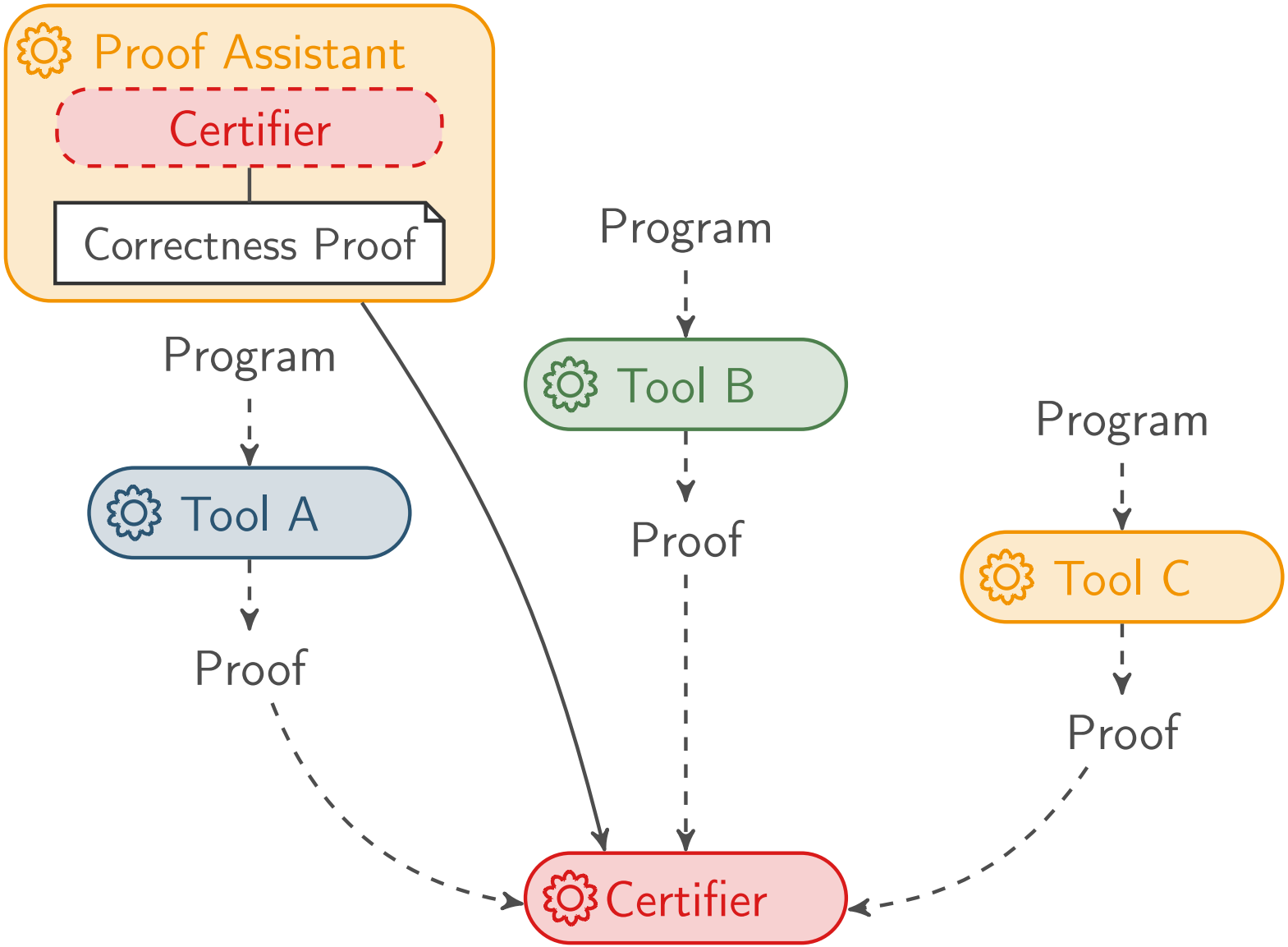
Automation


Correctness
Proof A
Program
Tool A
Proof
Correctness
Proof B
Program
Tool B
Proof
Correctness
Proof C
Program
Tool C
Proof

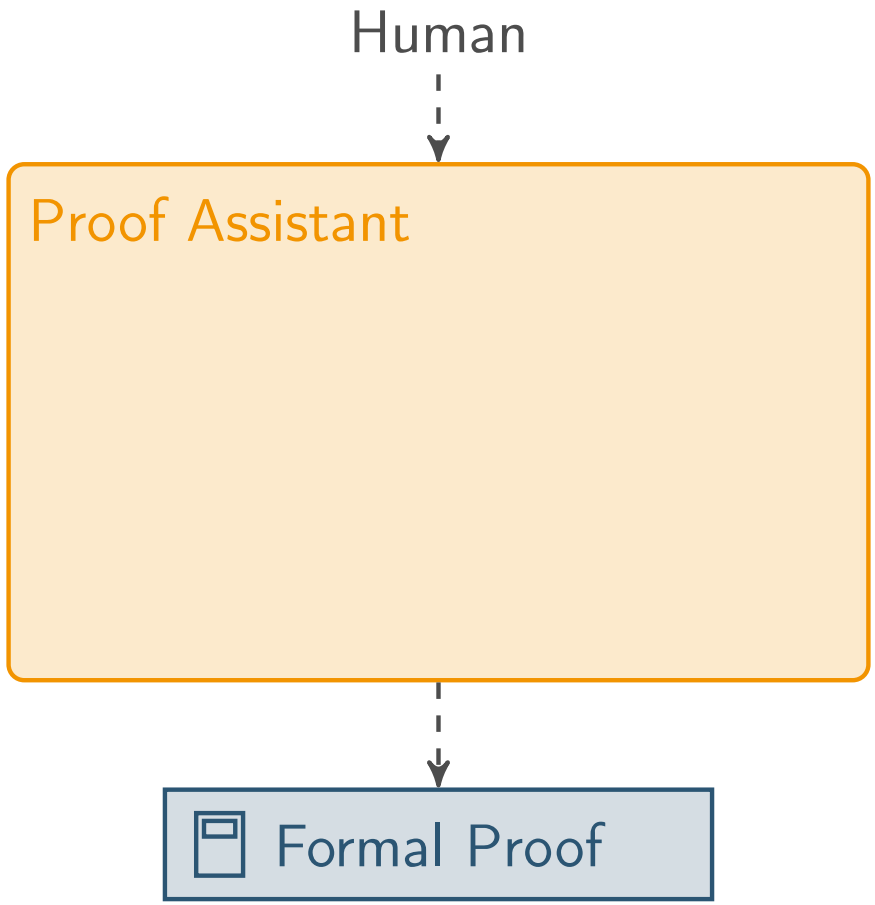
Automation


Program
Tool A
Proof
Program
Tool B
Proof
Program
Tool C
Proof
Certifier

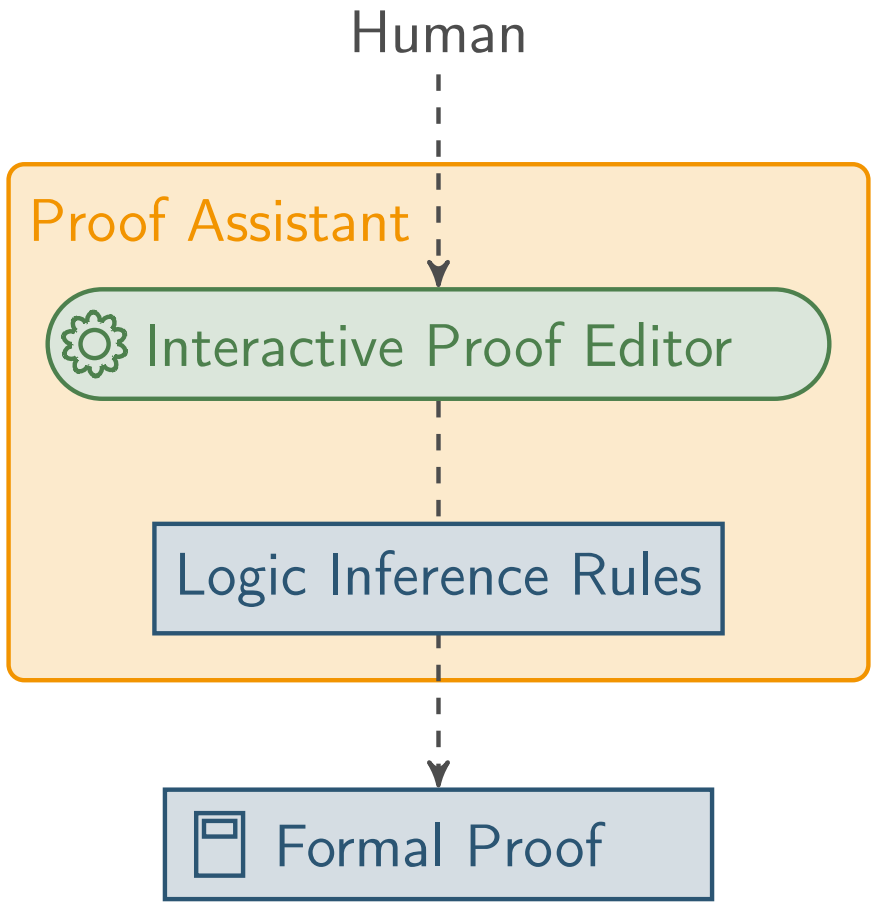
Automation


Proof Assistant
Certifier
Correctness Proof
Program
Tool A
Proof
Program
Tool B
Proof
Program
Tool C
Proof
Certifier

