DECIDABLE FRAGMENTS OF LTL$_f$ MODULO THEORIES

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The Big Picture

Satisfiability checking of $\mathrm{LTL}_f\mathrm{MT}$

$$
\begin{aligned}
& \mathit{dist} = 100 \wedge \\
& \mathsf{G}\,(\neg \mathit{near\_goal}(\mathit{dist}) \longrightarrow (\bigcirc\, \mathit{dist} < \mathit{dist})) \wedge \\
& \mathsf{F}\,(\mathit{goal\_reached})
\end{aligned}
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- LTL over finite traces with arbitrary decidable logical theories

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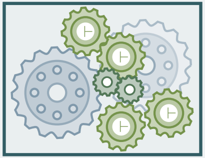

→ satisfiable

⇢ timeout

→ unsatisfiable

- LTL over finite traces with arbitrary decidable logical theories
- satisfiability can be checked by tableau method

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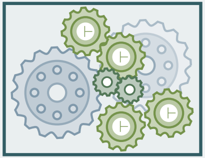

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- unsatisfiability can be rarely detected

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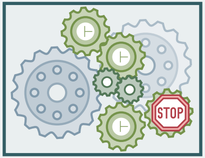

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This talk

- sound and complete PRUNE rule for tableau method

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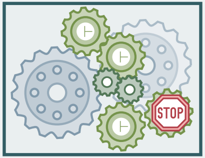

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- LTL over finite traces with arbitrary decidable logical theories
- satisfiability can be checked by tableau method
- unsatisfiability can be rarely detected

This talk

- sound and complete PRUNE rule for tableau method
- new, very general decidable fragments: generalizes results from literature

LTL_f modulo theories ($\mathsf{LTL}_f\mathsf{MT}$)

$$\varphi ::= \psi \mid \neg\varphi \mid \varphi_1 \vee \varphi_2 \mid \varphi_1 \wedge \varphi_2 \mid \mathsf{X}\,\varphi \mid \widetilde{\mathsf{X}}\,\varphi \mid \mathsf{F}\,\varphi \mid \mathsf{G}\,\varphi \mid \varphi_1\,\mathsf{U}\,\varphi_2$$

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- first-order atoms over arbitrary theories

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- existential and universal quantification

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- function application

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- first-order constants

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- quantified first-order variables z

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- first-order variables x — $x \in V$ for fixed, finite set of variables V

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- next-value variables $\bigcirc x$ — $x \in V$ for V fixed, finite set of data variables

  i.e. the value of x at the next state
- weak next-value variables

  i.e. the value of x at the next state, if it exists

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Examples

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  - variables V: *dist* (type *real*)   *goal-reached* (type *bool*)

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Examples

- robot motion planning
  - variables V: *dist* (type *real*) *goal-reached* (type *bool*)
  - $\mathit{dist} = 100$

initially, *dist* has value 100

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Examples

- robot motion planning
  - variables V: *dist* (type *real*) *goal-reached* (type *bool*)
  - $dist = 100 \wedge \mathsf{F}\,(goal\text{-}reached)$

at some point, *goal-reached* is true

LTL$_f$ modulo theories (LTL$_f$MT)

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Examples

- robot motion planning
  - variables V: *dist* (type *real*)   *goal-reached* (type *bool*)
  - $dist = 100 \wedge \mathsf{F}\,(goal\text{-}reached) \wedge \mathsf{G}\,(\neg near\text{-}goal(dist) \longrightarrow (\bigcirc\, dist < dist))$

while *dist* is not near goal, *dist* decreases monotonically

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Definitions

- trace is finite sequence of valuations of V

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Definitions

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  - variables V: dist (type *real*) $\mathit{goal\text{-}reached}$ (type *bool*)
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  - trace

