Confluence by Decreasing Diagrams, Converted

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Theoretical Philosophy
Utrecht University
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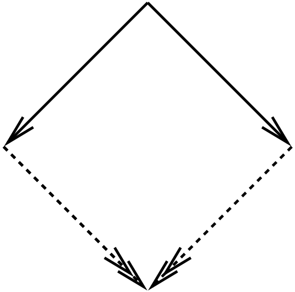
research conducted at LIX, Paris, the Republic of France

16:00 – 16:30, Thursday July 17, RTA 2008

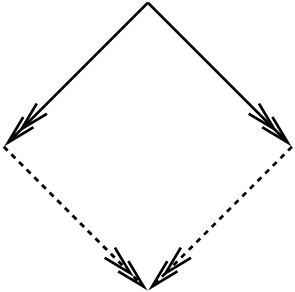
Lemma of Newman/Pous

Theorem (Newman 1942)

local confluence implies confluence, if $\rightarrow$ terminating



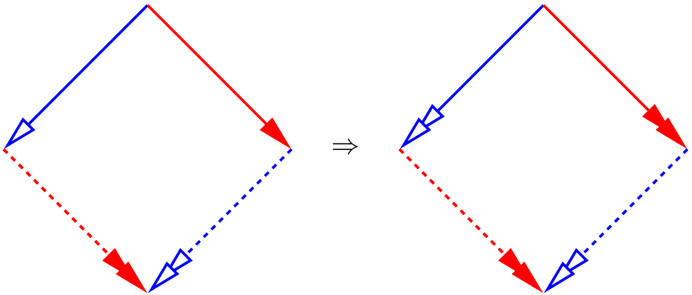
$\Rightarrow$



Lemma of Newman/Pous

Theorem (folklore ?)

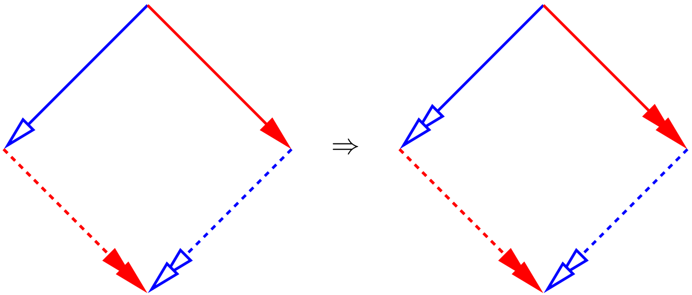
local commutation implies commutation, if $\triangleright \cup \blacktriangleright$ *terminating*



Lemma of Newman/Pous

Theorem (Pous 2007)

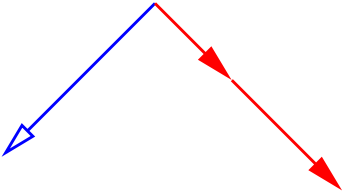
local commutation implies commutation, if $\triangleright^+ ; \blacktriangleright^+$ *terminating*



Lemma of Newman/Pous

Proof.

intuition: tiling terminates since splitting bounded by termination.



repeat: fill in local peak with local diagram □

Lemma of Newman/Pous

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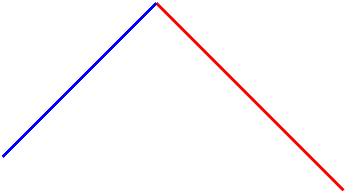

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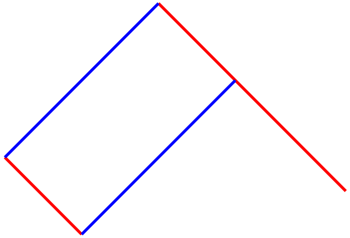

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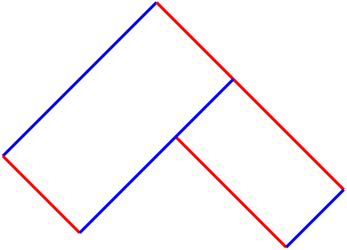

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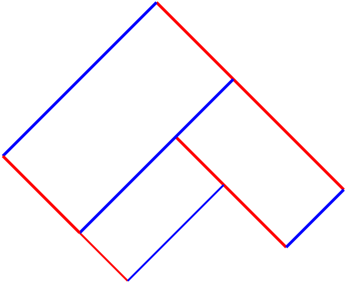

