Monitoring Arithmetic Temporal Properties on Finite Traces

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37th AAAI Conference on Artificial Intelligence
10 February 2023, Washington, DC

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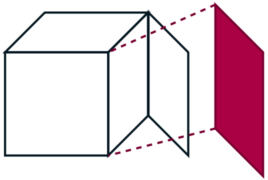
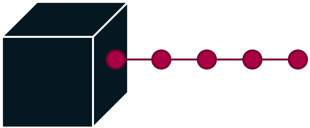
Checking properties of dynamic systems





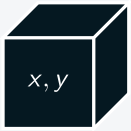
- system **fully known**, **specification** available
- analyze **all** executions, or all execution trees

analysis task:
model checking

- system **unknown**, or properties **inaccessible**
- analyze **running execution** and its possible continuations

analysis task:
monitoring

Overview



- can access finite set of numeric **process variables** V

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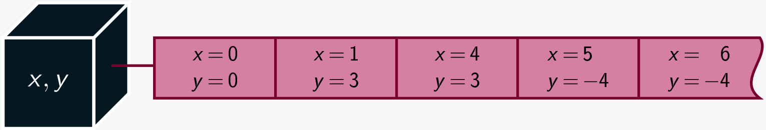

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- **trace** is finite sequence of assignments to V

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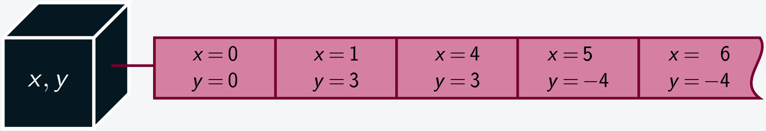

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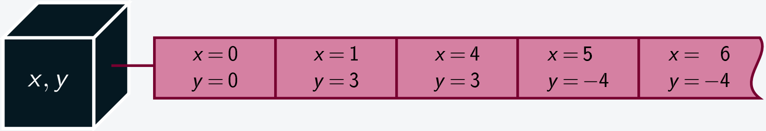

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“y is nonnegative until x is always greater than y”

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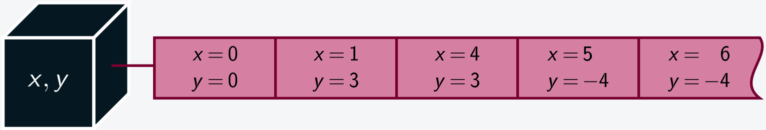

- $\boxed{\psi_1 = (y \geqslant 0)\ \mathsf{U}\ (\mathsf{G}(x > y))}$
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x' is value of x looking one trace instant ahead

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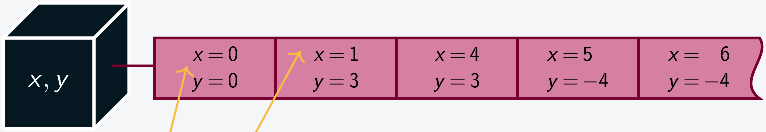

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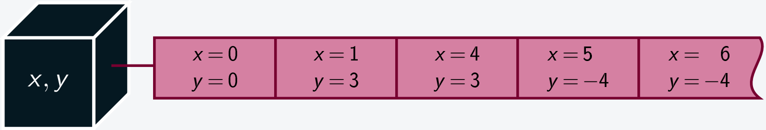

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"the current value of x is always less than the next one, and at some point x has value 2"

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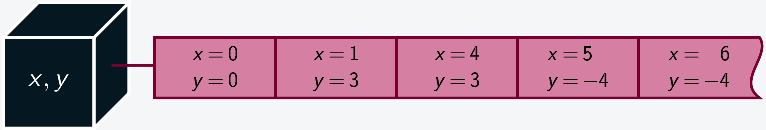

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- **anticipatory monitoring**: determine current and future satisfaction

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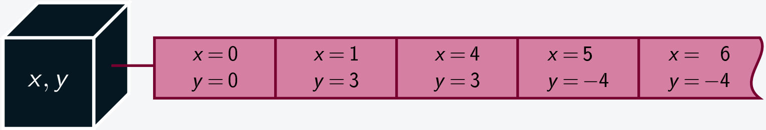
"ψ_1 holds but could get violated in the future"