$\mathit{goal\text{-}reached}=\mathit{false}$, $\mathit{dist}=10$	$\mathit{goal\text{-}reached}=\mathit{false}$, $\mathit{dist}=7.1$	$\mathit{goal\text{-}reached}=\mathit{false}$, $\mathit{dist}=5.6$	$\mathit{goal\text{-}reached}=\mathit{false}$, $\mathit{dist}=2.1$	$\mathit{goal\text{-}reached}=\mathit{false}$, $\mathit{dist}=0.01$

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Definitions

- trace is finite sequence of valuations of V
- LTL_fMT formula is satisfiable if $\exists$ first-order model and trace that satisfy it

Examples

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  - variables V: *dist* (type *real*)   *goal-reached* (type *bool*)
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goal-reached=*false* *dist*=10	*goal-reached*=*false* *dist*=7.1	*goal-reached*=*false* *dist*=5.6	*goal-reached*=*false* *dist*=2.1	*goal-reached*=*false* *dist*=0.01

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Examples

- robot motion planning
  - variables V: *dist* (type *real*) *goal-reached* (type *bool*)
  - $dist = 100 \wedge \mathsf{F}\,(goal\text{-}reached) \wedge \mathsf{G}\,(\neg near\text{-}goal(dist) \longrightarrow (\bigcirc\, dist < dist))$ satisfiable
  - trace

LRA + $near\text{-}goal_{\mathcal{M}} = \begin{cases} true & \text{if } dist \leqslant 0.01 \\ false & \text{otherwise} \end{cases}$

goal-reached=false *dist=10*	*goal-reached=false* *dist=7.1*	*goal-reached=false* *dist=5.6*	*goal-reached=false* *dist=2.1*	*goal-reached=false* *dist=0.01*

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Definitions

undecidable

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- can describe business processes that operate over data and access read-only database

Tableau method for LTL_fMT

- semi-decision procedure to check satisfiability
- can construct model if it exists
- implemented in LTL/LTL_f satisfiability checker BLACK, using SMT solver as backend

[Geatti, Gianola & Gigante, IJCAI 2022]

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[Geatti, Gianola & Gigante, IJCAI 2022]

Example (Tableau)

- $\psi := (x = 1) \wedge ((\bigcirc x > x) \mathsf{U} (x = 5))$

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$\{\psi\}$

root node labeled with ψ

...

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Example (Tableau)

- $\psi := (x = 1) \wedge ((\bigcirc x > x) \;\mathsf{U}\; (x = 5))$

$$\{\psi\}$$

rule to split conjunction

$$\{x = 1, (\bigcirc x > x) \;\mathsf{U}\; (x = 5)\}$$

...

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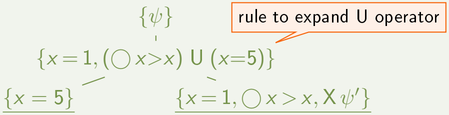

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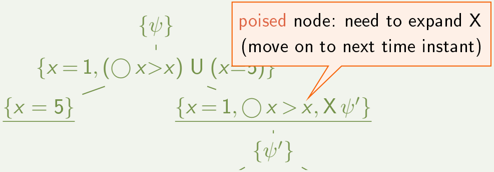

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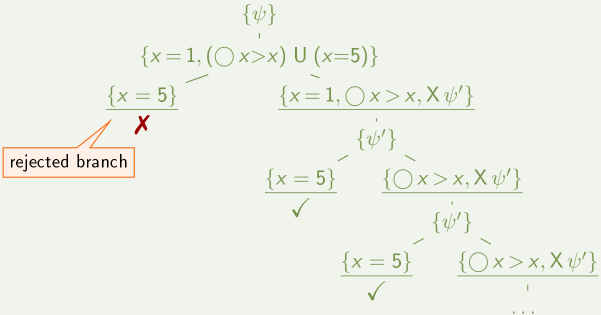

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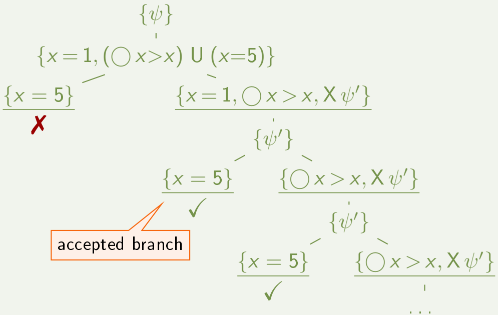

