Learning Strategy Design: First Lessons

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16 September 2020

Automated Theorem Proving

$Q(0,0,0,0)$
$\neg Q(x,y,z,0) \vee Q(x,y,z,1)$
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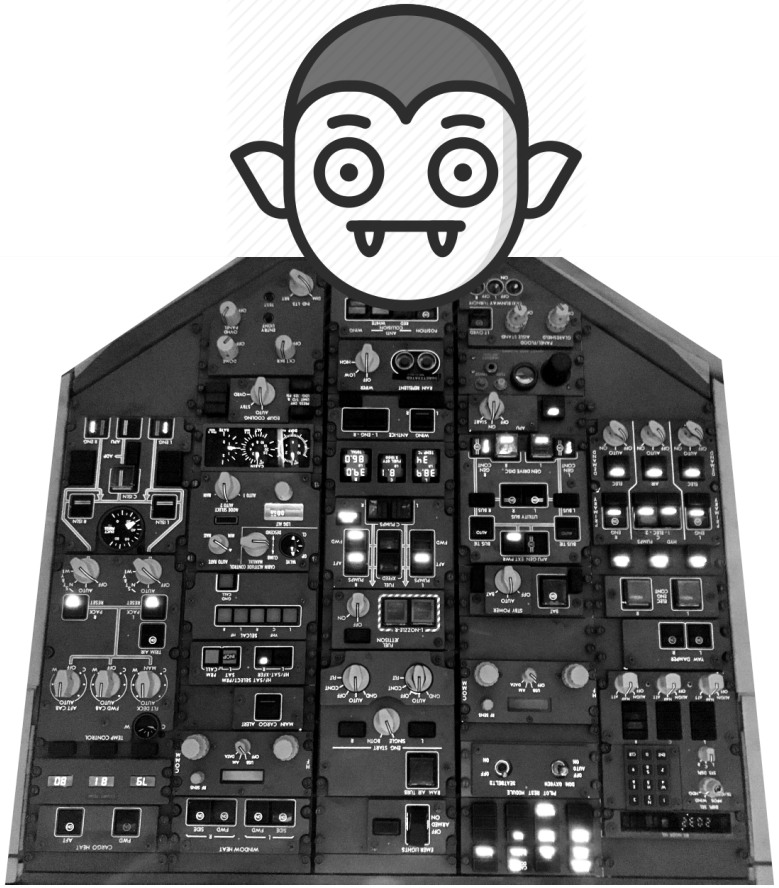

SAT
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UNSAT

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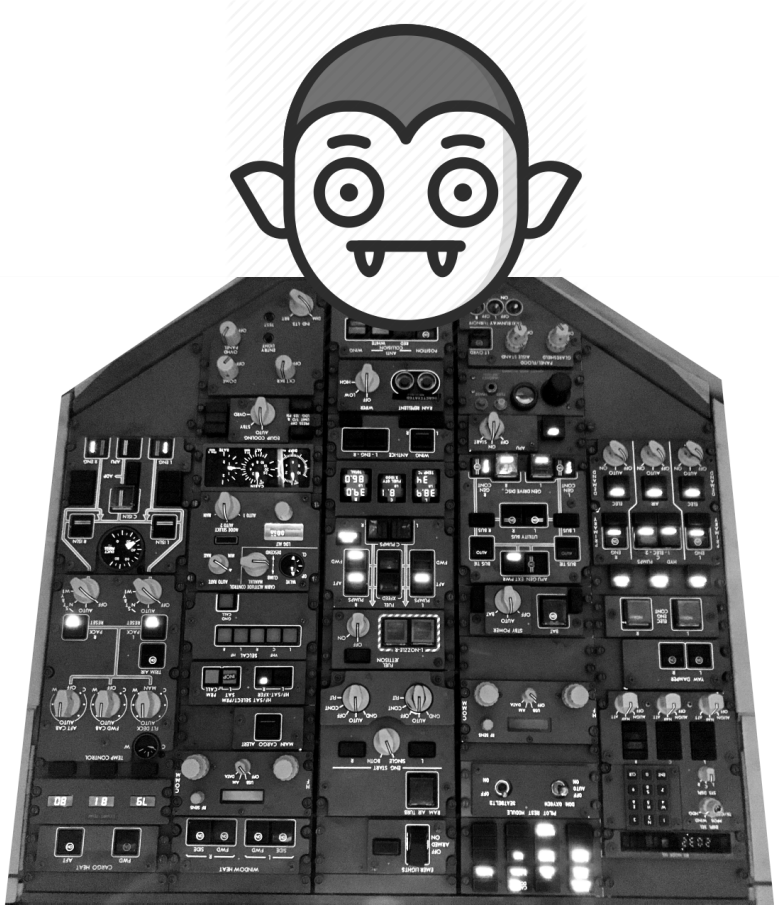

