

Transforming SAT into Termination of Rewriting

Harald Zankl Christian Sternagel Aart Middeldorp

Institute of Computer Science
University of Innsbruck

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Motivation

- termination competition
 - termination methods via SAT
 - generate large testbench of *challenging* TRSs
- unsymmetry
 - SAT: dedicated (NP) [Coo71] vs termination (undecidable) [HL78]
 - KBO: dedicated (P) [KV03] vs SAT (NP) [ZM07]



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 - **KBO: dedicated (P) [KV03] vs SAT (NP) [ZM07]**



Propositional Logic

Syntax

propositional atoms \mathcal{A} , **propositional formulas** $\varphi \in \mathcal{P}(\mathcal{A})$ with

$$\varphi ::= p \in \mathcal{A} \mid (\varphi \wedge \varphi) \mid (\neg \varphi)$$



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Semantics

- **assignment** $\alpha: \mathcal{A} \rightarrow \mathbb{B} := \{0, 1\}$
- $[\alpha]: \mathcal{P}(\mathcal{A}) \rightarrow \mathbb{B}$ with

$$[\alpha](\varphi) = \begin{cases} \alpha(\varphi) & \text{if } \varphi = p \in \mathcal{A} \\ [\alpha](\psi) \cdot [\alpha](\chi) & \text{if } \varphi = \psi \wedge \chi \\ \overline{[\alpha](\psi)} & \text{if } \varphi = \neg\psi \end{cases}$$

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with $a \cdot b = 1$ iff $a = b = 1$

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with $a \cdot b = 1$ iff $a = b = 1$ and $\bar{a} = 1$ iff $a = 0$

SAT to Nontermination

Formulas as Terms

$\lceil * \rceil: \mathcal{P}(\mathcal{A}) \rightarrow \mathcal{T}(\mathcal{F}, \mathcal{V})$

$$\lceil \varphi \rceil = \begin{cases} p \in \mathcal{V} & \text{if } \varphi = p \in \mathcal{A} \\ \lceil \psi \rceil \star \lceil \chi \rceil & \text{if } \varphi = \psi \wedge \chi \\ -(\lceil \psi \rceil) & \text{if } \varphi = \neg \psi \end{cases}$$



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Example

$$p_1 \wedge \neg p_2 \rightarrow p_1 \star (-p_2)$$

TRS Simp

$$\begin{array}{l} -\perp \rightarrow \top \quad -\top \rightarrow \perp \\ \perp * \perp \rightarrow \perp \quad \perp * \top \rightarrow \perp \quad \top * \perp \rightarrow \perp \quad \top * \top \rightarrow \top \end{array}$$



TRS Simp

$$\begin{array}{l}
 -\perp \rightarrow \top \quad -\top \rightarrow \perp \\
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 \end{array}$$

Theorem

Let $\varphi \in \mathcal{P}(\mathcal{A}_n)$. The TRS U^φ consisting of Simp and

$$\text{unsat}(p_1, \dots, p_n, \top) \rightarrow \text{unsat}(p_1, \dots, p_n, \lceil \varphi \rceil)$$

is *terminating* iff φ is *unsatisfiable*



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Example ($\varphi = p_1 \wedge \neg p_2$)

- **satisfying case:** $\alpha(p_1) = 1, \alpha(p_2) = 0$

$$t := \text{unsat}(\top, \perp, \top \star (-\perp)) \rightarrow \text{unsat}(\top, \perp, \top \star \top) \rightarrow \text{unsat}(\top, \perp, \top) \rightarrow t$$

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Example ($\varphi = p_1 \wedge \neg p_2$)

- satisfying case: $\alpha(p_1) = 1, \alpha(p_2) = 0$

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- **nonsatisfying case:** $\alpha(p_1) = 0, \alpha(p_2) = 0$

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Formal Proof?



Formal Proof?

Automated Proof?



Formal Proof?