Tableau method for LTL$_f$MT

- semi-decision procedure to check satisfiability
- can construct model if it exists
- implemented in LTL/LTL$_f$ satisfiability checker BLACK, using SMT solver as backend

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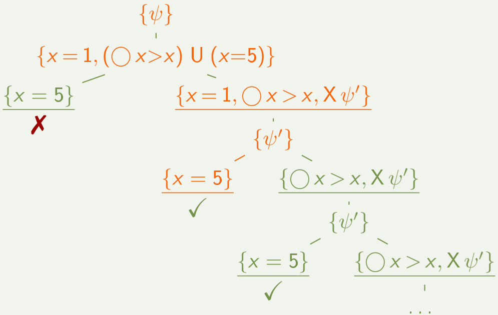

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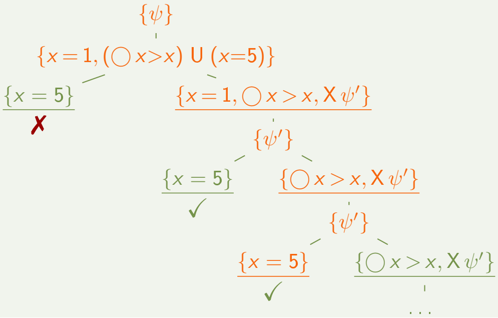
  - $\langle \{x = 1\}, \{x = 3\}, \{x = 5\} \rangle$



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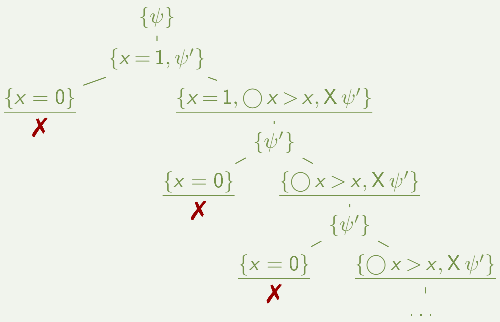

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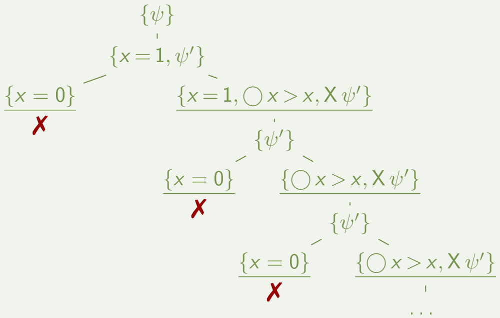
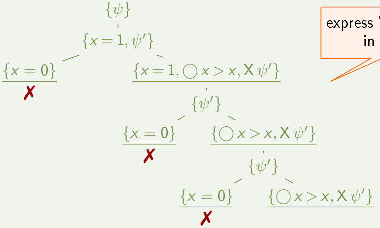
- tableau is infinite,
  no accepted branch



Key idea



express "situation of variables" in node, as formula

Definition (History constraints)

for tableau branch $\overline{\pi}$ where poised nodes have first-order formulas $F_0, \dots, F_{m-1}$,

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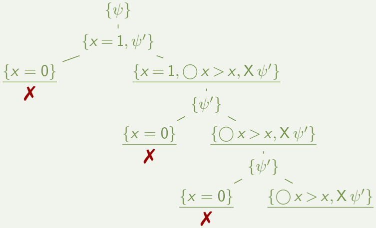

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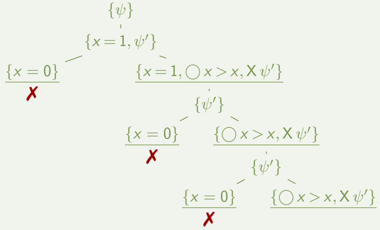