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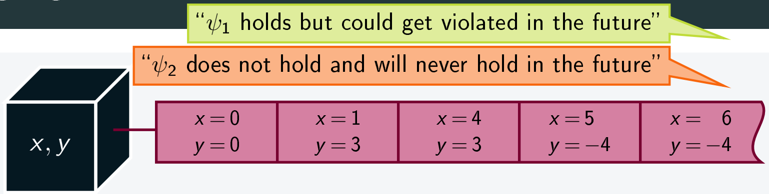

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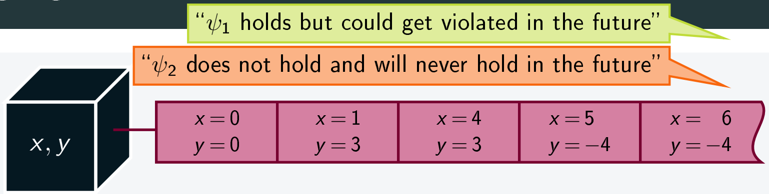

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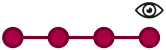
- what is **decidable/solvable**? how to construct **monitors**?

Anticipatory monitoring task

given trace and ALTL_f property, determine monitoring state [BLS2010]:

ps: permanent satisfaction



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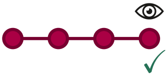

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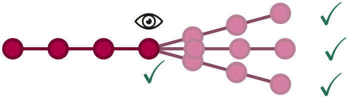

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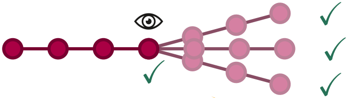

consider all finite continuations of unbounded length

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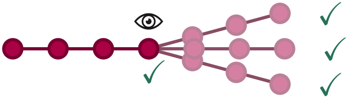
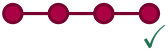


cs: current satisfaction



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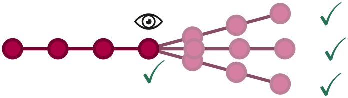

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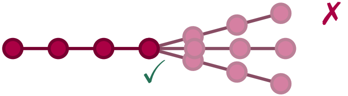

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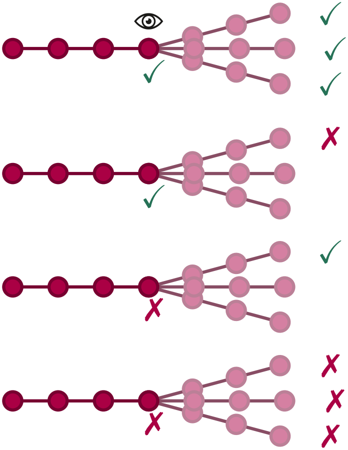
ps: permanent satisfaction

cs: current satisfaction

cv: current violation

pv: permanent violation



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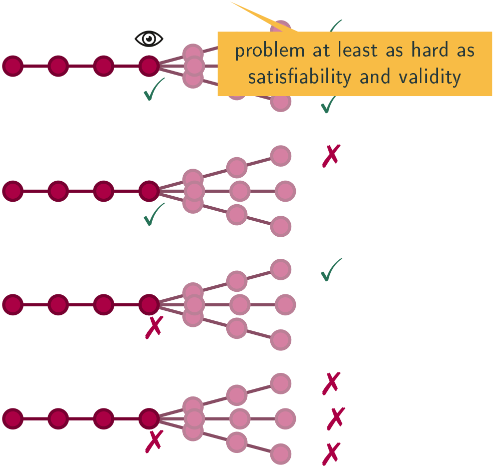

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Monitoring without lookahead

Theorem

monitoring of lookahead-free properties is solvable

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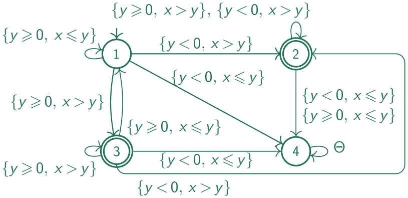
Example

- construct DFA for $\boxed{(y \geqslant 0)\ \mathsf{U}\ (\mathsf{G}(x > y))}$, treating constraints as propositions



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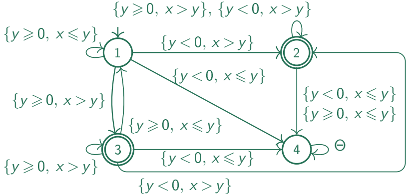

