Automating the Conversion Version of Decreasing Diagrams for First-Order Rewrite Systems

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Abstract

While determining suitable labels is alike for the valley and the conversion version of decreasing diagrams, a key issue for the conversion version is to find promising conversions (without labels) in a first step. In this note we propose to use (recording) completion for this endeavor.

The main task when automating the conversion version of decreasing diagrams [2] is to find conversions that close critical peaks decreasingly:

$$\xleftarrow[C]{} \cdot \xrightarrow[D]{} \subseteq \xleftrightarrow[\curlyvee C]{*} \cdot \xrightarrow[D]{=} \cdot \xleftrightarrow[\curlyvee CD]{*} \cdot \xleftarrow[C]{=} \cdot \xleftrightarrow[\curlyvee D]{*} \tag{1}$$

In this note we focus on the search for suitable conversions that close the critical peaks, as the search for suitable labels is similar to the case when valleys are considered. Our method improves upon the heuristic proposed in [1, Section 4] as it does neither restrict the use of variable-perserving nor of collapsing rewrite rules.

In the case of success, (recording) completion [3] transforms an equational system $\mathcal{E}_0$ into an equivalent, confluent and terminating TRS $\mathcal{R}_n$ in a number of n steps:

$$(\mathcal{E}_0, \mathcal{R}_0) \rightsquigarrow (\mathcal{E}_1, \mathcal{R}_1) \rightsquigarrow \cdots \rightsquigarrow (\mathcal{E}_n, \mathcal{R}_n)$$

While for completion $\mathcal{R}_0$ and $\mathcal{E}_n$ are required to be empty, in our setup we use slightly different starting configurations and do not require that $\mathcal{R}_n$ is locally confluent.

Let $\mathcal{R}$ be a TRS. Given a critical peak $t \; {}^{l_1 \to r_1}\!\!\leftarrow s \to^{l_2 \to r_2} u$ we want to find a conversion $t \xleftrightarrow[\mathcal{R} \cup (\mathcal{R} \setminus \{l_1 \to r_1, l_2 \to r_2\})^{-1}]{*} u$, as such a conversion is likely to fit the shape of (1). Hence we propose to perform recording completion as follows:

$$(\mathcal{R} \cup (\mathcal{R} \setminus \{l_1 \to r_1, l_2 \to r_2\})^{-1}, \{l_1 \to r_1, l_2 \to r_2\}) \rightsquigarrow^i (\mathcal{E}_i, \mathcal{R}_i) \tag{2}$$

In each step i we can test if $t \to^*_{\mathcal{R}_i} \cdot {}_{\mathcal{R}_i}^{*}\!\!\leftarrow u$, compute the related conversion $t \xleftrightarrow[\mathcal{R}]{*} u$ (in the TRS $\mathcal{R}$), and stop once a sufficient amount of conversions has been produced.

Clearly, more advanced heuristics will improve upon the the initial choice for $\mathcal{E}_0$ and $\mathcal{R}_0$ as proposed in (2). These heuristics may also depend on the labeling functions that will be used to establish decreasingness.

References

1 N. Hirokawa and A. Middeldorp. Decreasing diagrams and relative termination. *JAR*, 47(4):481–501, 2011.

2 V. van Oostrom. Confluence by decreasing diagrams – converted. In *Proc. 19th RTA*, volume 5117 of *LNCS*, pages 306–320, 2008.

3 T. Sternagel, S. Winkler, and H. Zankl. Recording completion for certificates in equational reasoning. In *Proc. 4th CPP*, pages 41–47. ACM, 2015.