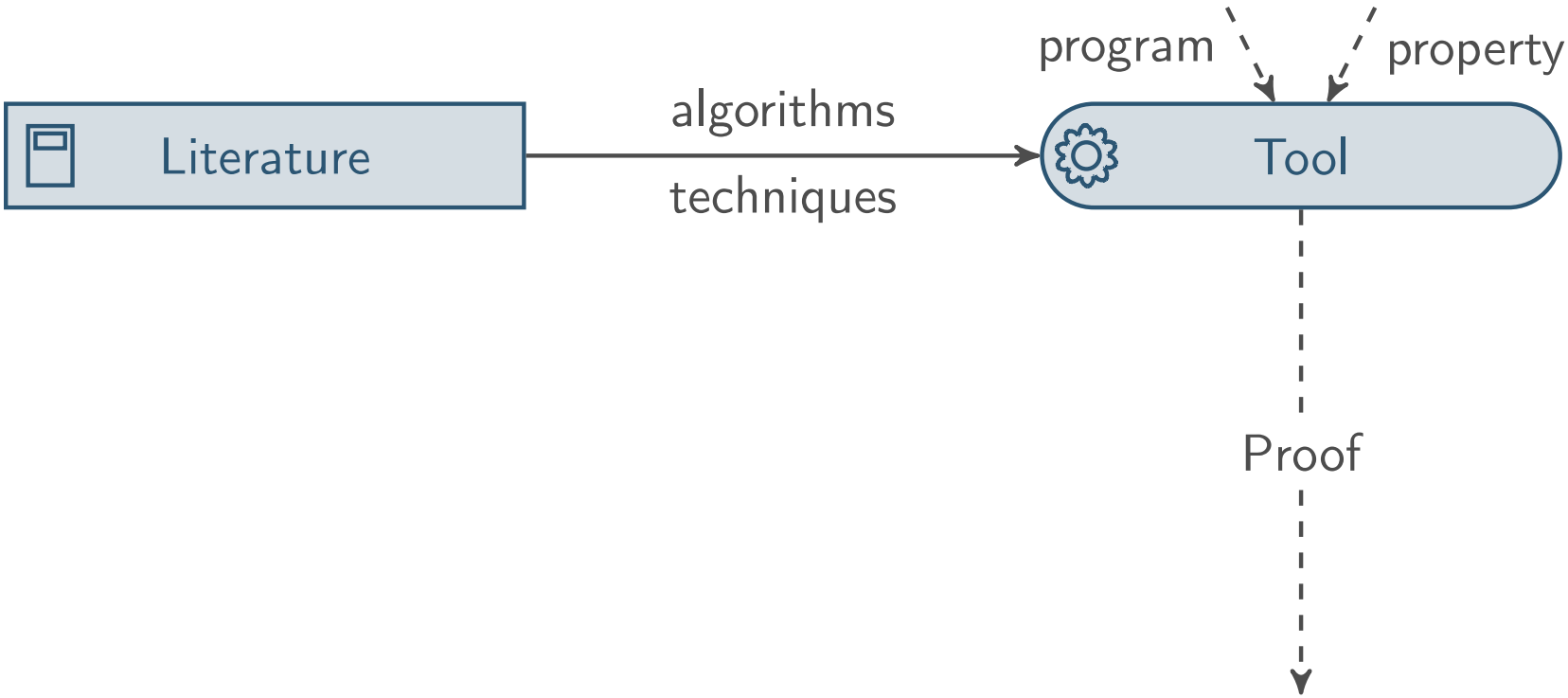
Proof Assistant



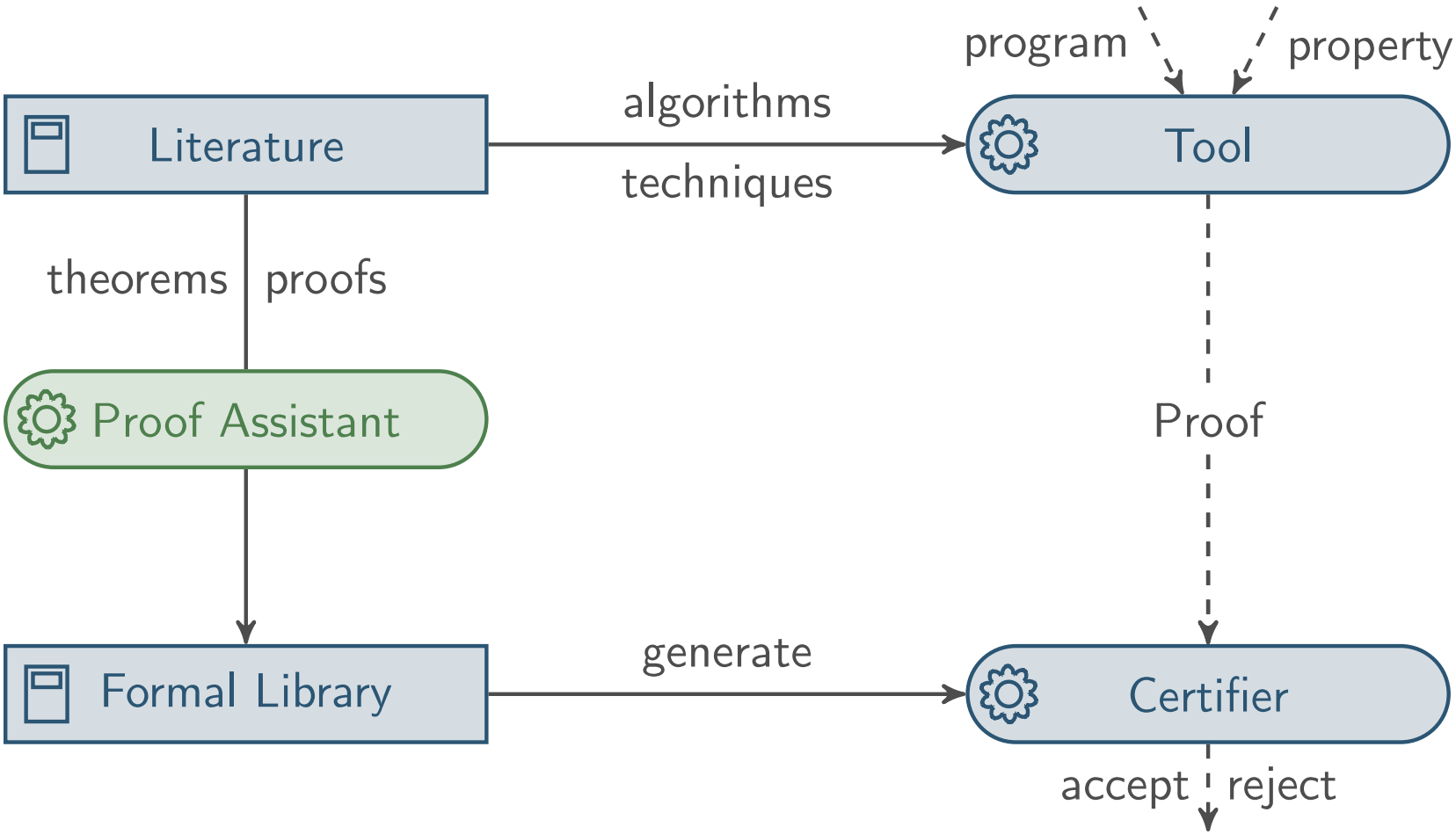
Proof Assistant



Putting it all together



Putting it all together



Conditional Term Rewriting

```
split x [] = ([], [])
split x (y : ys)
  | x ⩽ y = (xs, y : zs)
  | otherwise = (y : xs, zs)
  where (xs, zs) = split x ys

qsort [] = []
qsort (x : xs) = qsort ys ++ (x : qsort zs)
  where (ys, zs) = split x xs
```

Conditional Term Rewriting

$$
\begin{aligned}
&\mathbf{split}(\mathbf{x},\ [])\ &&\to ([],\ [])\\
&\mathbf{split}(\mathbf{x},\ \mathbf{y} : \mathbf{ys})\ &&\to (\mathbf{xs},\ \mathbf{y} : \mathbf{zs})\\
&&&\Leftarrow \mathbf{x} \leqslant \mathbf{y} \to \mathbf{true},\ \mathbf{split}(\mathbf{x},\ \mathbf{ys}) \to (\mathbf{xs},\ \mathbf{zs})\\
&\mathbf{split}(\mathbf{x},\ \mathbf{y} : \mathbf{ys})\ &&\to (\mathbf{y} : \mathbf{xs},\ \mathbf{zs})\\
&&&\Leftarrow \mathbf{x} \leqslant \mathbf{y} \to \mathbf{false},\ \mathbf{split}(\mathbf{x},\ \mathbf{ys}) \to (\mathbf{xs},\ \mathbf{zs})\\
\\
&\mathbf{qsort}([])\ &&\to []\\
&\mathbf{qsort}(\mathbf{x} : \mathbf{xs})\ &&\to \mathbf{qsort}(\mathbf{ys}) +\!\!+ (\mathbf{x} : \mathbf{qsort}(\mathbf{zs}))\\
&&&\Leftarrow \mathbf{split}(\mathbf{x},\ \mathbf{xs}) \to (\mathbf{ys},\ \mathbf{zs})
\end{aligned}
$$

Conditional Term Rewriting

$$
\begin{array}{lll}
\mathbf{split}(\mathbf{x},\ [\,]) & \to ([\,],\ [\,]) & \\
\mathbf{split}(\mathbf{x},\ \mathbf{y} : \mathbf{ys}) & \to (\mathbf{xs},\ \mathbf{y} : \mathbf{zs}) & \\
 & \Leftarrow \mathbf{x} \leqslant \mathbf{y} \to \mathbf{true},\ \mathbf{split}(\mathbf{x},\ \mathbf{ys}) \to (\mathbf{xs},\ \mathbf{zs}) & \\
\mathbf{split}(\mathbf{x},\ \mathbf{y} : \mathbf{ys}) & \to (\mathbf{y} : \mathbf{xs},\ \mathbf{zs}) & \\
 & \Leftarrow \mathbf{x} \leqslant \mathbf{y} \to \mathbf{false},\ \mathbf{split}(\mathbf{x},\ \mathbf{ys}) \to (\mathbf{xs},\ \mathbf{zs}) & \\
 & & \\
\mathbf{qsort}([\,]) & \to [\,] & \\
\mathbf{qsort}(\mathbf{x} : \mathbf{xs}) & \to \mathbf{qsort}(\mathbf{ys}) +\!\!+ (\mathbf{x} : \mathbf{qsort}(\mathbf{zs})) & \\
 & \Leftarrow \mathbf{split}(\mathbf{x},\ \mathbf{xs}) \to (\mathbf{ys},\ \mathbf{zs}) &
\end{array}
$$

$$\mathbf{qsort}\,(2{:}1{:}[\,]) \ \to$$

Conditional Term Rewriting

$$
\begin{aligned}
&\mathbf{split}(\mathbf{x},\ [])\ &&\to ([],\ [])\\
&\mathbf{split}(\mathbf{x},\ \mathbf{y} : \mathbf{ys})\ &&\to (\mathbf{xs},\ \mathbf{y} : \mathbf{zs})\\
&&&\Leftarrow \mathbf{x} \leqslant \mathbf{y} \to \mathbf{true},\ \mathbf{split}(\mathbf{x},\ \mathbf{ys}) \to (\mathbf{xs},\ \mathbf{zs})\\
&\mathbf{split}(\mathbf{x},\ \mathbf{y} : \mathbf{ys})\ &&\to (\mathbf{y} : \mathbf{xs},\ \mathbf{zs})\\
&&&\Leftarrow \mathbf{x} \leqslant \mathbf{y} \to \mathbf{false},\ \mathbf{split}(\mathbf{x},\ \mathbf{ys}) \to (\mathbf{xs},\ \mathbf{zs})\\
\\
&\mathbf{qsort}([])\ &&\to []\\
&\mathbf{qsort}(\mathbf{x} : \mathbf{xs})\ &&\to \mathbf{qsort}(\mathbf{ys}) +\!\!+ (\mathbf{x} : \mathbf{qsort}(\mathbf{zs}))\\
&&&\Leftarrow \mathbf{split}(\mathbf{x},\ \mathbf{xs}) \to (\mathbf{ys},\ \mathbf{zs})
\end{aligned}
$$

$$
\begin{aligned}
&\mathbf{qsort}\ (2{:}1{:}[])\ \to\\
&\qquad\qquad \mathbf{split}\ (2,\ \ 1{:}[])\ \to
\end{aligned}
$$

Conditional Term Rewriting

$$
\begin{aligned}
&\mathbf{split}(\mathbf{x},\ [])\ &&\to ([],\ [])\\
&\mathbf{split}(\mathbf{x},\ \mathbf{y} : \mathbf{ys})\ &&\to (\mathbf{xs},\ \mathbf{y} : \mathbf{zs})\\
&&&\Leftarrow \mathbf{x} \leqslant \mathbf{y} \to \mathbf{true},\ \mathbf{split}(\mathbf{x},\ \mathbf{ys}) \to (\mathbf{xs},\ \mathbf{zs})\\
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&&&\Leftarrow \mathbf{x} \leqslant \mathbf{y} \to \mathbf{false},\ \mathbf{split}(\mathbf{x},\ \mathbf{ys}) \to (\mathbf{xs},\ \mathbf{zs})
\end{aligned}
$$

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\begin{aligned}
&\mathbf{qsort}([])\ &&\to []\\
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&&&\Leftarrow \mathbf{split}(\mathbf{x},\ \mathbf{xs}) \to (\mathbf{ys},\ \mathbf{zs})
\end{aligned}
$$

$$
\begin{aligned}
&\mathbf{qsort}\,(2{:}1{:}[])\ \to\\
&\qquad\mathbf{split}\,(2,\ \ 1{:}[])\ \to\\
&\qquad\qquad 2 \leqslant 1 \to \mathbf{false},\\
&\qquad\qquad \mathbf{split}\,(2,\ \ [])\ \to ([],\ [])
\end{aligned}
$$

Conditional Term Rewriting

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\begin{aligned}
&\mathbf{split}(\mathbf{x},\ [])\ &&\to ([],\ [])\\
&\mathbf{split}(\mathbf{x},\ \mathbf{y} : \mathbf{ys})\ &&\to (\mathbf{xs},\ \mathbf{y} : \mathbf{zs})\\
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&\mathbf{split}(\mathbf{x},\ \mathbf{y} : \mathbf{ys})\ &&\to (\mathbf{y} : \mathbf{xs},\ \mathbf{zs})\\
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\end{aligned}
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&&&\Leftarrow \mathbf{split}(\mathbf{x},\ \mathbf{xs}) \to (\mathbf{ys},\ \mathbf{zs})
\end{aligned}
$$

$$
\begin{aligned}
&\mathbf{qsort}\,(2{:}1{:}[])\ \to\\
&\qquad\qquad \mathbf{split}\,(2,\ \ 1{:}[])\ \to (1{:}[],\ [])
\end{aligned}
$$

Conditional Term Rewriting

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\begin{aligned}
&\mathbf{split}(\mathbf{x},\ [])\ &&\to ([],\ [])\\
&\mathbf{split}(\mathbf{x},\ \mathbf{y} : \mathbf{ys})\ &&\to (\mathbf{xs},\ \mathbf{y} : \mathbf{zs})\\
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&\mathbf{split}(\mathbf{x},\ \mathbf{y} : \mathbf{ys})\ &&\to (\mathbf{y} : \mathbf{xs},\ \mathbf{zs})\\
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&&&\Leftarrow \mathbf{split}(\mathbf{x},\ \mathbf{xs}) \to (\mathbf{ys},\ \mathbf{zs})
\end{aligned}
$$

$$
\mathbf{qsort}\,(2{:}1{:}[])\ \to \mathbf{qsort}(1{:}[]) +\!\!+ (2{:}\mathbf{qsort}([]))
$$