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1. predict one of Vampire's 801 CASC strategies for given problem

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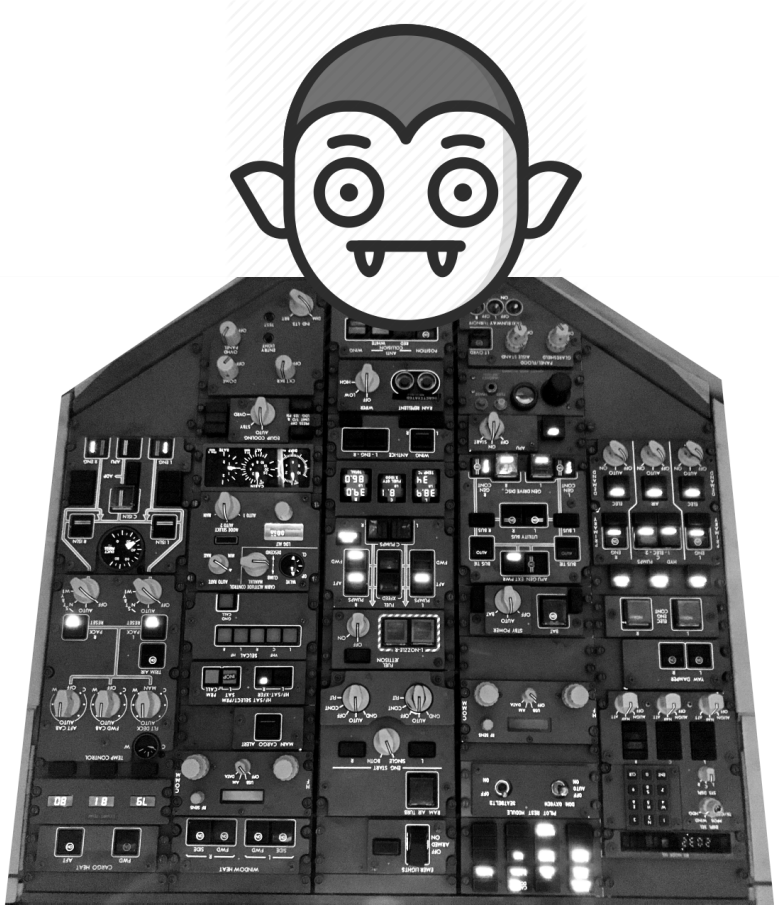

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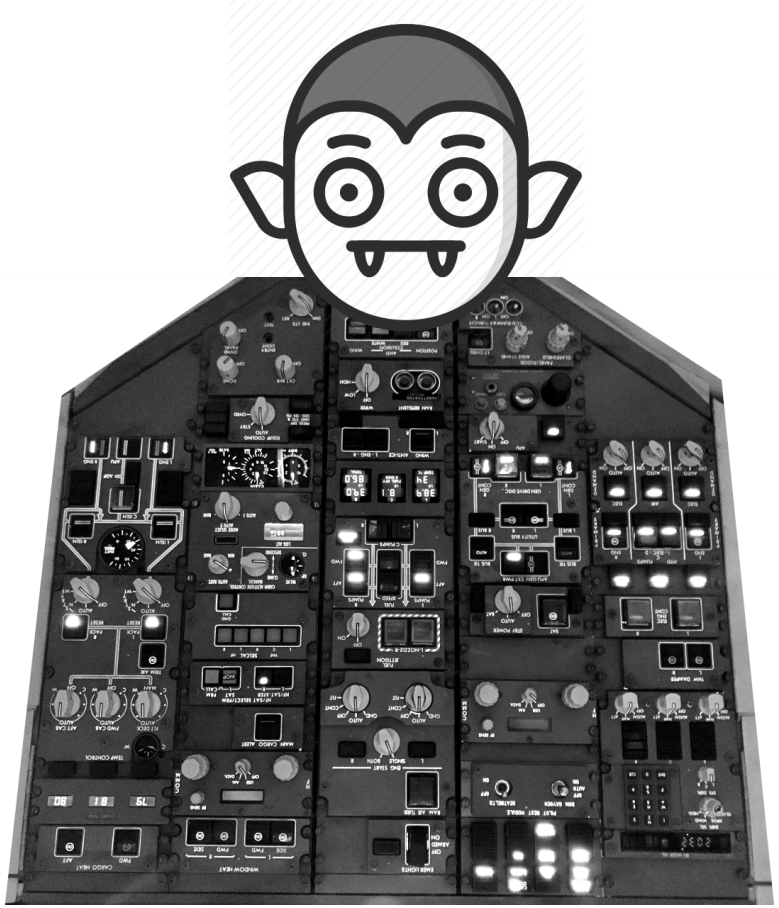

