Nontermination of String Rewriting using SAT

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Overview

Preliminaries

SAT encoding

Experimental Results

Conclusion

Definition

string rewrite system (SRS) is TRS with $\mathit{arity}(f) = 1 \ \forall f \in \mathcal{F}$

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Example

$$\mathsf{a}(\mathsf{b}(x)) \to \mathsf{b}(\mathsf{b}(\mathsf{a}(\mathsf{a}(x))))$$

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Example

$$\mathsf{ab} \rightarrow \mathsf{bbaa}$$

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SRS is looping if $\exists t, u, v \colon t \to^{+} u \cdot t \cdot v$

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Theorem
TRS looping ⇒ nonterminating

Idea

encode rewrite sequence in matrix of dimension $n \times m$

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do we need context?

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Question

do we need context?

Answer

not in DP setting

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(Dis-)Advantages

+ no worry about context

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- \+ no worry about context
- \+ can be combined with recursive SCC algorithm

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(Dis-)Advantages

- \+ no worry about context
- \+ can be combined with recursive SCC algorithm
- − more rules to consider

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Exactly one Function Symbol

$$\alpha_{ij} = \bigvee_{a \in X} M_{ij}^{a} \wedge \bigwedge_{a \in X} (M_{ij}^{a} \to \bigwedge_{b \in X \setminus \{a\}} \neg M_{ij}^{b})$$

for $X = \mathcal{F}$ if $j > 0$ and $X = \mathcal{F}^{\sharp} \setminus \mathcal{F}$ otherwise

Rule Application

$$\beta_i^{l \to r} = R_i^{l \to r} \to (applies_i^{l \to r} \wedge \bigwedge_{0 \leqslant j < m} copy_{ij}^{l \to r})$$

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if $l \to r \in \mathcal{S}$

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Example (ab → bbaa, Ab → Aa, Ab → A})

Abb → Aab

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Rule Application (cont'd)

$copy_{ij}^{l \to r} = \top$ if $j + \max\{|l|, |r|\} \geqslant m$

Rule Application (cont'd)

$copy_{ij}^{l\to r} = \top$ if $j + \max\{|l|, |r|\} \geqslant m$ and otherwise

$$\Bigl((j < \mathbf{p}_i) \wedge \bigwedge_{a\in X} \bigl(M^a_{ij} \leftrightarrow M^a_{(i+1)j}\bigr)\Bigr) \vee \Bigl((j \geqslant \mathbf{p}_i) \wedge \bigwedge_{a\in \mathcal{F}} \bigl(M^a_{i(j+|l|)} \leftrightarrow M^a_{(i+1)(j+|r|)}\bigr)\Bigr)$$

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if $l \to r \in \mathcal{S}$ and in case $l \to r \in \mathrm{DP}(\mathcal{S})$

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$$copy_{00}^{\mathsf{Ab} \to \mathsf{Aa}} = (M_{0(\mathbf{p_0}+2)}^{\mathsf{a}} \leftrightarrow M_{1(\mathbf{p_0}+2)}^{\mathsf{a}}) \wedge (M_{0(\mathbf{p_0}+2)}^{\mathsf{b}} \leftrightarrow M_{1(\mathbf{p_0}+2)}^{\mathsf{b}})$$

Find Loop

$$\gamma = \bigvee_{0<i<n} \left((\mathbf{e}_i \geqslant \mathbf{e}_0) \wedge \bigwedge_{\substack{0 \leqslant j < m \\ a \in X}} \left(M^a_{0j} \leftrightarrow M^a_{ij} \right) \right)$$

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Putting all together

The formula $NT_{\mathcal{S}}$ is defined as

$$\left(\bigwedge_{0 \leqslant i < n} \left(\bigwedge_{0 \leqslant j < m} \alpha_{ij} \right) \wedge (\mathbf{e}_i < m) \wedge \beta_i \right) \wedge \gamma$$

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with

$$\beta_i = \bigvee_{l \to r \in \mathcal{S} \cup \mathrm{DP}(\mathcal{S})} R_i^{l \to r} \wedge \bigwedge_{l \to r \in \mathcal{S} \cup \mathrm{DP}(\mathcal{S})} \beta_i^{l \to r}$$

Lemma

If $NT_{\mathcal{S}}$ *is satisfiable then* $\mathcal{S}$ *admits a looping reduction.* □

Problem

Concrete vs Abstract variables

M^a_{ij} vs $M^a_{i(\mathbf{pi}+j)}$

Lemma

If $NT_{\mathcal{S}}$ is satisfiable then $\mathcal{S}$ admits a looping reduction. □

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M^a_{ij} vs $M^a_{i(\mathbf{pi}+j)}$ denote different variables even if $\mathbf{pi} = 0$

Solution

$$\varphi_{aux} = \bigwedge_{0 \leqslant i < n} \bigwedge_{0 \leqslant j < m} \bigwedge_{M^a_{i\mathbf{p}}} \left(\mathbf{p} = j \rightarrow (M^a_{ij} \leftrightarrow M^a_{i\mathbf{p}})\right)$$

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Overview

322 SRSs from TPDB 3.2 (2006 termination competition)

Timeout 60 sec

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tool	no
Jambox	25

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tool	no
Jambox	25
Matchbox	12

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Jambox	25
Matchbox	12
AProVE	12

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Matchbox	12
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TORPA	4

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$\mathsf{T_TTbox}$	0

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TPA	0
$\mathsf{T_TTbox}$	0
CiME	0

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Jambox	25
$\mathsf{T_TT_2}$	24
Matchbox	12
AProVE	12
TORPA	5
TEPARLA	1
MultumNonMulta	0
TPA	0
$\mathsf{T_TT}$box	0
CiME	0

335 SRSs from Waldmann/size-12/alpha-3

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NTI	64

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Matchbox	78
NTI	64
$\mathsf{T_TT_2}$	33

335 SRSs from Waldmann/size-12/alpha-3

Timeout 120 sec

tool	no
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NTI	64
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num − 11.srs num − 195.srs num − 217.srs num − 328.srs

Overview

Preliminaries

SAT encoding

Experimental Results

Conclusion

Remarks

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- if you know how, please tell me