Conditional Term Rewriting

$$
\begin{aligned}
&\mathbf{split}(\mathbf{x},\ [])\ &&\to ([],\ [])\\
&\mathbf{split}(\mathbf{x},\ \mathbf{y} : \mathbf{ys})\ &&\to (\mathbf{xs},\ \mathbf{y} : \mathbf{zs})\\
&&&\Leftarrow \mathbf{x} \leqslant \mathbf{y} \to \mathbf{true},\ \mathbf{split}(\mathbf{x},\ \mathbf{ys}) \to (\mathbf{xs},\ \mathbf{zs})\\
&\mathbf{split}(\mathbf{x},\ \mathbf{y} : \mathbf{ys})\ &&\to (\mathbf{y} : \mathbf{xs},\ \mathbf{zs})\\
&&&\Leftarrow \mathbf{x} \leqslant \mathbf{y} \to \mathbf{false},\ \mathbf{split}(\mathbf{x},\ \mathbf{ys}) \to (\mathbf{xs},\ \mathbf{zs})\\
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&\mathbf{qsort}([])\ &&\to []\\
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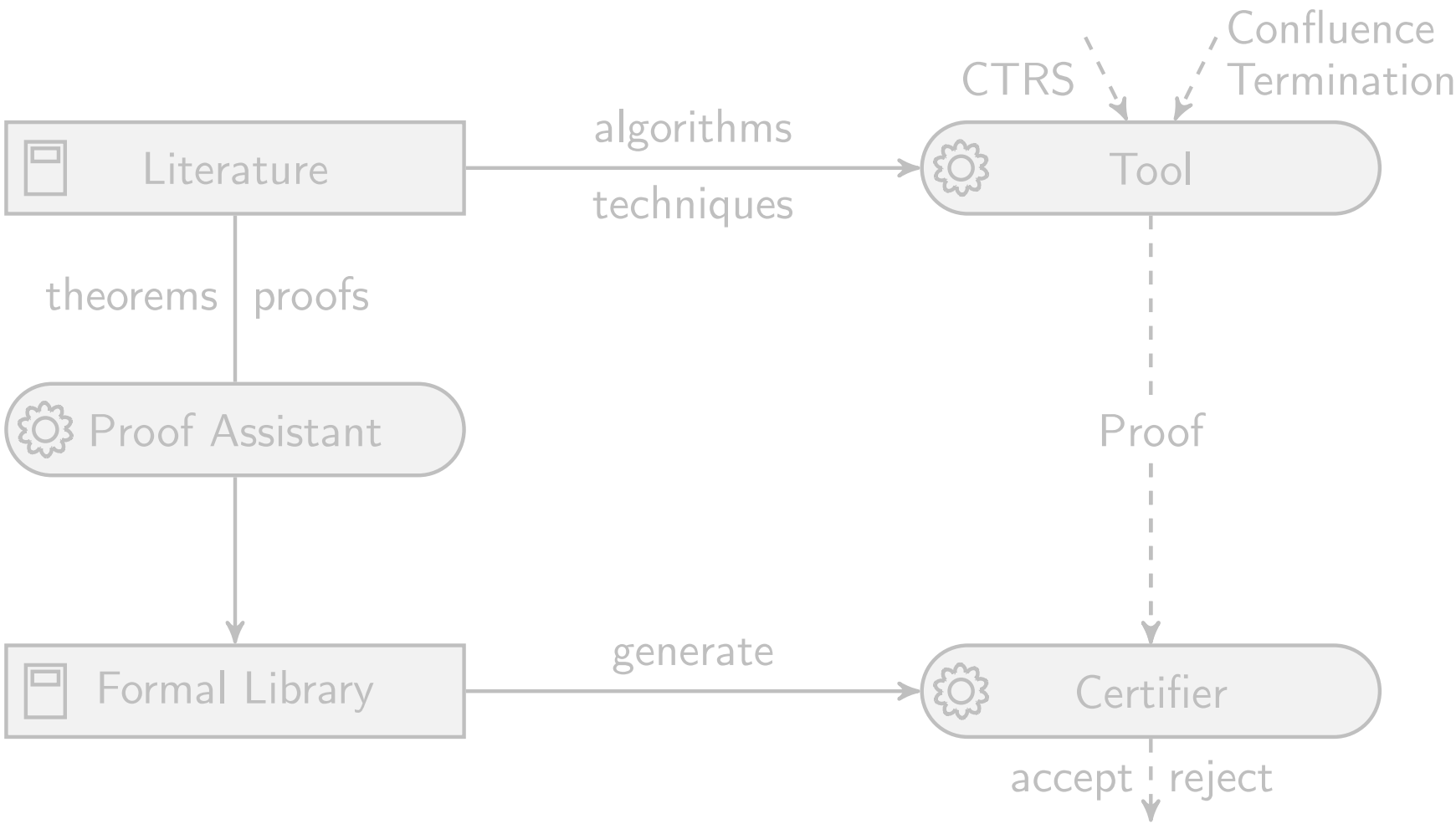
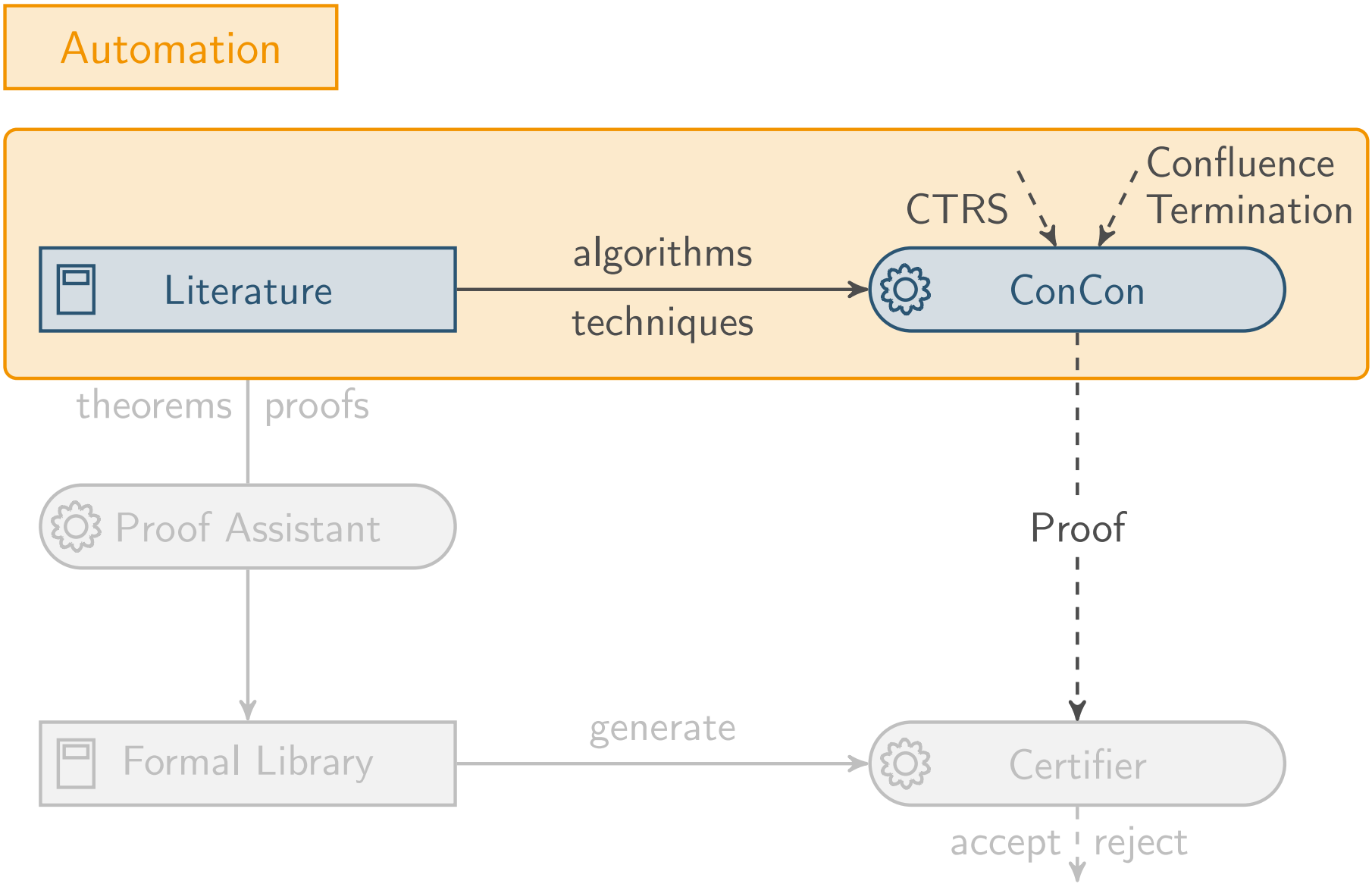
&&&\Leftarrow \mathbf{split}(\mathbf{x},\ \mathbf{xs}) \to (\mathbf{ys},\ \mathbf{zs})
\end{aligned}
$$

$$
\mathbf{qsort}\,(2{:}1{:}[]) \ \to \mathbf{qsort}(1{:}[]) +\!\!+ (2{:}\mathbf{qsort}([])) \to \cdots \to 1{:}2{:}[]
$$