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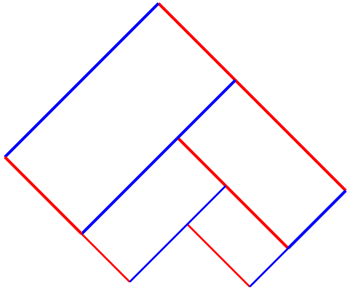

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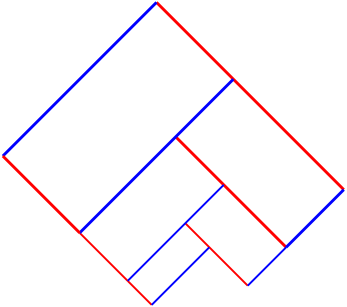

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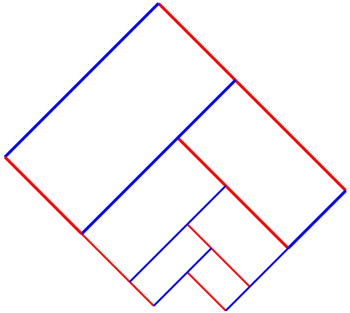

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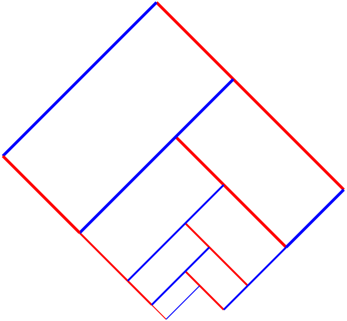

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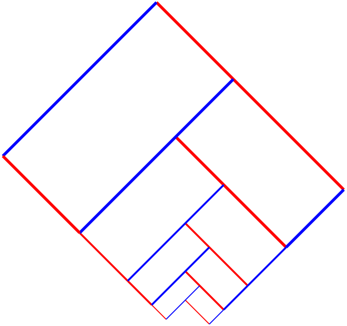

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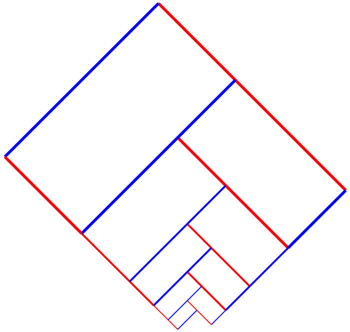

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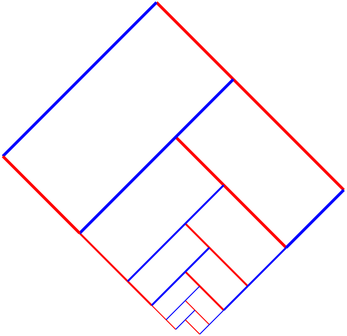

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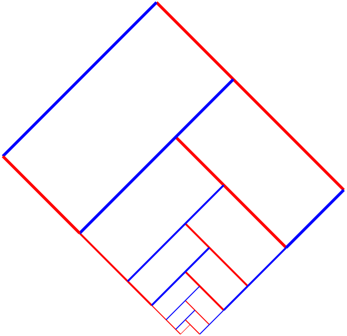

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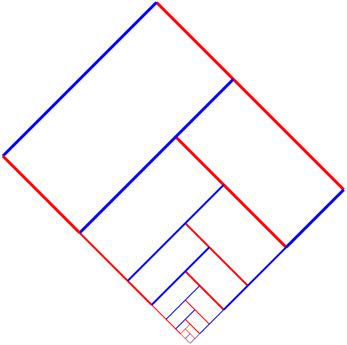

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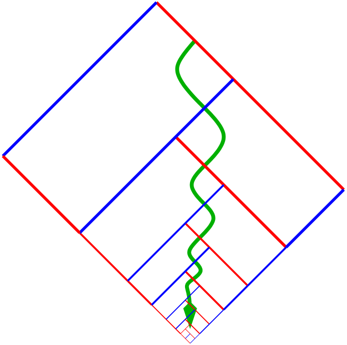
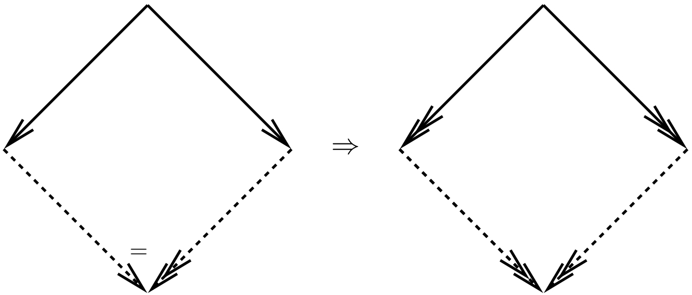


must stop: infinite tiling $\Rightarrow$ infinite $\triangleright^+ ; \blacktriangleright^+$ reduction. $\square$