- every trace prefix leads to unique DFA state

$x = 0$ $y = 0$	$x = 1$ $y = 3$	$x = 4$ $y = 3$	$x = 5$ $y = 4$	$x = 6$ $y = -4$

A A C C B

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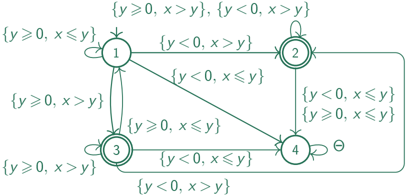

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Monitoring without lookahead

every DFA state q corresponds to unique monitoring state

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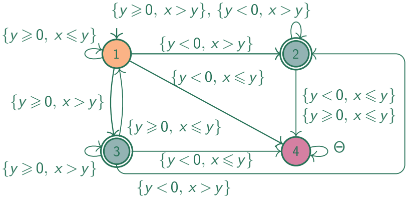

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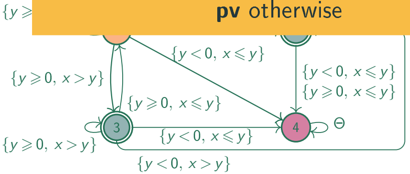
Example

- construct DFA for $(y$ … s

if q final:	**cs** if non-final state reachable from q	
	ps otherwise	
if q not final:	**cv** if final state reachable from q	
	pv otherwise	



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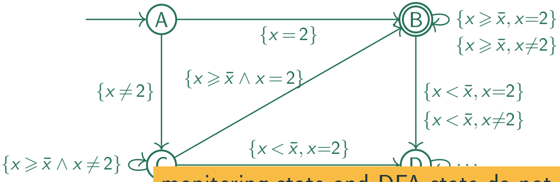
Example (DFAs are not monitors)

- DFAs construction for $\boxed{\mathsf{G}(x' > x) \wedge \mathsf{F}(x=2)}$



monitoring state and DFA state do not correspond

- sequence of monitoring states and DFA states

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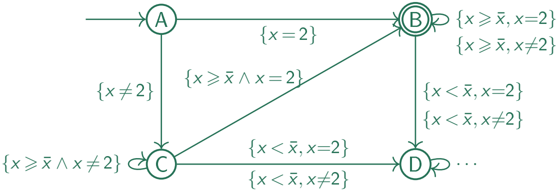

- sequence of monitoring states and DFA states

$x=0$	$x=1$	$x=4$	$x=5$	$x=6$
$y=0$	$y=3$	$y=3$	$y=4$	$y=-4$
cv C	**cv** C	**pv** C	**pv** C	**pv** C

Fact

Monitoring with lookahead is not solvable: reduction from reachability in 2CM

Monitoring with lookahead is not solvable

problem: state reachability depends on assignment

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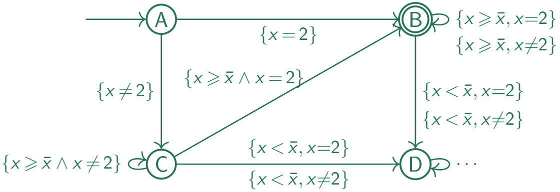

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given DFA state q reached by trace τ, find **condition** whether final DFA state is reachable from q after τ

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Approach: Symbolic finite state abstraction

- **history constraints** are constraints accumulated along paths in DFA:

$$h(\mathsf{A} \to \mathsf{C}) = (x = x_0 \wedge x \neq 2)$$
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by quantifier elimination

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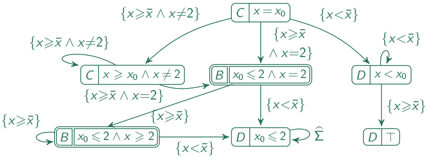
- **constraint graph** CG(q) represents history constraints for all paths from q



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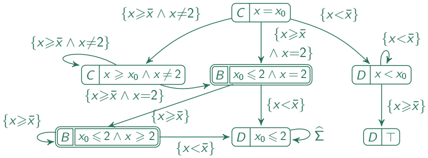

can be infinite

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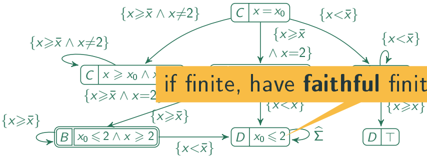
- **constraint graph** $\mathsf{CG}(q)$ represents history constraints for all paths from q

can be infinite



if finite, have **faithful** finite state abstraction

Aim

given DFA state q reached by trace τ, find **condition** whether final DFA state is reachable from q after τ