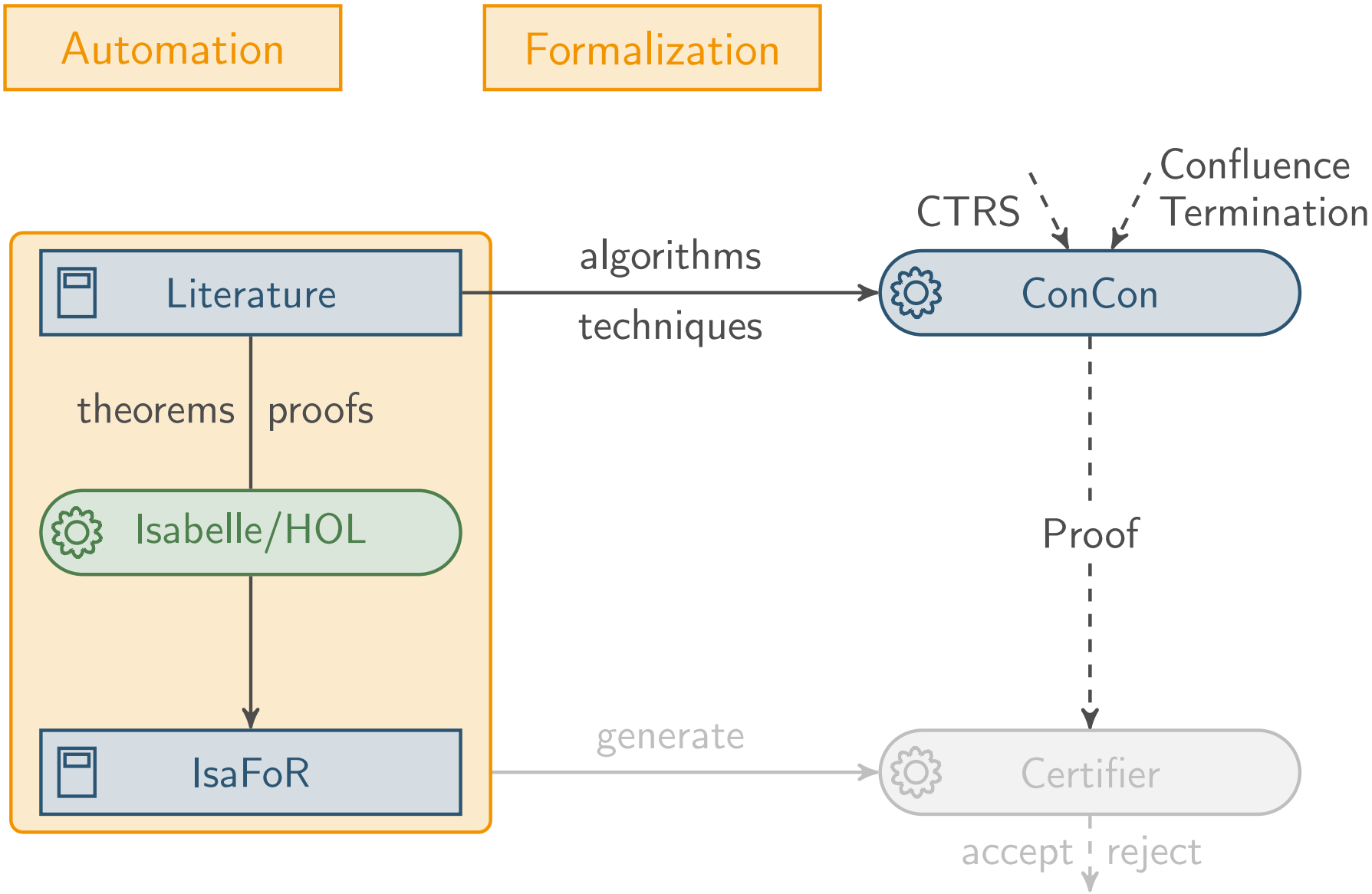
Our Contribution - Overview



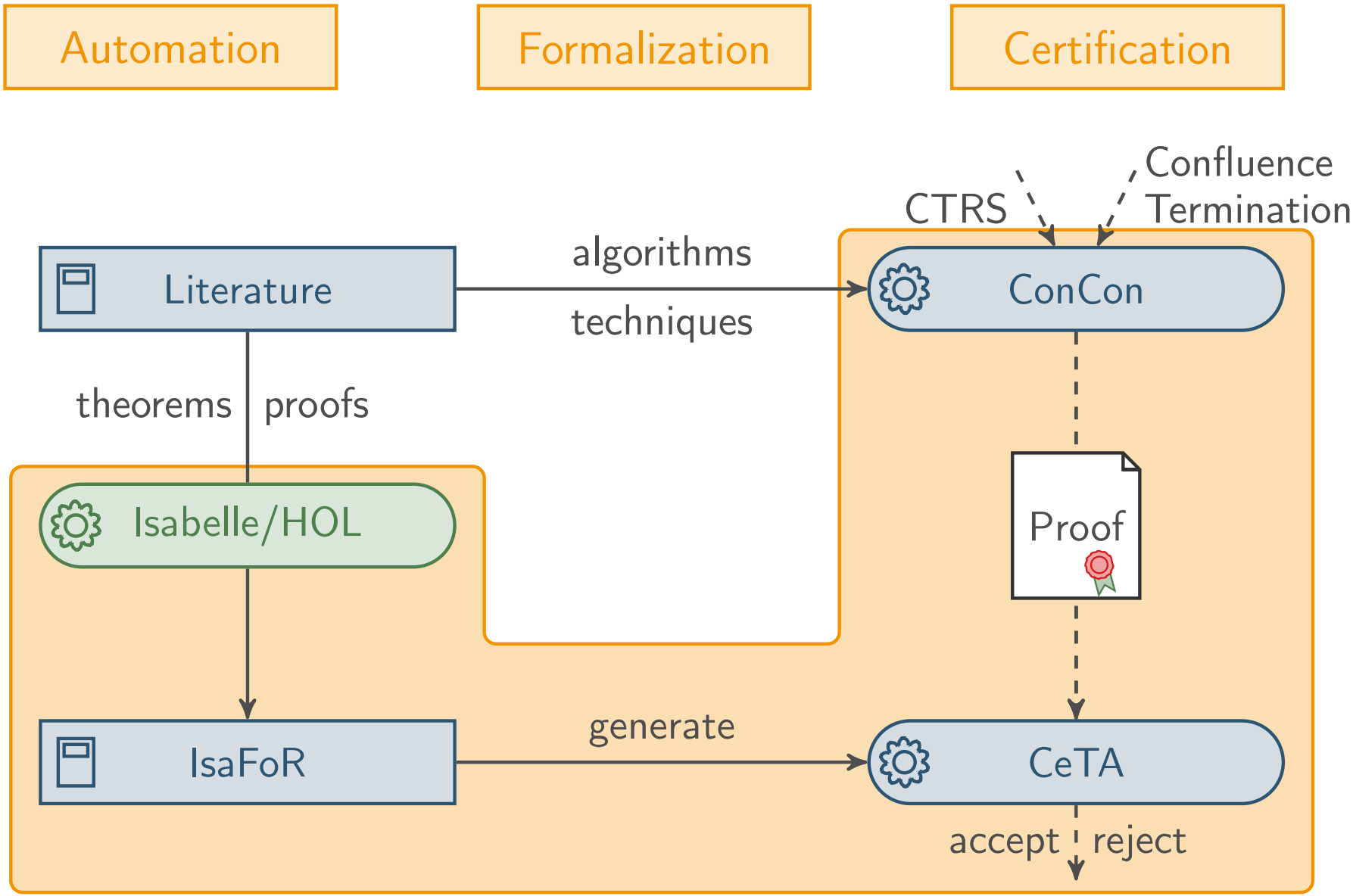
Our Contribution - Overview



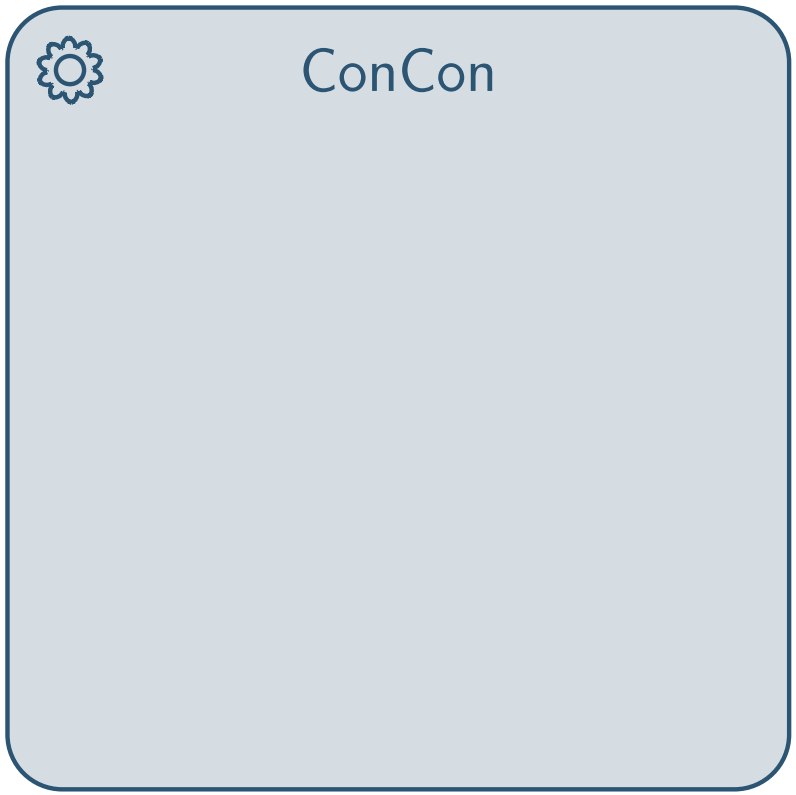
Our Contribution - Overview



Our Contribution - Overview



Our Contribution I - Automation



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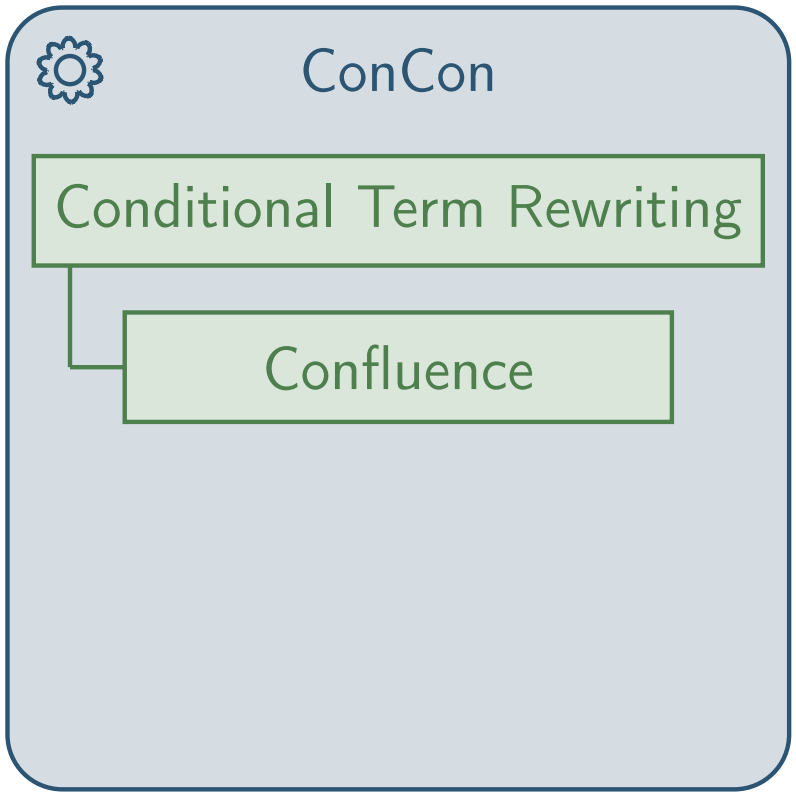

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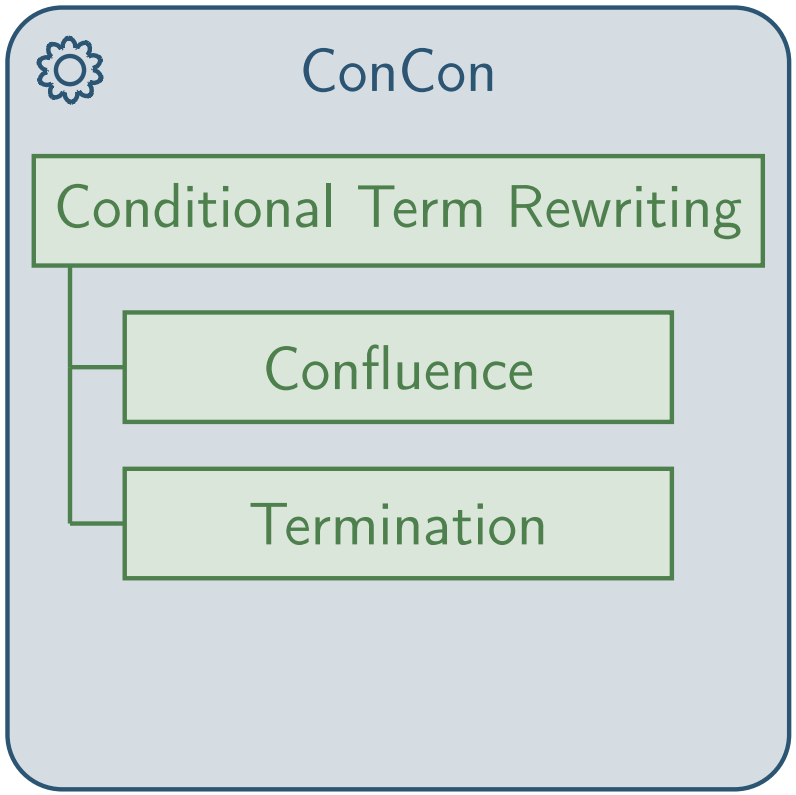

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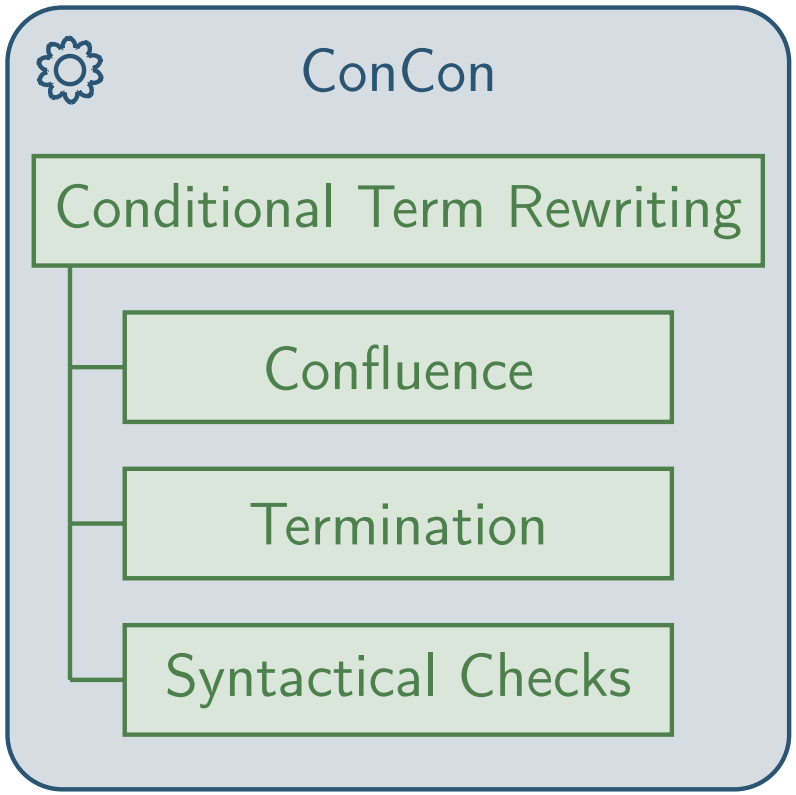

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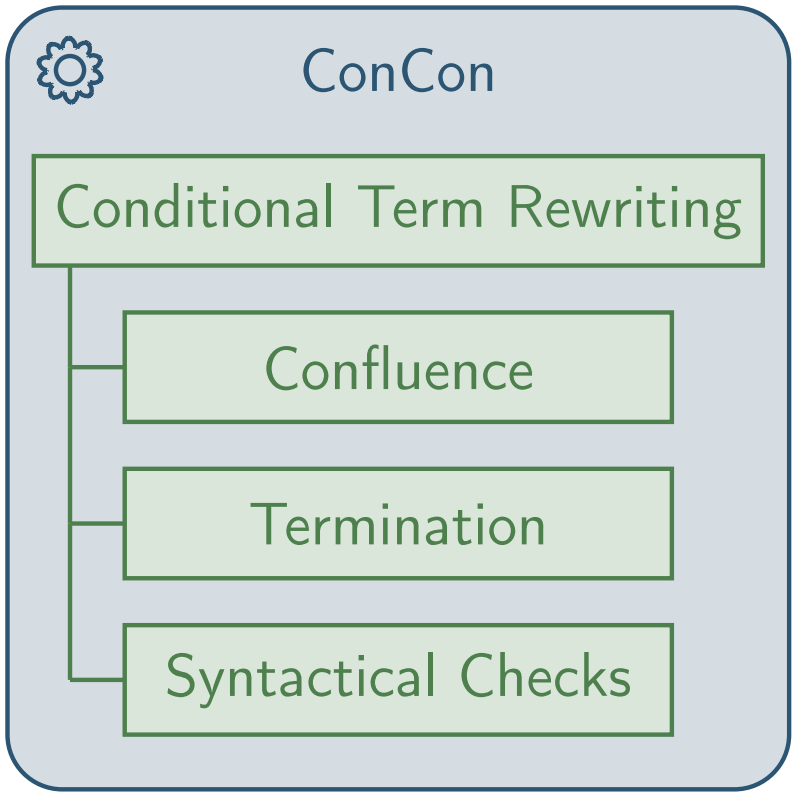

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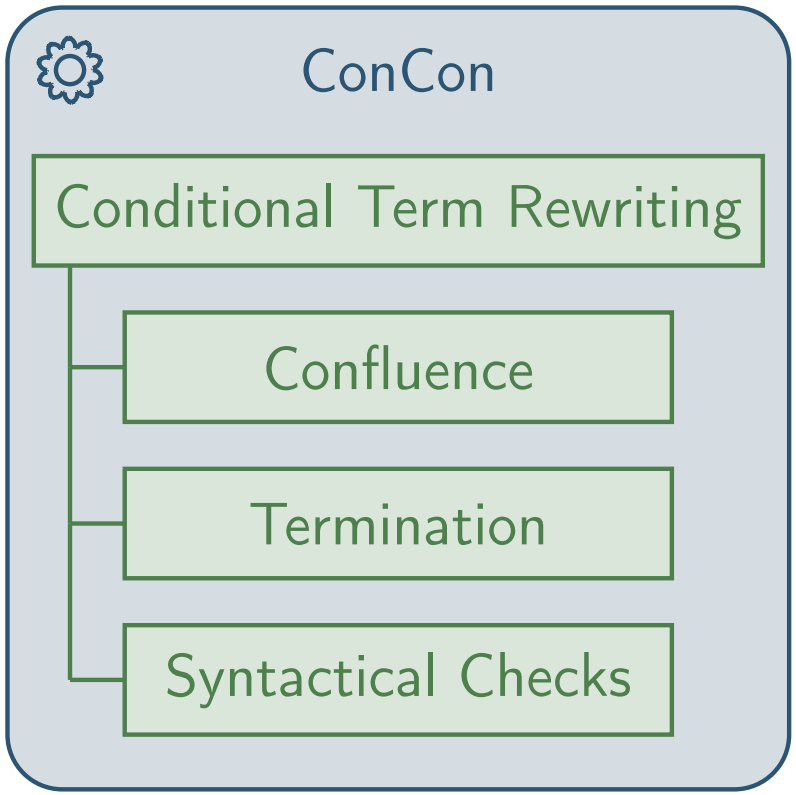
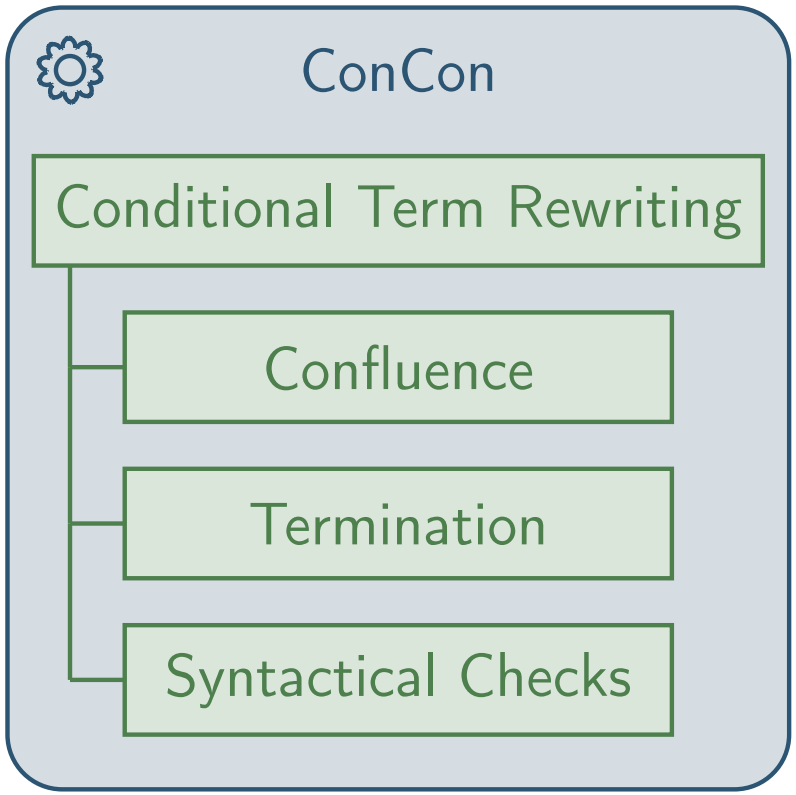