Automated Proof?

Semantic Labeling!



Algebras and Models

Definition (\mathcal{F} -Algebras)

- signature \mathcal{F}
- \mathcal{F} -algebra \mathcal{A}
 - carrier A
 - interpretation $f_{\mathcal{A}}: A^n \rightarrow A$



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$$[\alpha]_{\mathcal{A}}(l) = [\alpha]_{\mathcal{A}}(r)$$

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Labelings

Definition

- set of labels L_f
- labeled signature $\mathcal{F}_{\text{lab}} = \{f_a \mid f \in \mathcal{F} \wedge a \in L_f\} \cup \{f \mid L_f = \emptyset\}$
- labeling ℓ for \mathcal{A} : $\ell_f: A^n \rightarrow L_f$ if $L_f \neq \emptyset$



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- labeled signature $\mathcal{F}_{\text{lab}} = \{f_a \mid f \in \mathcal{F} \wedge a \in L_f\} \cup \{f \mid L_f = \emptyset\}$
- **labeling** l for \mathcal{A} : $l_f: A^n \rightarrow L_f$ if $L_f \neq \emptyset$



Labelings

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Definition

$$\text{lab}_\alpha(t) = \begin{cases} t & t \in \mathcal{V} \\ f(\text{lab}_\alpha(t_1), \dots, \text{lab}_\alpha(t_n)) & t = f(t_1, \dots, t_n) \wedge L_f = \emptyset \\ f_a(\text{lab}_\alpha(t_1), \dots, \text{lab}_\alpha(t_n)) & \text{otherwise} \end{cases}$$

with $a = \ell_f([\alpha]_{\mathcal{A}}(t_1), \dots, [\alpha]_{\mathcal{A}}(t_n))$

Semantic Labeling

Definition

semantic labeling transformation $(\cdot)_{\text{lab}}$

$$\mathcal{R}_{\text{lab}} = \{\text{lab}_{\alpha}(l) \rightarrow \text{lab}_{\alpha}(r) \mid l \rightarrow r \in \mathcal{R} \wedge \alpha \in A^{\nu}\}$$



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Theorem ([Zan95])

\mathcal{R} terminating iff \mathcal{R}_{lab} terminating

φ unsatisfiable $\Rightarrow \mathcal{U}^\varphi$ terminating

\mathcal{U}^φ

$-\perp \rightarrow \top \quad -\top \rightarrow \perp \quad \perp * \perp \rightarrow \perp \quad \perp * \top \rightarrow \perp \quad \top * \perp \rightarrow \perp \quad \top * \top \rightarrow \top$

$\text{unsat}(p_1, \dots, p_n, \top) \rightarrow \text{unsat}(p_1, \dots, p_n, \lceil \varphi \rceil)$



φ unsatisfiable $\Rightarrow \mathcal{U}^\varphi$ terminating

\mathcal{U}^φ

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$$\text{unsat}(p_1, \dots, p_n, \top) \rightarrow \text{unsat}(p_1, \dots, p_n, \lceil \varphi \rceil)$$

Model

$\mathcal{B} = (\mathbb{B}, \{\perp_{\mathcal{B}}, \top_{\mathcal{B}}, -_{\mathcal{B}}, \star_{\mathcal{B}}, \text{unsat}_{\mathcal{B}}\})$ with $\text{unsat}_{\mathcal{B}}(p_1, \dots, p_n, x) = 0$

$$\perp_{\mathcal{B}} = 0 \quad \top_{\mathcal{B}} = 1 \quad -_{\mathcal{B}}(x) = \bar{x} \quad \star_{\mathcal{B}}(x, y) = x \cdot y$$