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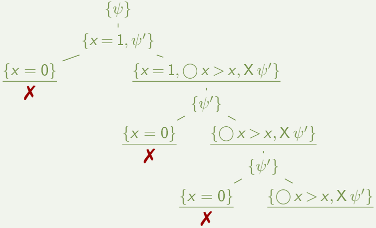

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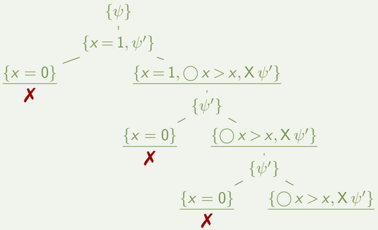

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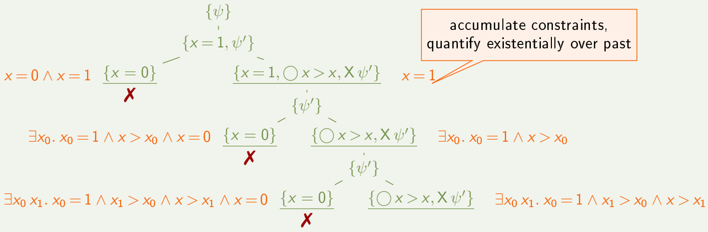

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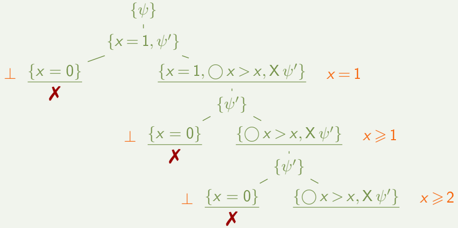

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for branch with poised nodes $\overline{\pi} = \langle \pi_0, \dots, \pi_i, \dots, \pi_{m-1} \rangle$, if π_i and π_{m-1} have same labels and $h(\overline{\pi}) \models_{\mathcal{T}} h(\overline{\pi}_{\leqslant i})$ then branch is rejected

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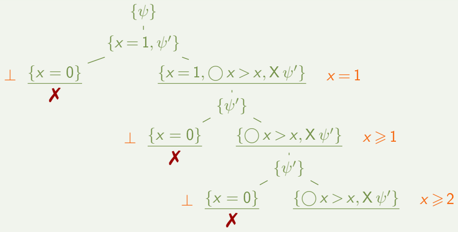

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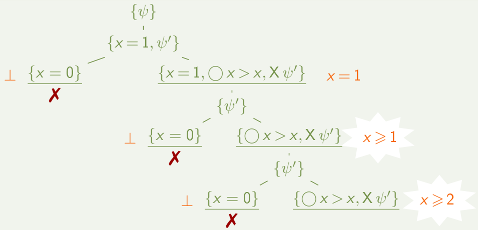

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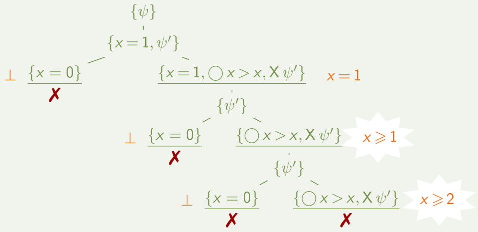

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Theorem (Soundness and completeness)

for LTL$_f$MT formula ψ, tableau with PRUNE rule has accepted branch iff ψ is satisfiable

Decidability Results

Observation: Satisfiability can be decidable

tableau with PRUNE halts on classes of formulas where set of history constraints is finite up to $\equiv$

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generalizes Demri & D'Souza (2007):
An automata-theoretic approach to constraint LTL

generalizes Demri (2006):
LTL over integer periodicity constraints

Conclusion

Summary

- tableau method to check satisfiability of LTL_f modulo theories: propose sound and complete PRUNE rule
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Future work

- implementation in BLACK
- in traces, allow interpretations of relations and functions to change over time
- extension to branching time logics
- LTL_fMT monitoring