Scala

LGPL
Free Software
Free as in Freedom

Our Contribution I - Automation

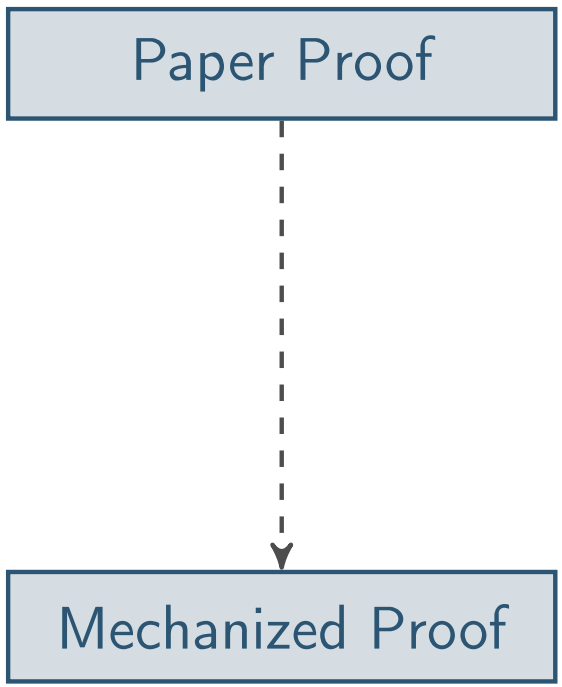


Our Contribution II - Formalization

Our Contribution II - Formalization

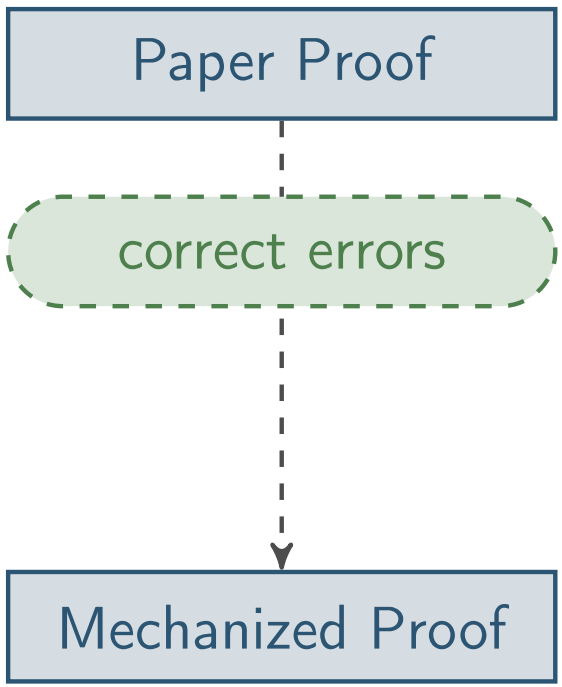
Our Contribution II - Formalization

Challenges



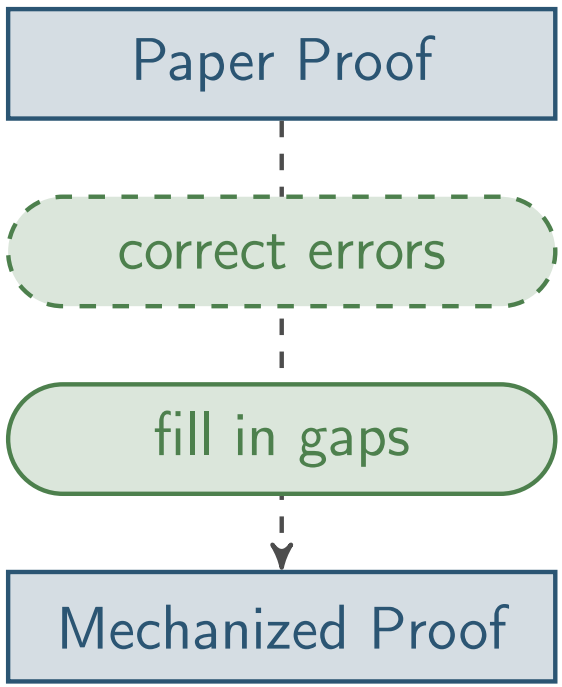
Our Contribution II - Formalization

Challenges



Our Contribution II - Formalization

Challenges



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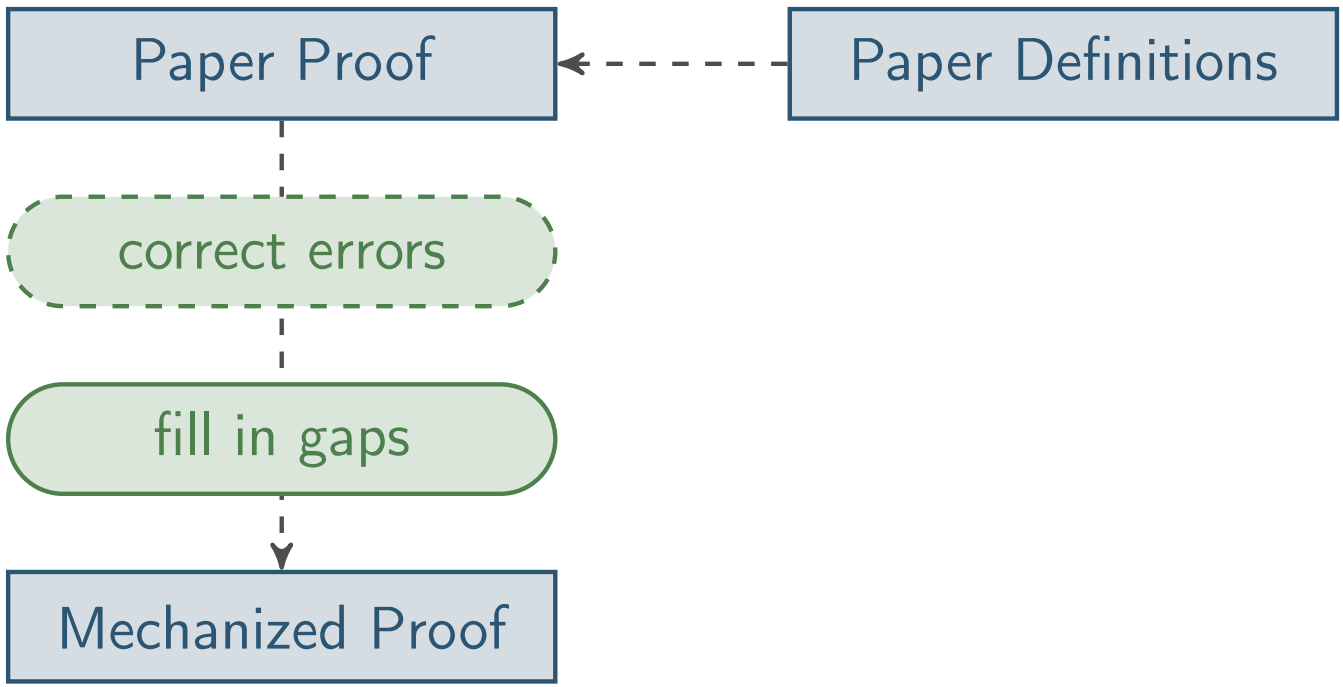
Challenges



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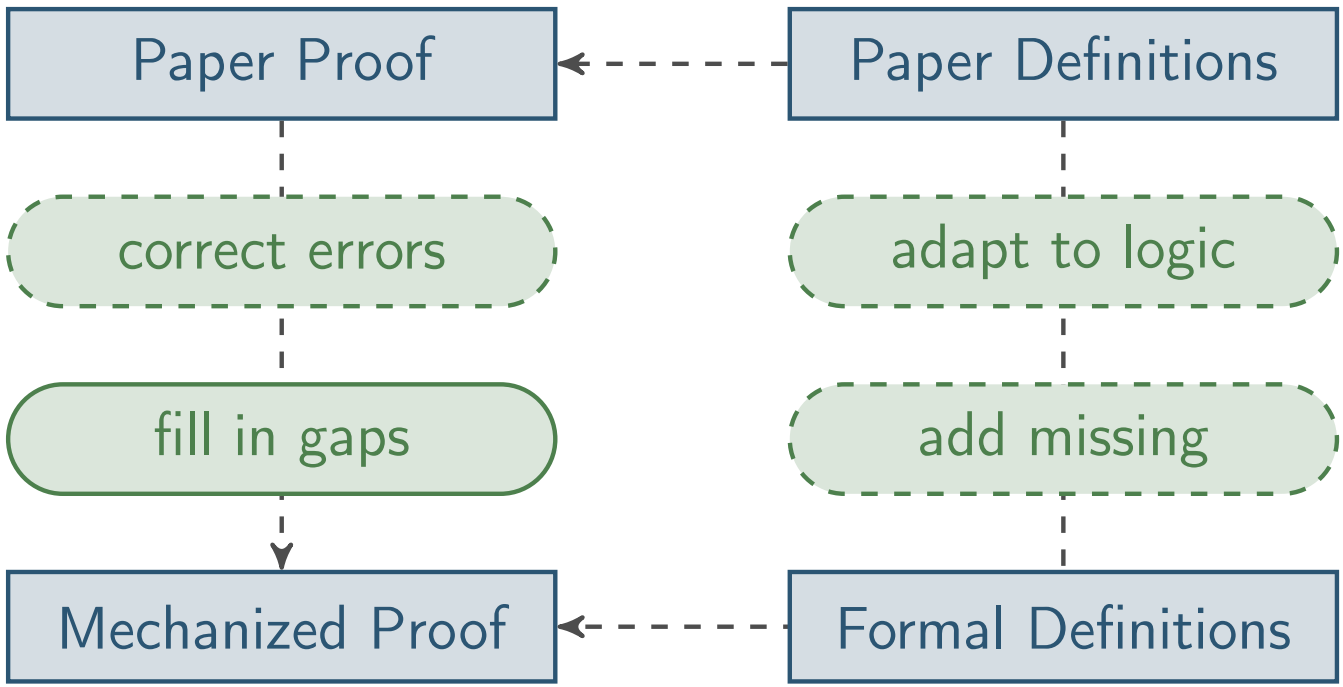
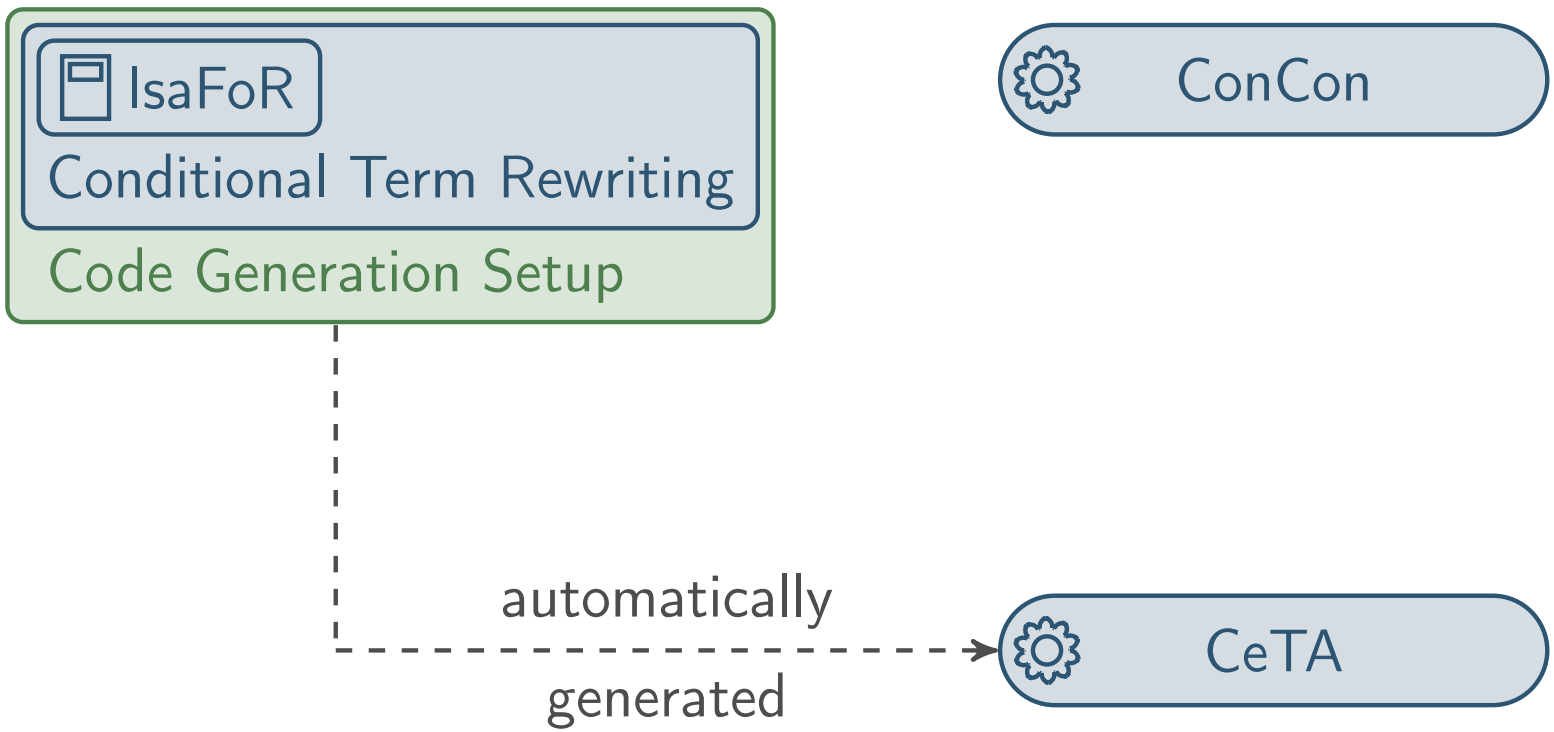
Challenges