φ unsatisfiable $\Rightarrow \mathcal{U}^\varphi$ terminating

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Labeling

$L_{\text{unsat}} = \mathbb{B}, \ell_{\text{unsat}}(p_1, \dots, p_n, x) = x$

φ **unsatisfiable** $\Rightarrow \mathcal{U}^\varphi$ terminating

$\mathcal{U}_{\text{lab}}^\varphi$

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$$\text{unsat}_1(p_1, \dots, p_n, \top) \rightarrow \text{unsat}_0(p_1, \dots, p_n, \lceil \varphi \rceil)$$

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$\mathcal{U}_{\text{lab}}^\varphi$ terminating (LPO)

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$\mathcal{B} = (\mathbb{B}, \{\perp_{\mathcal{B}}, \top_{\mathcal{B}}, -_{\mathcal{B}}, \star_{\mathcal{B}}, \text{unsat}_{\mathcal{B}}\})$ with $\text{unsat}_{\mathcal{B}}(p_1, \dots, p_n, x) = 0$

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$\mathcal{U}_{\text{lab}}^\varphi$ terminating (LPO) $\Rightarrow \mathcal{U}^\varphi$ terminating

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SAT iff Termination



SAT iff Termination

Problem

generate/test **all assignments** consecutively



SAT iff Termination

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generate/test all assignments consecutively

TRS \mathcal{N}_{ext}

$$\text{next}(\text{nil}) \rightarrow \text{nil}$$

$$\text{next}(\perp :: xs) \rightarrow \top :: xs$$

$$\text{next}(\top :: xs) \rightarrow \perp :: \text{next}(xs)$$



SAT iff Termination

Problem

generate/test all assignments consecutively

TRS \mathcal{N}_{ext}

$$\text{next}(\text{nil}) \rightarrow \text{nil}$$

$$\text{next}(\perp :: \underline{xs}) \rightarrow \top :: \underline{xs}$$

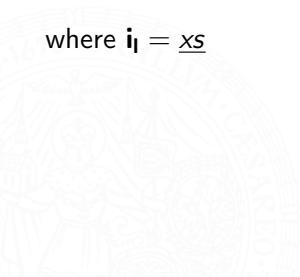
$$\text{next}(\top :: \underline{xs}) \rightarrow \perp :: \text{next}(\underline{xs})$$

$$\underline{\text{nil}} = \mathbf{0}_0$$

$$\underline{\perp :: \underline{xs}} = (\mathbf{2i})_{l+1}$$

$$\underline{\top :: \underline{xs}} = (\mathbf{1 + 2i})_{l+1}$$

where $\mathbf{i}_l = \underline{xs}$



SAT iff Termination

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generate/test all assignments consecutively

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$$\underline{\top :: \underline{xs}} = (\mathbf{1 + 2i})_{i+1}$$

where $\mathbf{i}_i = \underline{xs}$

Lemma

If $\underline{xs} = \mathbf{i}_i$ then $\text{next}(\underline{xs}) \rightarrow_{\mathcal{N}_{\text{ext}}}^* \underline{xs}'$ with $\underline{xs}' = (\mathbf{i + 1 \bmod 2^i})_i$

SAT iff Termination

Problem

generate/test all assignments consecutively

TRS \mathcal{N}_{ext}

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 \text{next}(\top :: \underline{xs}) \rightarrow \perp :: \text{next}(\underline{xs}) & \underline{\top :: \underline{xs}} = (\mathbf{1} + \mathbf{2i})_{\mathbf{l}+1}
 \end{array}$$

where $\mathbf{i}_l = \underline{xs}$

Lemma

If $\underline{xs} = \mathbf{i}_l$ then $\text{next}(\underline{xs}) \rightarrow_{\mathcal{N}_{\text{ext}}}^* \underline{xs}'$ with $\underline{xs}' = (\mathbf{i} + \mathbf{1} \bmod \mathbf{2}^l)_l$

Ensure *correct* terms by sorts, i.e.,

$\perp, \top : \text{bool}$, $(::) : \text{bool} \times \text{list} \rightarrow \text{list}$, $\text{next} : \text{list} \rightarrow \text{list}$, ...

