Transforming SAT into Termination of Rewriting

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Motivation

- termination competition
  - termination methods via SAT
  - generate large testbench of *challenging* TRSs
- unsymmetry
  - SAT: dedicated (NP) [Coo71] vs termination (undecidable) [HL78]
  - KBO: dedicated (P) [KV03] vs SAT (NP) [ZM07]

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Propositional Logic

Syntax

propositional atoms $\mathcal{A}$, propositional formulas $\varphi \in \mathcal{P}(\mathcal{A})$ with

$$\varphi ::= p \in \mathcal{A} \mid (\varphi \land \varphi) \mid (\neg\varphi)$$

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Semantics

- assignment $\alpha \colon \mathcal{A} \to \mathbb{B} := \{0, 1\}$
- $[\alpha] \colon \mathcal{P}(\mathcal{A}) \to \mathbb{B}$ with

$$[\alpha](\varphi) = \begin{cases} \alpha(\varphi) & \text{if } \varphi = p \in \mathcal{A} \\ [\alpha](\psi) \cdot [\alpha](\chi) & \text{if } \varphi = \psi \land \chi \\ \overline{[\alpha](\psi)} & \text{if } \varphi = \neg \psi \end{cases}$$

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with $a \cdot b = 1$ iff $a = b = 1$

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with $a \cdot b = 1$ iff $a = b = 1$ and $\overline{a} = 1$ iff $a = 0$

SAT to Nontermination

Formulas as Terms

$\ulcorner * \urcorner \colon \mathcal{P}(\mathcal{A}) \to \mathcal{T}(\mathcal{F}, \mathcal{V})$

$$\ulcorner \varphi \urcorner = \begin{cases} p \in \mathcal{V} & \text{if } \varphi = p \in \mathcal{A} \\ \ulcorner \psi \urcorner \star \ulcorner \chi \urcorner & \text{if } \varphi = \psi \wedge \chi \\ -(\ulcorner \psi \urcorner) & \text{if } \varphi = \neg \psi \end{cases}$$

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Example

$p_1 \wedge \neg p_2 \to p_1 \star (-\, p_2)$

TRS $\mathcal{S}$imp

$$-\bot \to \top \qquad -\top \to \bot$$

$$\bot \star \bot \to \bot \qquad \bot \star \top \to \bot \qquad \top \star \bot \to \bot \qquad \top \star \top \to \top$$

TRS $\mathcal{S}\mathsf{imp}$

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Theorem

Let $\varphi \in \mathcal{P}(\mathcal{A}_n)$. The TRS $\mathcal{U}^\varphi$ consisting of $\mathcal{S}\mathsf{imp}$ and

$$\mathsf{unsat}(p_1, \ldots, p_n, \top) \to \mathsf{unsat}(p_1, \ldots, p_n, \ulcorner\varphi\urcorner)$$

is terminating iff φ is unsatisfiable

TRS $\mathcal{S}$imp

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Example ($\varphi = p_1 \wedge \neg p_2$)

- satisfying case: $\alpha(p_1) = 1$, $\alpha(p_2) = 0$

$$t := \mathsf{unsat}(\top, \perp, \top \star (-\perp)) \to \mathsf{unsat}(\top, \perp, \top \star \top) \to \mathsf{unsat}(\top, \perp, \top) \to t$$

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Formal Proof?

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Automated Proof?

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Automated Proof?

Semantic Labeling!

Algebras and Models

Definition ($\mathcal{F}$-Algebras)

- signature $\mathcal{F}$
- $\mathcal{F}$-algebra $\mathcal{A}$
  - carrier A
  - interpretation $f_{\mathcal{A}} \colon \mathsf{A}^n \to \mathsf{A}$

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$$[\alpha]_\mathcal{A}(l) = [\alpha]_\mathcal{A}(r)$$

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Labelings

Definition

- set of labels L_f
- labeled signature $\mathcal{F}_{\mathsf{lab}} = \{f_a \mid f \in \mathcal{F} \wedge a \in \mathsf{L}_f\} \cup \{f \mid \mathsf{L}_f = \varnothing\}$
- labeling ℓ for $\mathcal{A}$: $\ell_f \colon \mathsf{A}^n \to \mathsf{L}_f$ if $\mathsf{L}_f \neq \varnothing$

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Definition

$$\mathsf{lab}_\alpha(t) = \begin{cases} t & t \in \mathcal{V} \\ f(\mathsf{lab}_\alpha(t_1), \ldots, \mathsf{lab}_\alpha(t_n)) & t = f(t_1, \ldots, t_n) \wedge \mathsf{L}_f = \varnothing \\ f_a(\mathsf{lab}_\alpha(t_1), \ldots, \mathsf{lab}_\alpha(t_n)) & \text{otherwise} \end{cases}$$

with $a = \ell_f([\alpha]_{\mathcal{A}}(t_1), \ldots, [\alpha]_{\mathcal{A}}(t_n))$