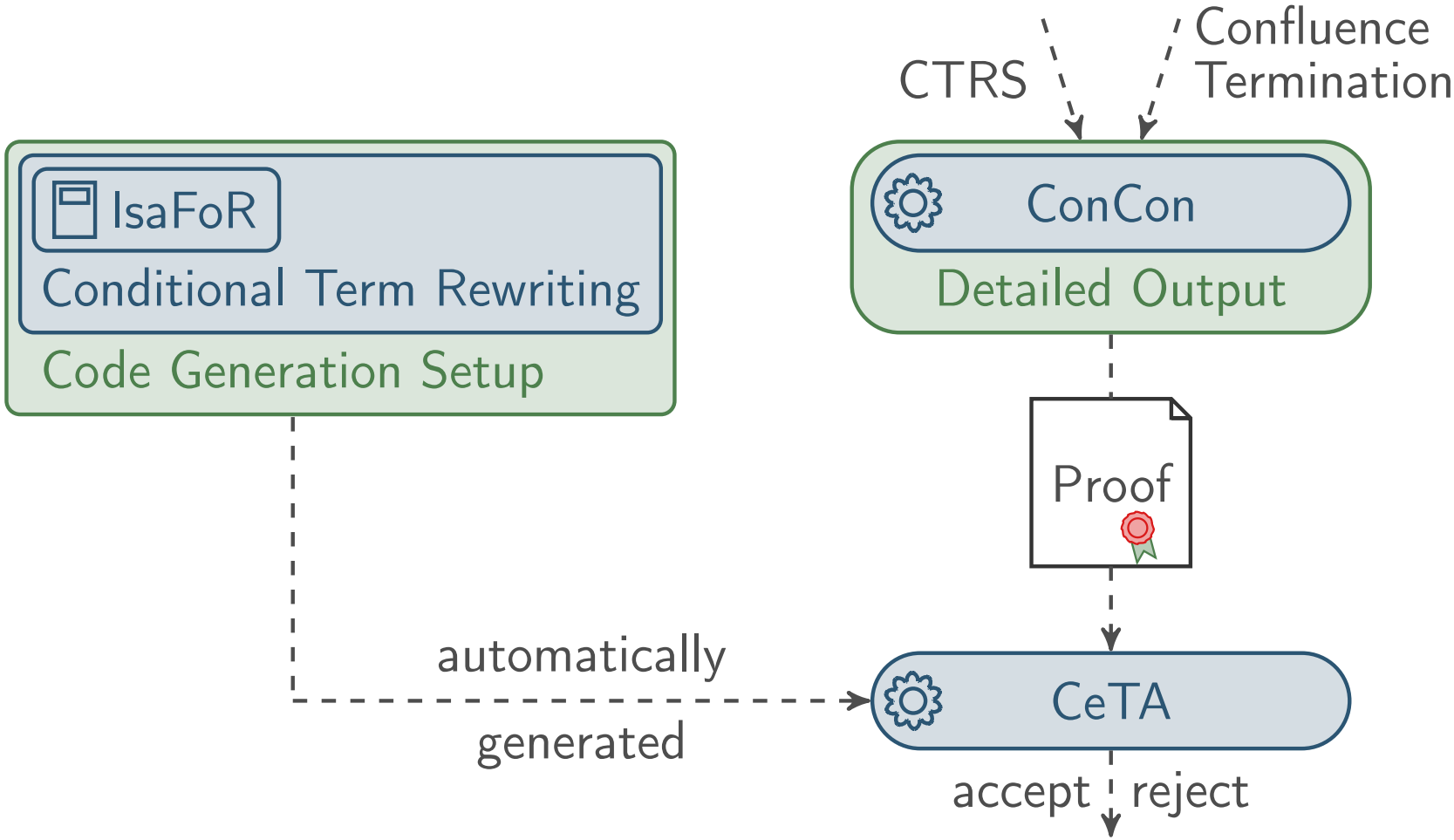


Our Contribution III - Certification

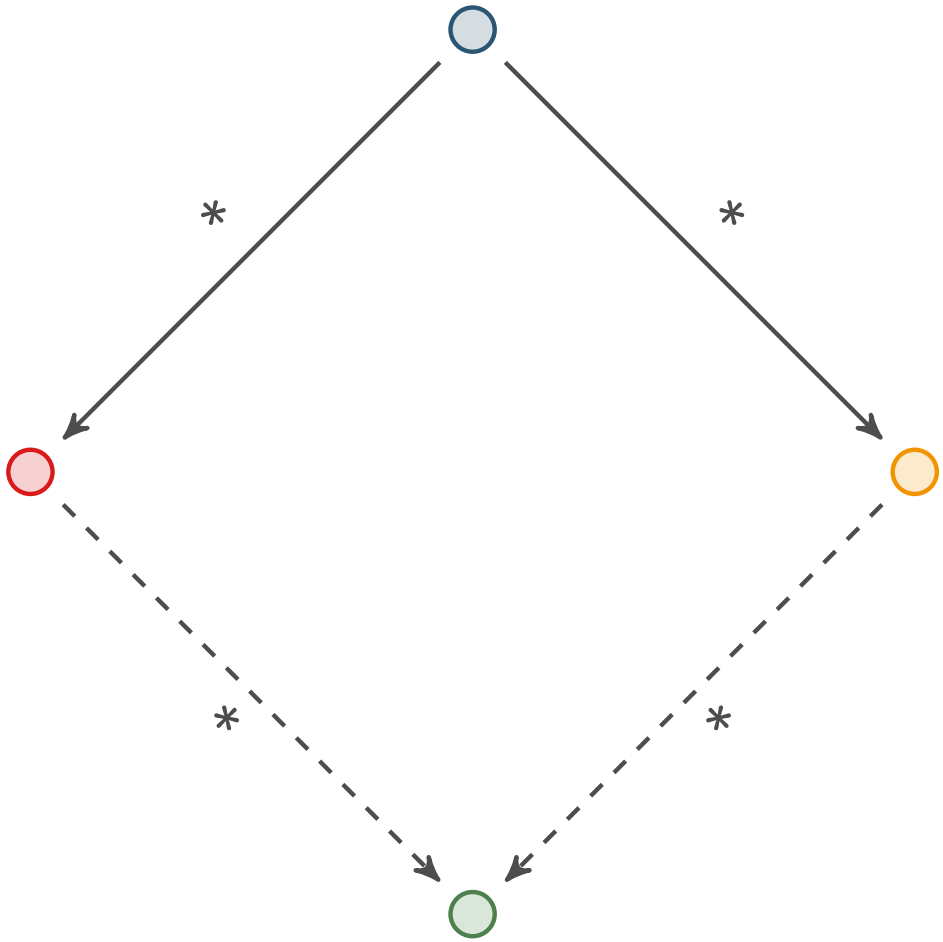
Our Contribution III - Certification



Our Contribution III - Certification



Analyzing (Non-)Confluence



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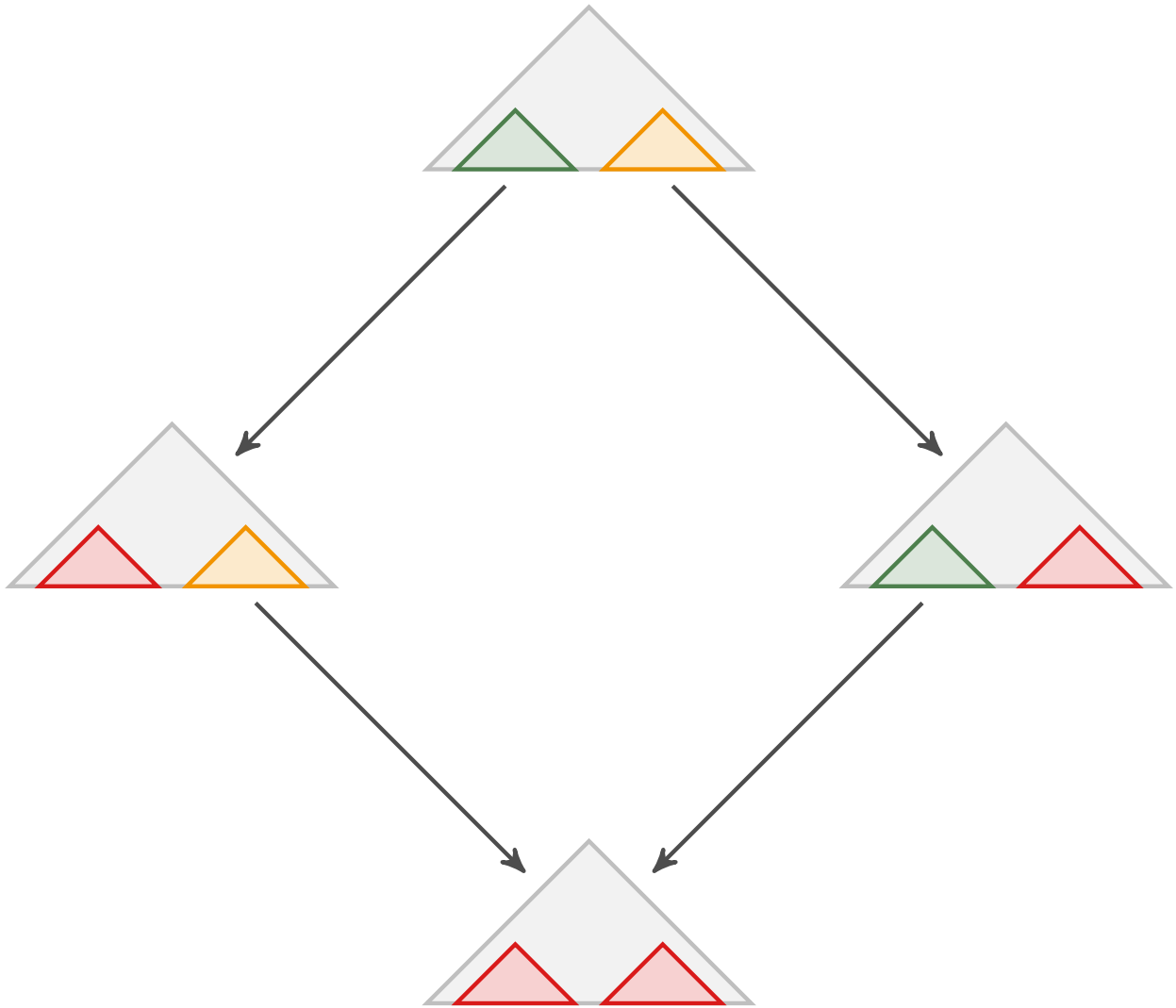

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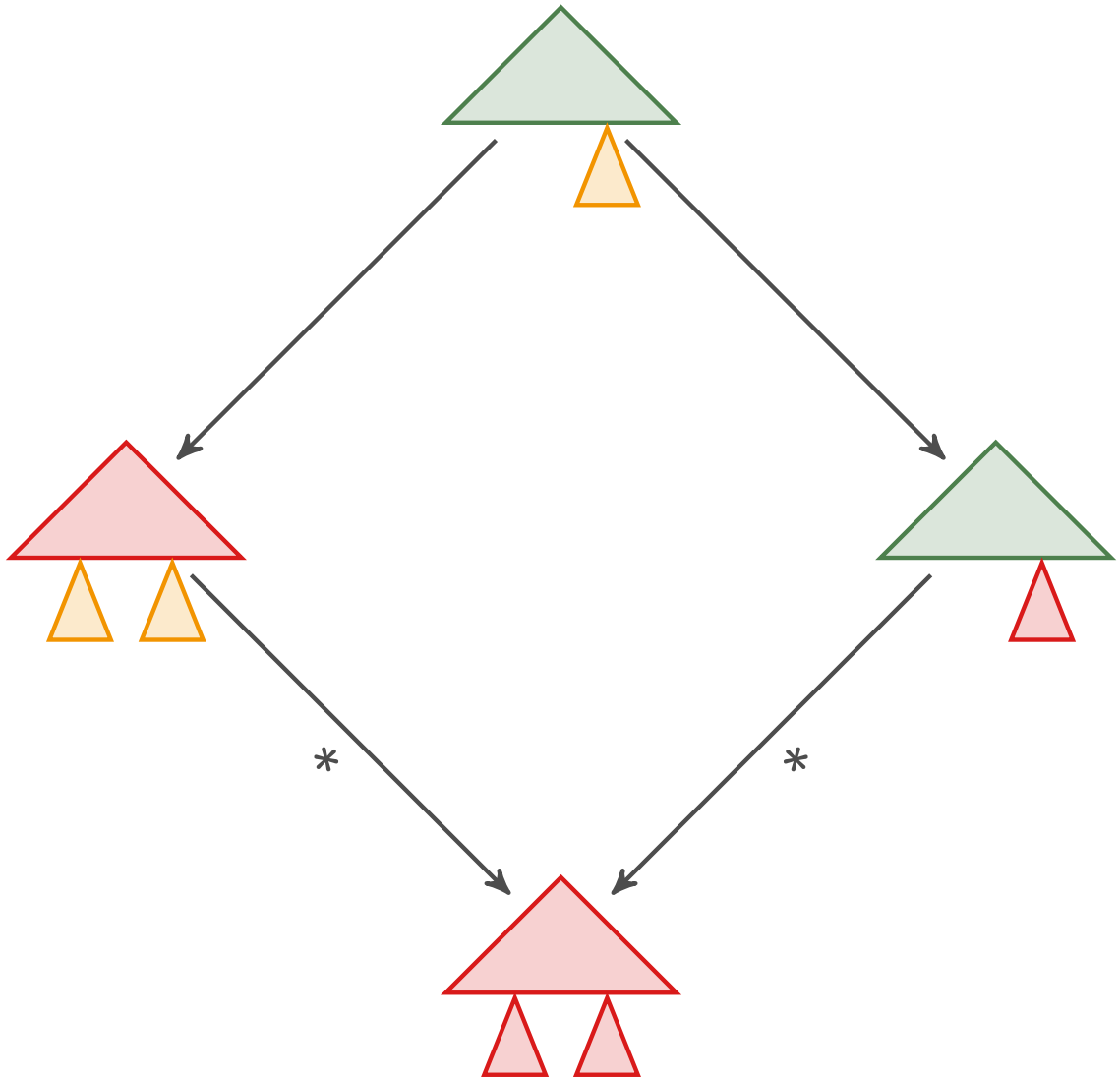