Theorem

Let $\varphi \in \mathcal{P}(\mathcal{A}_n)$. The TRS \mathcal{S}^φ consisting of *Simp*, *Next* and

$$\text{sat}([p_1; \dots; p_n], \perp) \rightarrow \text{sat}(\text{next}([p_1; \dots; p_n]), \lceil \varphi \rceil)$$

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Example

- **satisfying case** ($[\top; \perp]$ satisfies φ):

$$\text{sat}([\perp; \perp], \perp) \rightarrow \text{sat}(\text{next}([\perp; \perp]), \lceil \varphi \rceil_{[\perp; \perp]}) \rightarrow^* \text{sat}([\top; \perp], \lceil \varphi \rceil_{[\perp; \perp]}) \rightarrow^*$$

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Theorem

Let $\varphi \in \mathcal{P}(\mathcal{A}_n)$. The TRS \mathcal{S}^φ consisting of *Simp*, *Next* and

$$\text{sat}([p_1; \dots; p_n], \perp) \rightarrow \text{sat}(\text{next}([p_1; \dots; p_n]), \lceil \varphi \rceil)$$

is terminating iff φ is satisfiable

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- satisfying case ($[\top; \perp]$ satisfies φ):

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(many-sorted) semantic labeling



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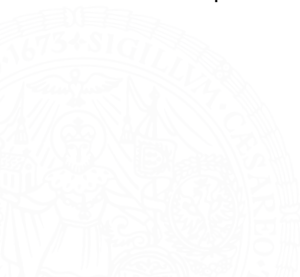
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(many-sorted) semantic labeling

Theorem [Zan94]

Termination is persistent for **non-collapsing** TRSs



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Alternative Encoding because

- variables p_1, \dots, p_n must be specified
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Formulas as Terms

$$\lceil * \rceil: \mathcal{P}(\mathcal{A}) \rightarrow \mathcal{T}(\mathcal{F}, \mathcal{V})$$

$$\lceil \varphi \rceil = \begin{cases} p \in \mathcal{V} & \text{if } \varphi = p \in \mathcal{A} \\ \lceil \psi \rceil * \lceil \chi \rceil & \text{if } \varphi = \psi \wedge \chi \\ -(\lceil \psi \rceil) & \text{if } \varphi = \neg \psi \end{cases}$$

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Formulas as Terms²

$$\ulcorner * \urcorner: \mathcal{P}(\mathcal{A}_n) \rightarrow \mathcal{T}(\mathcal{F})$$

$$\ulcorner \varphi \urcorner = \begin{cases} v_i \in \mathcal{F} & \text{if } \varphi = p_i \in \mathcal{A}_n \quad (1 \leq i \leq n) \\ \text{and}(\ulcorner \psi \urcorner, \ulcorner \chi \urcorner) & \text{if } \varphi = \psi \wedge \chi \\ \text{not}(\ulcorner \psi \urcorner) & \text{if } \varphi = \neg \psi \end{cases}$$

Theorem

Let $\varphi \in \mathcal{P}(\mathcal{A}_n)$. The TRS \mathcal{S}^φ consisting of *Simp*, *Next* and

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TRS Assign

$$\text{assign}(xs, \text{and}(x, y)) \rightarrow \text{assign}(xs, x) \star \text{assign}(xs, y)$$
$$\text{assign}(xs, \text{not}(x)) \rightarrow - \text{assign}(xs, x)$$
$$\text{assign}(xs, v_i) \rightarrow \text{nth}(xs, s^i(0))$$
$$1 \leq i \leq n$$


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$$\text{nth}(\perp :: xs, 0) \rightarrow \perp$$

$$\text{nth}(\top :: xs, 0) \rightarrow \top$$

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TRS $\mathcal{A}_{\text{assign}}$