Approach: Symbolic finite state abstraction

- **history constraints** are constraints accumulated along paths in DFA:

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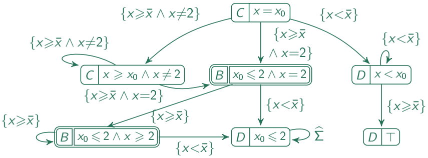

- formulas in final nodes of CG give **condition** for reachability of final DFA states: captured by $\mathsf{FSat}(\mathsf{CG}(q))$ (similarly $\mathsf{FUns}(\mathsf{CG}(q))$)

Monitoring procedure

all monitoring structures can be computed upfront
(DFA, CGs, FSat, FUns)

procedure $\textsc{Monitor}(\psi, \tau)$
  compute DFA for ψ
  $w \leftarrow$ word over constraints consistent with τ
  $q \leftarrow$ DFA state in such that $\{q_0\} \rightarrow^*_w q$
  $\alpha \leftarrow$ last assignment in τ
  if q accepting in DFA **then**
    return (**cs if** $\alpha \models$ FUns(CG(q)) **else ps**)
  else return (**cv if** $\alpha \models$ FSat(CG(q)) **else pv**)

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all monitoring structures can be computed upfront (DFA, CGs, FSat, FUns)

```
1: procedure MONITOR(ψ, τ)
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3:    w ← word over constraints consistent with τ
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5:    α ← last assignment in τ
6:    if q accepting in DFA then
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8:    else return (cv if α ⊨ FSat(CG(q)) else pv)
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Theorem (Correctness)

if $\textsc{monitor}(\psi, \tau) = s$ *then* s *is monitoring state for* ψ *and* τ

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does not terminate if CGs infinite

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previously used in context of model checking [FMW22]

Definition (Finite summary)

property ψ has finite summary if paths in DFA for ψ are covered by finitely many history constraints

[FMW22] P. Felli, M. Montali, S. Winkler. Linear-time verification of data-aware dynamic systems with arithmetic. AAAI-36(5), 5642-5650, 2022

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Theorem
monitoring task is solvable for any ψ that has finite summary, and MONITOR *is monitoring procedure*

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Concrete solvable property classes

Property classes that enjoy finite summary

- **monotonicity constraint** properties over $\mathbb{Q}$ or $\mathbb{Z}$ $\mathsf{G}(x' > x) \wedge \mathsf{F}(x = 2)$
  (all constraints are variable-to-variable/constant comparisons)

S. Demri and D. D'Souza: An automata-theoretic approach to constraint LTL. Inform. Comput., 205(3): 380-415, 2007.

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E. Damaggio, A. Deutsch and V. Vianu: Artifact systems with data dependencies and arithmetic. ACM Trans. Database Syst., 37(3): 22:1–22:36, 2012

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2. gener
   termi
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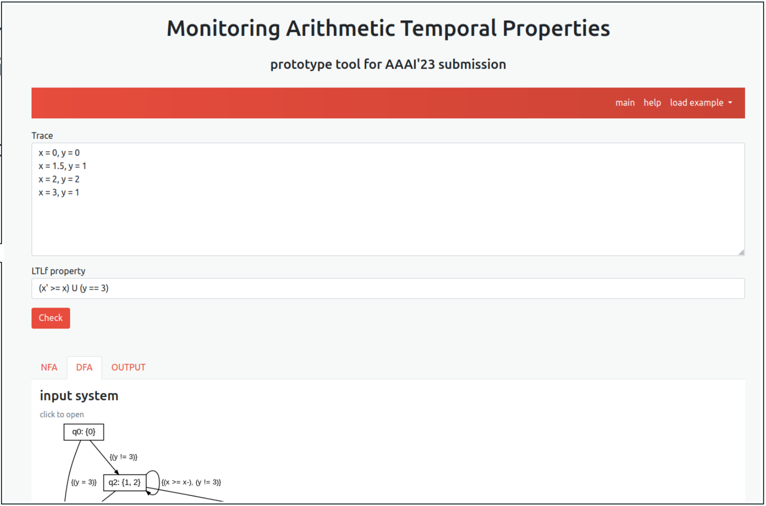

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Future work

- lift approach to richer properties equipped with full-fledged relations
- possibly study more general, controlled first-order quantification across time