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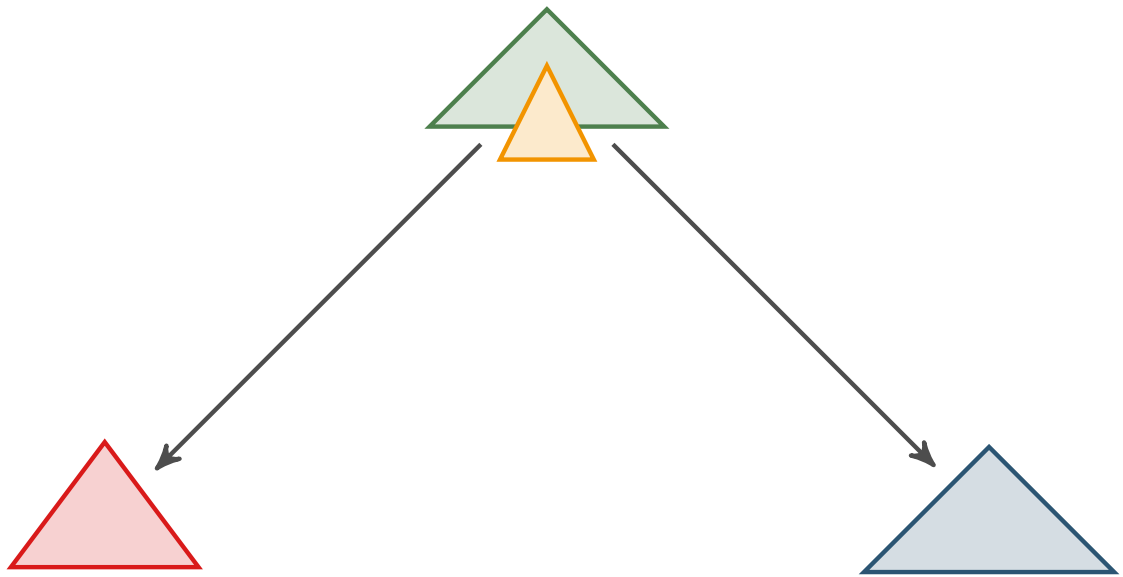


Confluence Methods

Theorem I

A DTRS is confluent if its unraveling is left-linear and confluent.

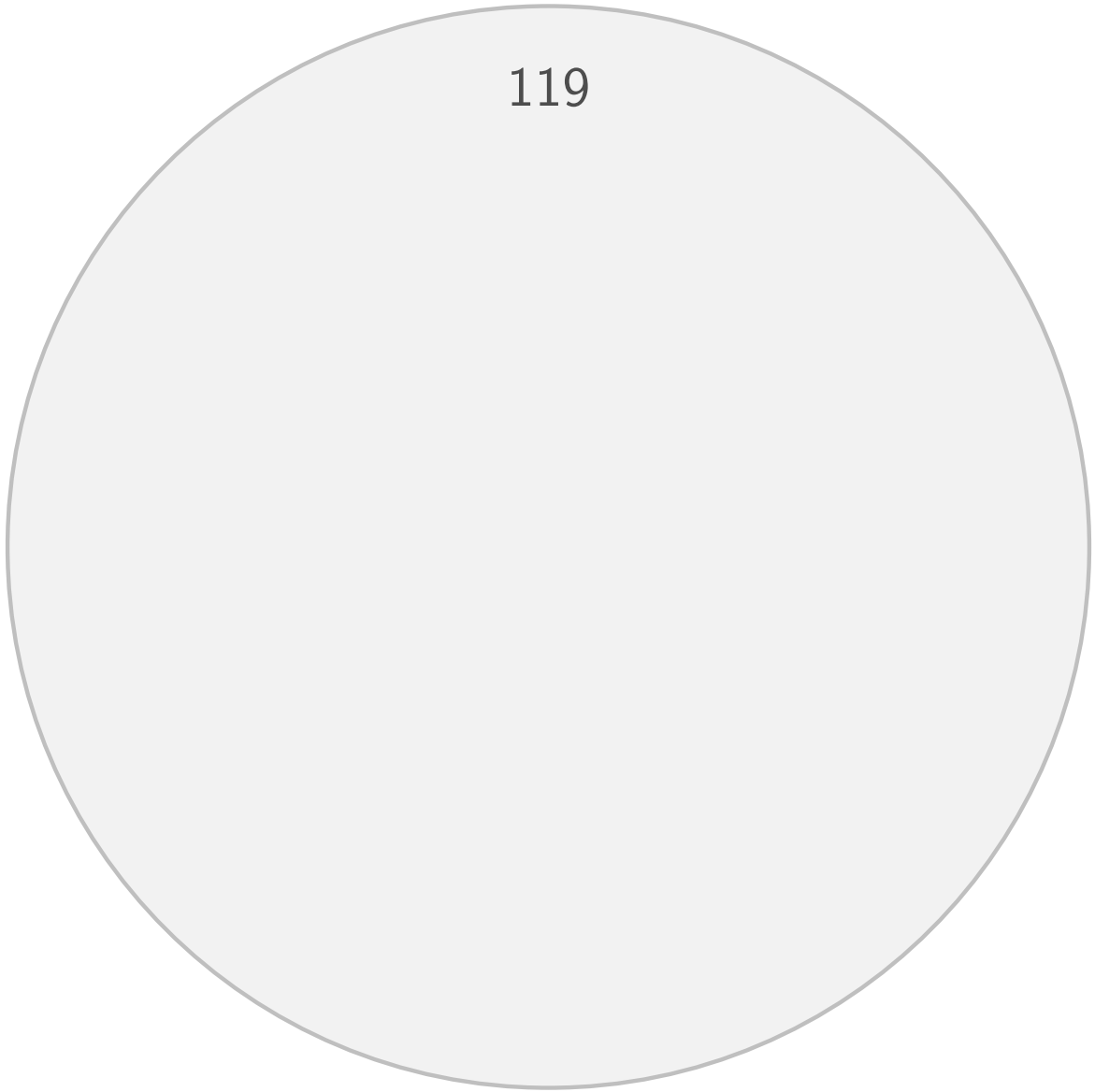
Theorem II

An orthogonal, properly oriented, right-stable CTRS is confluent.

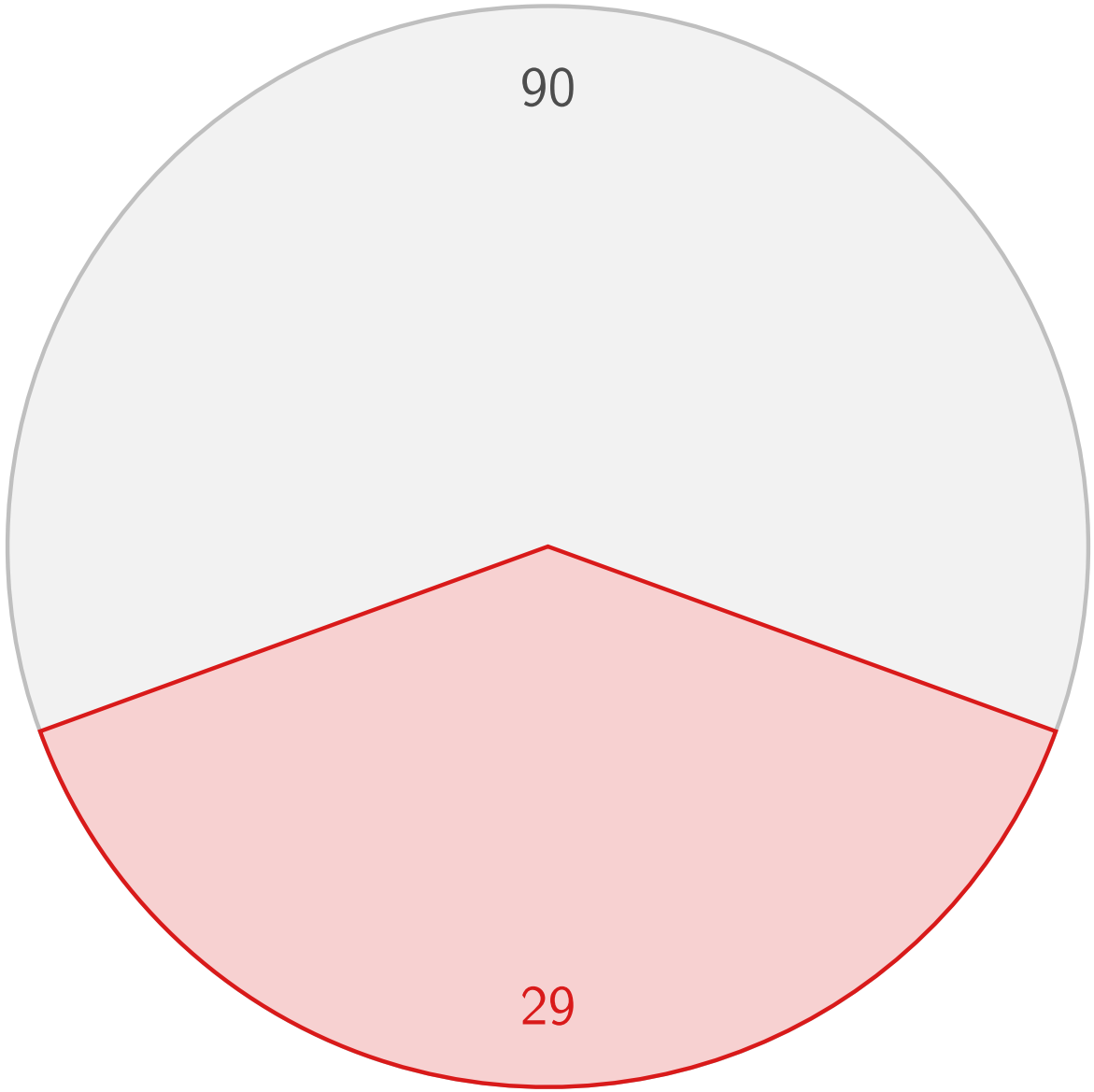
Theorem III

A quasi-decreasing SDTRS is confluent if all its CCPs are joinable.