Semantic Labeling

Definition

semantic labeling transformation $(\cdot)_{\mathsf{lab}}$

$$\mathcal{R}_{\mathsf{lab}} = \{\mathsf{lab}_\alpha(l) \to \mathsf{lab}_\alpha(r) \mid l \to r \in \mathcal{R} \wedge \alpha \in \mathsf{A}^{\mathcal{V}}\}$$

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Theorem ([Zan95])

$\mathcal{R}$ terminating iff $\mathcal{R}_{\mathsf{lab}}$ terminating

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$\mathcal{U}^\varphi$

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$$\mathsf{unsat}\ (p_1, \ldots, p_n, \top) \to \mathsf{unsat}\ (p_1, \ldots, p_n, \ulcorner\varphi\urcorner)$$

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Model

$\mathcal{B} = (\mathbb{B}, \{\bot_\mathcal{B}, \top_\mathcal{B}, -_\mathcal{B}, \star_\mathcal{B}, \mathsf{unsat}_\mathcal{B}\})$ with $\mathsf{unsat}_\mathcal{B}(p_1, \dots, p_n, x) = 0$

$$\bot_\mathcal{B} = 0 \quad \top_\mathcal{B} = 1 \quad -_\mathcal{B}(x) = \overline{x} \quad \star_\mathcal{B}\,(x, y) = x \cdot y$$

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$$\mathsf{unsat}_1(p_1, \ldots, p_n, \top) \to \mathsf{unsat}_0(p_1, \ldots, p_n, \ulcorner\varphi\urcorner)$$

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$\mathcal{U}^{\varphi}_{\mathsf{lab}}$ terminating (LPO)

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$\mathcal{U}^{\varphi}_{\mathsf{lab}}$ terminating (LPO) $\Rightarrow \mathcal{U}^{\varphi}$ terminating

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SAT iff Termination

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Problem

generate/test all assignments consecutively

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TRS $\mathcal{N}$ext

$$\begin{aligned}
\mathsf{next}(\mathsf{nil}) &\to \mathsf{nil} \\
\mathsf{next}(\bot :: xs) &\to \top :: xs \\
\mathsf{next}(\top :: xs) &\to \bot :: \mathsf{next}(xs)
\end{aligned}$$

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where $\mathbf{i_l} = \underline{xs}$

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If $\underline{xs} = \mathbf{i_l}$ *then* $\mathsf{next}(xs) \to^*_{\mathcal{N}\mathsf{ext}} xs'$ *with* $\underline{xs'} = (\mathbf{i} + \mathbf{1} \bmod \mathbf{2^l})_{\mathbf{l}}$

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Ensure *correct* terms by sorts, i.e.,

$\bot, \top : \mathsf{bool}$, $(::) : \mathsf{bool} \times \mathsf{list} \to \mathsf{list}$, $\mathsf{next} : \mathsf{list} \to \mathsf{list}$, ...

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Let $\varphi \in \mathcal{P}(\mathcal{A}_n)$*. The TRS* $\mathcal{S}^\varphi$ *consisting of* $\mathcal{S}\mathsf{imp}$, $\mathcal{N}\mathsf{ext}$ *and*

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is terminating iff φ *is satisfiable*

Example

- satisfying case ($[\top; \bot]$ satisfies φ):

  $\mathsf{sat}([\bot; \bot], \bot) \to \mathsf{sat}(\mathsf{next}([\bot; \bot]), \ulcorner\varphi\urcorner_{[\bot;\bot]}) \to^* \mathsf{sat}([\top; \bot], \ulcorner\varphi\urcorner_{[\bot;\bot]}) \to^*$
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(many-sorted) semantic labeling

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Theorem [Zan94]

Termination is persistent for non-collapsing TRSs

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Alternative Encoding because

- variables $p_1, \ldots, p_n$ must be specified
- no separation between syntax/semantic

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Formulas as Terms

$\ulcorner * \urcorner \colon \mathcal{P}(\mathcal{A}\ ) \to \mathcal{T}(\mathcal{F}, \mathcal{V})$

$$\ulcorner\varphi\urcorner = \begin{cases} p \in \mathcal{V} & \text{if } \varphi = p\ \in \mathcal{A} \\ \ulcorner\psi\urcorner \star \ulcorner\chi\urcorner & \text{if } \varphi = \psi \wedge \chi \\ -(\ulcorner\psi\urcorner) & \text{if } \varphi = \neg\psi \end{cases}$$