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1. predict one of Vampire's 801 CASC strategies for given problem
2. correlate problem features with beneficial strategy components
3. correlate problem features with success of CASC tools

Data

Vampire was run for 60sec on all 17574 FOL problems in TPTP 7.2.0 using all 801 strategies $\mathcal{S}$ used in CASC-27

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```
% File     : PUZ015-1 : TPTP v7.2.0. Released v1.0.0.
% Domain   : Puzzles
...
% Source   : [ANL]
% Status   : Satisfiable
% Rating   : 0.89 v7.1.0, 0.88 v7.0.0
% Syntax   : Number of clauses     :   21 (   0 non-Horn;  13 unit;  21 RR)
%            Number of atoms       :   29 (  11 equality)
%            Maximal clause size   :    2 (   1 average)
%            Number of predicates  :    2 (   0 propositional; 2-2 arity)
%            Number of functors    :   16 (  12 constant; 0-8 arity)
%            Number of variables   :   58 (   0 singleton)
%            Maximal term depth    :    3 (   2 average)
%------------------------------------------------------------------------------
cnf(cover_columns_1_and_2,axiom,
    ( ~ achievable(row(X),squares(not_covered,not_covered,Y3,Y4,Y5,Y6,Y7,Y8))
    | achievable(row(X),squares(covered,covered,Y3,Y4,Y5,Y6,Y7,Y8)) )).
cnf(cover_columns_2_and_3,axiom,
    ( ~ achievable(row(X),squares(Y1,not_covered,not_covered,Y4,Y5,Y6,Y7,Y8)) )).
```

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Problem Features

- all 92 problem properties collected by TPTP and Vampire:

  # clauses, # terms, # predicates, # functions, # variables, # connectives, # $\exists$, # $\forall$, # $\vee$, # $\wedge$, # $\neg$, # unit clauses, is EPR, is UEQ, is ground, # Horn clauses, # unit clauses, has sorts, has rationals, has reals, has groups, has rings, has equality, has arrays, has extensionality, max term depth, avg term depth, max predicate arity, avg predicate arity, max function arity, max # variables in clause, ...

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- three TPTP features: domain, source, rating
- three hand-crafted features (approximated):
  - # of unifiable positive and negative literals
  - # terms matching non-variable equation sides
  - # terms unifiable with non-variable equation sides

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- count how many test problems are solved by recommended strategy

Solved problems

features $\mathcal{F}$	all	no TPTP features	source	# terms	domain	rating	Vampire default single strategy
solved (of 3515)	2583	2548	2342	2180	2241	2166	2013

Solved problems

when predicting from single feature, source works best

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Feature importance (without rating)

1. # terms 6%
2. # unifiable pos/neg literals 4.7%
3. # variables 4.2%
4. # atoms 3.8%
5. # connectives 3.5%
6. # functions 3.4%
7. # terms unifiable with equations 3.4%
8. # negations 3.4%
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Side remark: regression quality $\neq$ prediction power

- for all features $r^2 = 0.71$, but source-only 0.28 and rating-only 0.41

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source	option value	advantage	surprise	#problems
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Sla93	`sa=fmb`	60%	82%	30
NV07a	`igrr=64/1`	67%	37%	72
WM89	`fmbsr=1.6`	63%	66%	20
Pel09	`age weight=16`	36%	53%	1017
ILTP	`uwa=all`	10%	56%	151

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# cnst > 1556	`age_weight=16`	22%	14%	722
# cnst > 1556	`ep=RST`	12%	21%	722

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vars/clause > 183	`stl`$\in \{150, 210\}$	20%	10%	178

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Overperformance

iProver, Z3, Zipperposition on `EPR`, versions of E on UEQ, ...

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Future investigations

- correlations for multiple features
- play with dimensionality reduction
- use such analysis to build good strategy schedules
- suggestions?