CSI – A Confluence Tool

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Quiz

Google Maps

C S I

- open source confluence tool (first-order rewrite systems)
- `http://cl-informatik.uibk.ac.at/software/csi`

Preliminaries

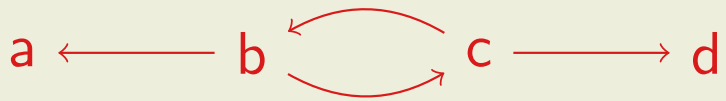
Definition (confluence)

$${}^*\!\leftarrow \cdot \rightarrow^* \subseteq \rightarrow^* \cdot {}^*\!\leftarrow$$

Definition (local confluence)

$$\leftarrow \cdot \rightarrow \subseteq \rightarrow^* \cdot {}^*\!\leftarrow$$

Example



confluence: × local confluence: ✓

Theorem (Newman, 1942 & van Oostrom, 1994)

local confluence & termination $\longrightarrow$ *confluence*

local confluence & decreasingness $\longrightarrow$ *confluence*

Non-Confluence

Procedure

① $t \overset{\leqslant n}{\leftarrow} s \overset{\leqslant n}{\rightarrow} u$ ② not $t \rightarrow^* \cdot {}^*\!\leftarrow u$

Definition

$\mathrm{red}(s) := \{s' \mid s \rightarrow^* s'\}$

How to test ②?

$$\underbrace{\mathrm{red}(t)}_{\textit{overapprox.}} \cap \underbrace{\mathrm{red}(u)}_{\textit{overapprox.}} = \varnothing \longrightarrow ②$$

Overapproximate by ...

tcap (termination analysis) (Giesl, Thiemann, Schneider-Kamp, 2005)
tree automata completion (Korp, Middeldorp 2007, Genet, 1998)

Confluence (Order-Sorted Decomposition)

Theorem (Felgenhauer, Z., Middeldorp, 2011)

A non-duplicating $(\mathcal{S}, \geqslant)$-sorted TRS $\mathcal{R}$ is confluent if

1. *for $\ell \to r \in \mathcal{R}$*
   - *ℓ, r order-sorted*
   - *$\mathsf{sort}(\ell) \geqslant \mathsf{sort}(r)$*
   - *sort of variables in ℓ matches sort of context*
2. *$\mathcal{R}$ is confluent on order-sorted terms*

Example

$1\colon \mathsf{f}(x, \mathsf{A}) \to \mathsf{G}(x)$ $2\colon \mathsf{f}(y, \mathsf{G}(y)) \to \mathsf{B}$ $3\colon \mathsf{G}(\mathsf{C}) \to \mathsf{C}$ $4\colon \mathsf{F}(z) \to \mathsf{F}(\mathsf{G}(z))$

$f : f_1 \times f_2 \to f$ $G : G_1 \to G$ $B : B$ $x : x$ $z : z$

$F : F_1 \to F$ $A : A$ $C : C$ $y : y$

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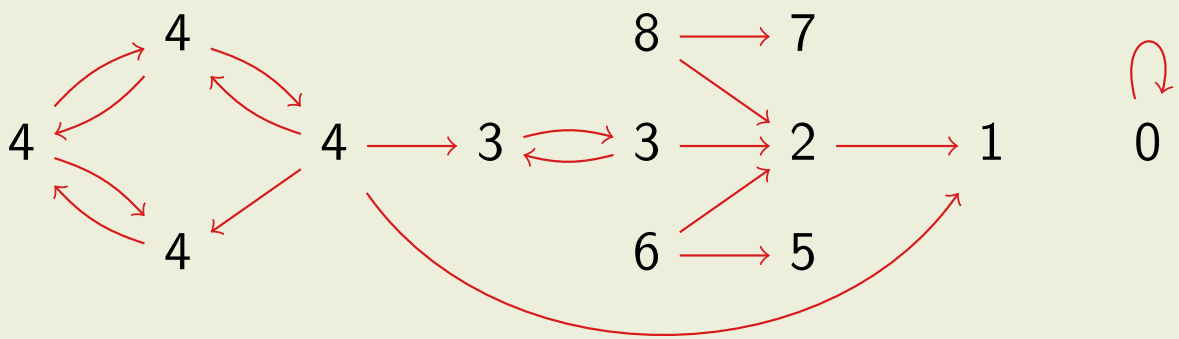