Theorem

Let $\varphi \in \mathcal{P}(\mathcal{A}_n)$. The TRS $\mathcal{S}^\varphi$ consisting of $\mathcal{S}$imp, $\mathcal{N}$ext *and*

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Formulas as Terms2

$\ulcorner * \urcorner \colon \mathcal{P}(\mathcal{A}_n) \to \mathcal{T}(\mathcal{F} \quad)$

$$\ulcorner\varphi\urcorner = \begin{cases} \mathsf{v_i} \in \mathcal{F} & \text{if } \varphi = p_i \in \mathcal{A}_n \quad (1 \leqslant i \leqslant n) \\ \mathsf{and}(\ulcorner\psi\urcorner,\ulcorner\chi\urcorner) & \text{if } \varphi = \psi \wedge \chi \\ \mathsf{not}(\ulcorner\psi\urcorner) & \text{if } \varphi = \neg\psi \end{cases}$$

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TRS $\mathcal{A}$ssign

$$
\begin{aligned}
\mathsf{assign}(xs, \mathsf{and}(x, y)) &\to \mathsf{assign}(xs, x) \star \mathsf{assign}(xs, y) \\
\mathsf{assign}(xs, \mathsf{not}(x)) &\to -\,\mathsf{assign}(xs, x) \\
\mathsf{assign}(xs, \mathsf{v_i}) &\to \mathsf{nth}(xs, \mathsf{s}^i(0)) \qquad 1 \leqslant i \leqslant n
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\mathsf{nth}(\bot :: xs, 0) &\to \bot \\
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\begin{aligned}
\mathsf{next}(\mathsf{nil}) &\to \mathsf{nil} \\
\mathsf{next}(\bot :: xs) &\to \top :: xs \\
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\end{aligned}
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TRS $\mathcal{N}$ext2

$$
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\mathsf{next}(\mathsf{nil}) &\to \top :: \mathsf{nil} \\
\mathsf{next}(\bot :: xs) &\to \top :: xs \\
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\end{aligned}$$

TRS $\mathcal{N}$ext2

$$\begin{aligned}
\mathsf{next}(\mathsf{nil}) &\to \top :: \mathsf{nil} && \underline{\mathsf{nil}} = \mathbf{0} \\
\mathsf{next}(\bot :: xs) &\to \top :: xs && \underline{\bot :: xs} = \mathbf{2\,i} \\
\mathsf{next}(\top :: xs) &\to \bot :: \mathsf{next}(xs) && \underline{\top :: xs} = \mathbf{1} + \mathbf{2\,i}
\end{aligned}$$

where $\underline{xs} = \mathbf{i}$

Lemma

If $\underline{xs} = \mathbf{i}$ *then* $\mathsf{next}(xs) \to^*_{\mathcal{N}\mathsf{ext2}} xs'$ *with* $\underline{xs'} = \mathbf{i} + \mathbf{1}$

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Formal Proof

(many-sorted) semantic labeling

100 (random) formulas of different shape

	2 variables, depth 3			3 variables, depth 4			4 variables, depth 5		
tool	$\mathcal{S}^{\varphi}$	$\mathcal{T}^{\varphi}$	$\mathcal{U}^{\varphi}$	$\mathcal{S}^{\varphi}$	$\mathcal{T}^{\varphi}$	$\mathcal{U}^{\varphi}$	$\mathcal{S}^{\varphi}$	$\mathcal{T}^{\varphi}$	$\mathcal{U}^{\varphi}$
	T/ N	T/N	T/ N	T/N	T/N	T/ N	T/N	T/N	T/ N
AProVE	81/19	0/ 0	19/81	34/ 0	0/ 0	10/88	14/ 0	0/ 0	5/79
Jambox	16/ 0	0/ 0	19/ 0	24/ 0	0/ 0	12/ 0	15/ 0	0/ 0	11/ 0
NTI	0/19	0/ 0	0/81	0/ 5	0/ 0	0/74	0/ 0	0/ 0	0/11
TPA	0/ 0	0/ 0	1/ 0	0/ 0	0/ 0	0/ 0	0/ 0	0/ 0	0/ 0
$\mathsf{T_TT_2}$	10/ 0	0/ 0	0/ 0	6/ 0	0/ 0	0/ 0	5/ 0	0/ 0	0/ 0

Table: Experimental Results

100 (random) formulas of different shape

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	T/ N	T/N	T/ N	T/N	T/N	T/ N	T/N	T/N	T/ N
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Remark

trivial for any SAT-solver

Summary

- φ satisfiable iff $\mathcal{U}^\varphi$ nonterminating
- φ satisfiable iff $\mathcal{S}^\varphi$, $\mathcal{T}^\varphi$ terminating
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- new termination methods

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