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TRS \mathcal{N}_{ext}

$$\text{next}(\text{nil}) \rightarrow \text{nil}$$

$$\text{next}(\perp :: xs) \rightarrow \top :: xs$$

$$\text{next}(\top :: xs) \rightarrow \perp :: \text{next}(xs)$$

TRS $\mathcal{A}ssign$

$$\text{assign}(xs, \text{and}(x, y)) \rightarrow \text{assign}(xs, x) \star \text{assign}(xs, y)$$

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TRS $\mathcal{N}ext2$

$$\text{next}(\text{nil}) \rightarrow \top :: \text{nil}$$

$$\text{next}(\perp :: xs) \rightarrow \top :: xs$$

$$\text{next}(\top :: xs) \rightarrow \perp :: \text{next}(xs)$$

TRS \mathcal{A} ssign

$$\text{assign}(xs, \text{and}(x, y)) \rightarrow \text{assign}(xs, x) \star \text{assign}(xs, y)$$

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TRS \mathcal{N} ext2

$$\text{next}(\text{nil}) \rightarrow \top :: \text{nil}$$

$$\underline{\text{nil}} = \mathbf{0}$$

$$\text{next}(\perp :: xs) \rightarrow \top :: xs$$

$$\underline{\perp :: xs} = \mathbf{2i}$$

$$\text{next}(\top :: xs) \rightarrow \perp :: \text{next}(xs)$$

$$\underline{\top :: xs} = \mathbf{1 + 2i}$$

where $\underline{xs} = \mathbf{i}$

Lemma

If $\underline{xs} = \mathbf{i}$ then $\text{next}(xs) \rightarrow_{\mathcal{N}_{\text{ext}2}}^* xs'$ with $\underline{xs'} = \mathbf{i} + \mathbf{1}$



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If xs encodes a *non-satisfying* assignment then

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is *terminating* iff φ is *satisfiable*

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Formal Proof

(many-sorted) **semantic labeling**

100 (random) formulas of different shape

tool	2 variables, depth 3			3 variables, depth 4			4 variables, depth 5		
	S^φ	T^φ	U^φ	S^φ	T^φ	U^φ	S^φ	T^φ	U^φ
	T/ N	T/N	T/ N	T/N	T/N	T/ N	T/N	T/N	T/ N
AProVE	81/19	0/0	19/81	34/0	0/0	10/88	14/0	0/0	5/79
Jambox	16/0	0/0	19/0	24/0	0/0	12/0	15/0	0/0	11/0
NTI	0/19	0/0	0/81	0/5	0/0	0/74	0/0	0/0	0/11
TPA	0/0	0/0	1/0	0/0	0/0	0/0	0/0	0/0	0/0
T _T T ₂	10/0	0/0	0/0	6/0	0/0	0/0	5/0	0/0	0/0

Table: Experimental Results

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	S^φ	T^φ	U^φ	S^φ	T^φ	U^φ	S^φ	T^φ	U^φ
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Remark

trivial for any SAT-solver

Summary

- φ satisfiable iff \mathcal{U}^φ nonterminating
- φ satisfiable iff $\mathcal{S}^\varphi, \mathcal{T}^\varphi$ terminating
- generate (hard) testbenches efficiently



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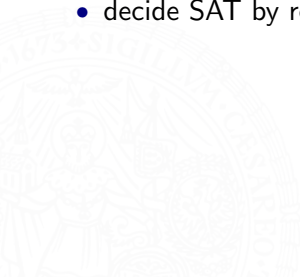


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Applications

- $\mathcal{S}^\varphi, \mathcal{T}^\varphi$, and \mathcal{U}^φ confluent
- decide SAT by rewriting

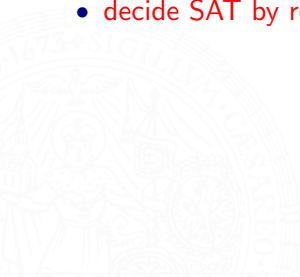


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