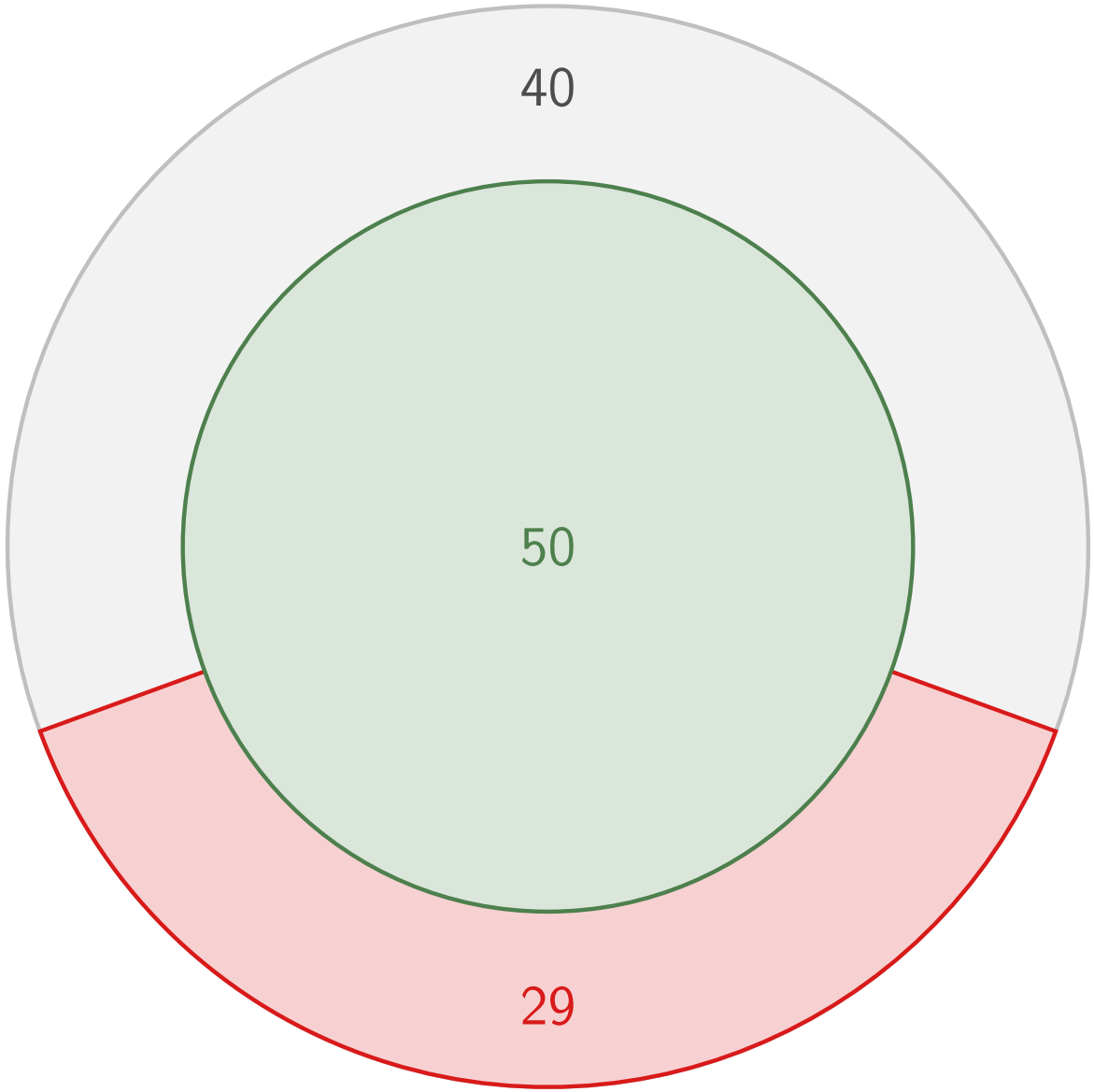
Experimental Results



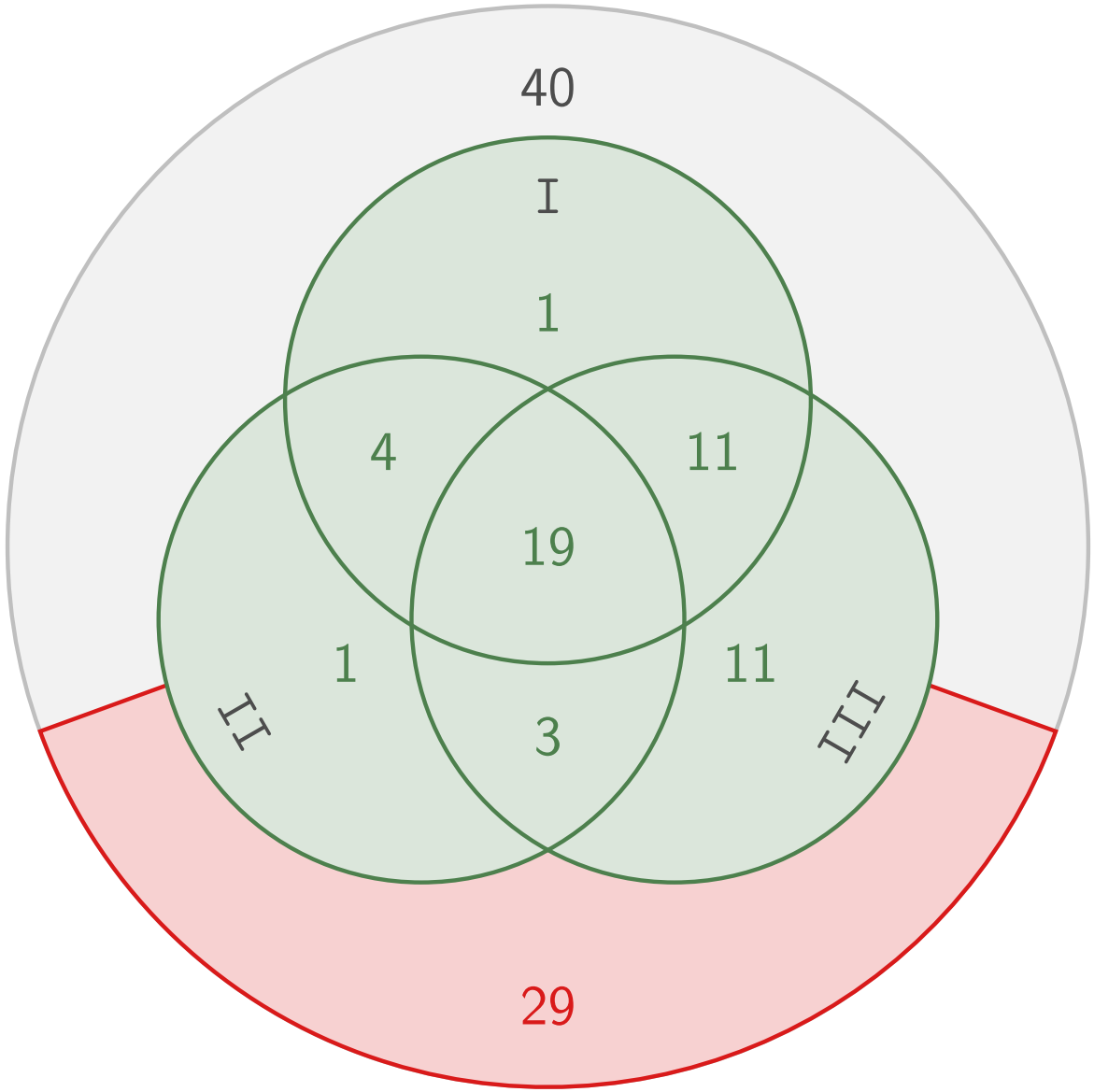
Experimental Results



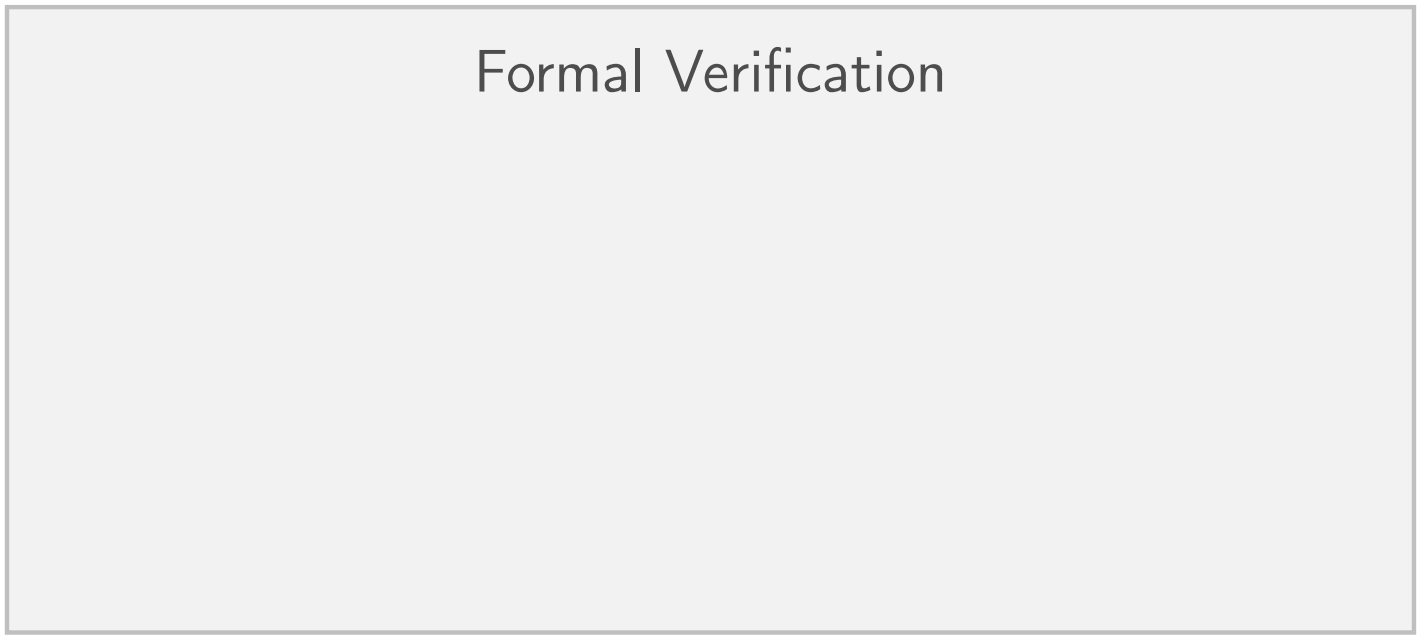
Experimental Results



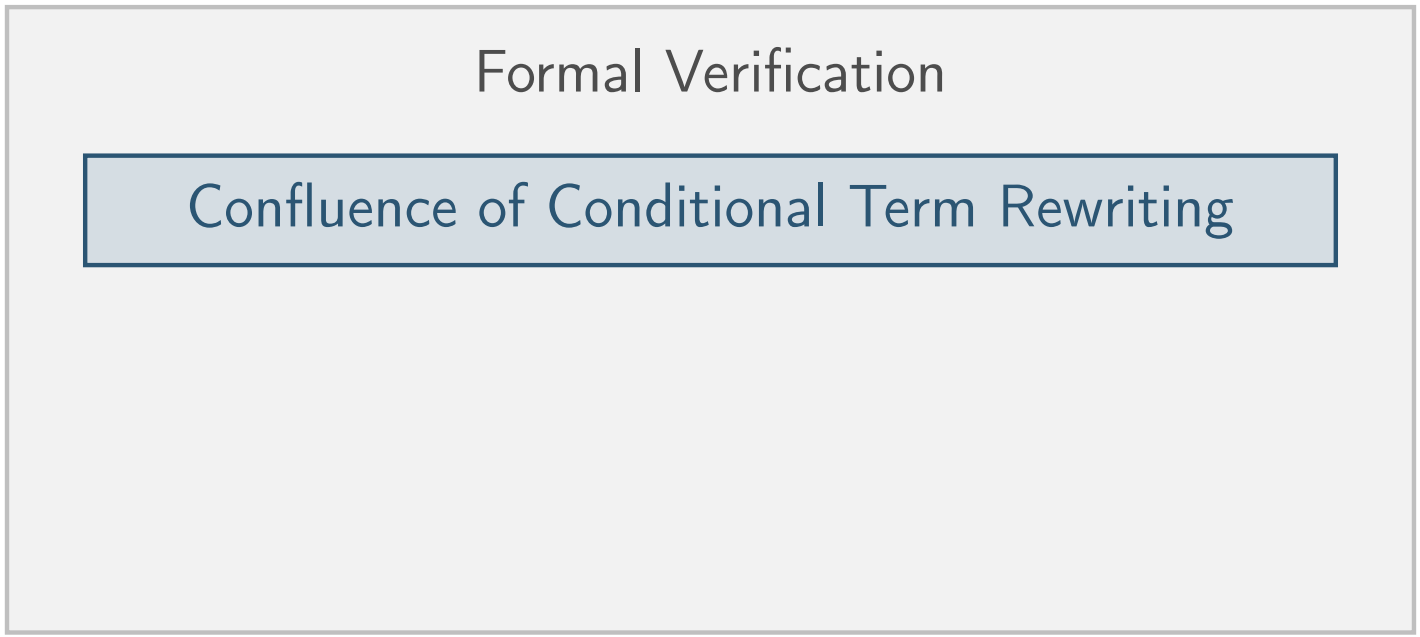
Experimental Results



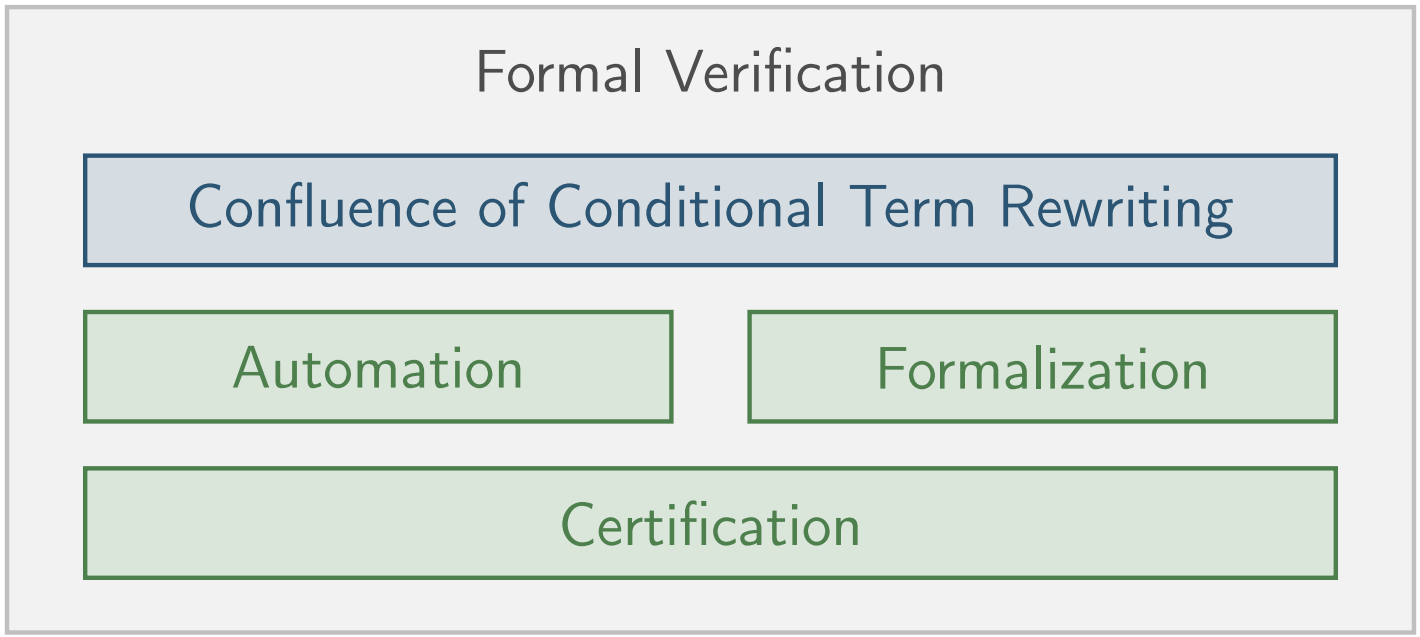
Conclusion



Conclusion


Formal Verification
Confluence of Conditional Term Rewriting

Conclusion


Formal Verification
Confluence of Conditional Term Rewriting
Automation
Formalization
Certification

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C. Sternagel and T. Sternagel
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T. Sternagel and C. Sternagel
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In *Proc. 4th IWC*, pages 13–17, 2015.

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In *Proc. Joint 25th RTA and 12th TLCA*, *LNCS*, pages 456–465, 2014.