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$f : 4 \times 8 \to 6$ $G : 4 \to 2$ $B : 5$ $x : 4$ $z : 3$

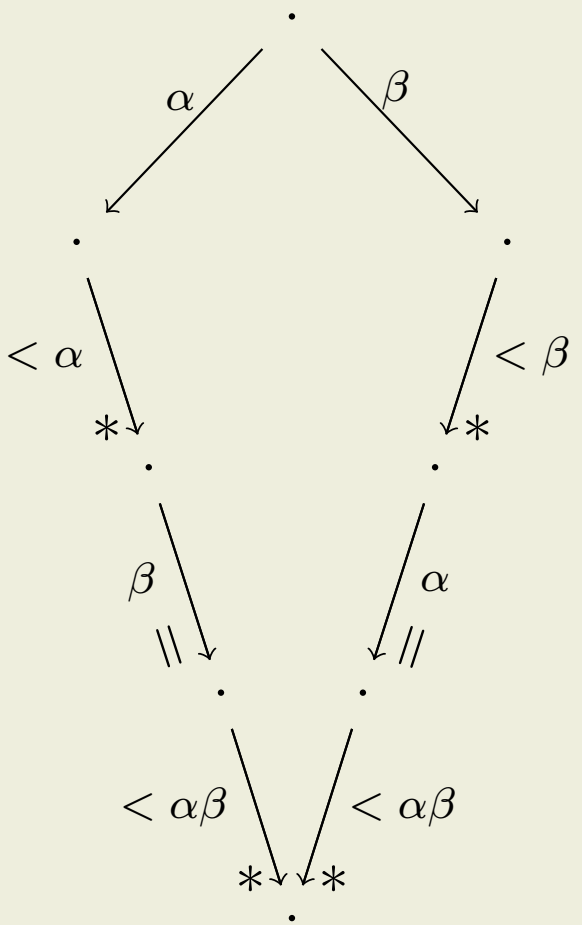
$F : 3 \to 0$ $A : 7$ $C : 1$ $y : 4$

$4 > 3 > 2 > 1$ $8 > 7, 2$ $6 > 5, 2$

$\mathcal{R}_1 = \{(1), (2), (3)\}$ $\mathcal{R}_2 = \{(3), (4)\}$

Confluence (Decreasing Diagrams)

Definition



Theorem (Z., Felgenhauer, Middeldorp, '11)

left-linear TRS $\mathcal{R}$ is confluent if

- *$\mathcal{R}_d/\mathcal{R}_{nd}$ terminating*
- *critical peaks decreasing (rule labeling)*

Decreasingness

- different labelings
- combine lexicographically
- modular approach

Strategy Language

Example

```
(KB || NOTCR || (((CLOSED || DD) | add)2*)!  || sorted -order)*
```

Strategy Language to

- control tool
- combine different methods (flexibly)

Evaluation

Experiments

	ACP	CSI	$\sum$	ACP	CSI	$\sum$
confluent	64	61	67	42	43	43
not confluent	18	20	21	47	47	47
	106 TRSs			99 TRSs		

Conclusion

Summary

- automated confluence tool
- non-confluence
- order-sorted decomposition
- decreasing diagrams
- full control by strategy language

Future Work

- non-confluence: heuristics for peaks
- decreasing diagrams: parallel steps (for labelings)
- certification of generated proofs

Confluence Competition 2012

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`http://coco.nue.riec.tohoku.ac.jp`