Lemma of Hindley/uet

Theorem (Huet 1980)

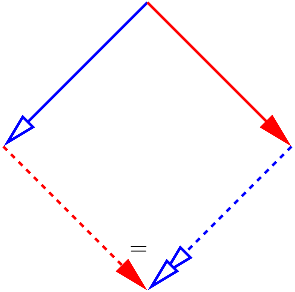
strong confluence implies confluence



Lemma of Hindley/uet

Theorem (Hindley 1964)

strong commutation implies commutation



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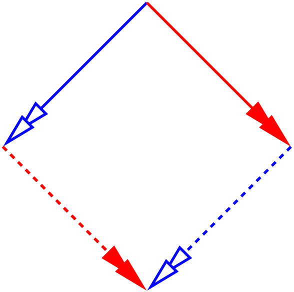
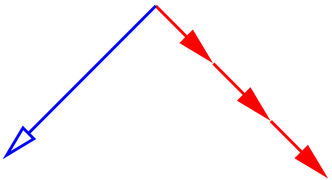


Lemma of Hindley/uet

Proof.

intuition: tiling terminates since only $\triangleright$ steps are split



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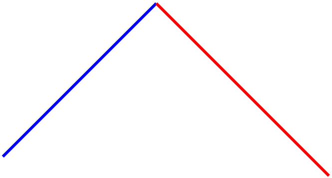

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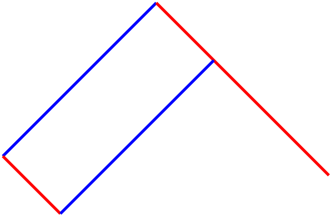

repeat: fill in local peak with local diagram $\square$

Lemma of Hindley/uet

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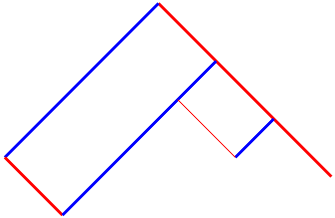

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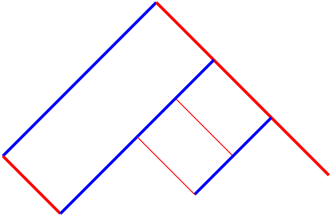
Lemma of Hindley/uet

Proof.

intuition: tiling terminates since only ▷ steps are split



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Lemma of Hindley/uet

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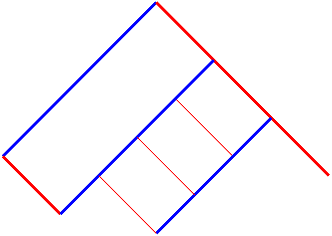

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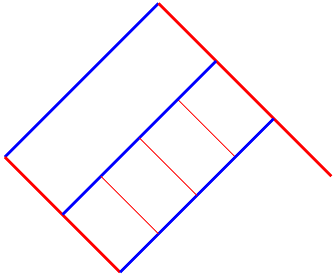

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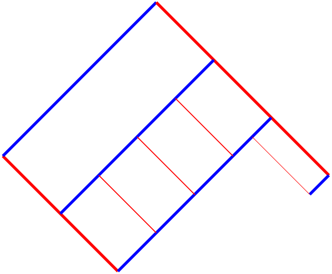

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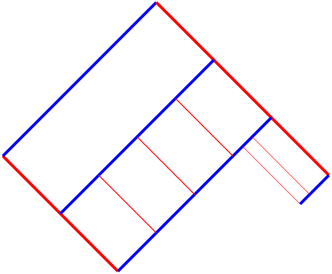

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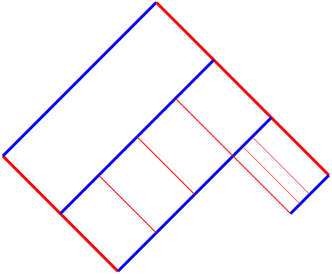

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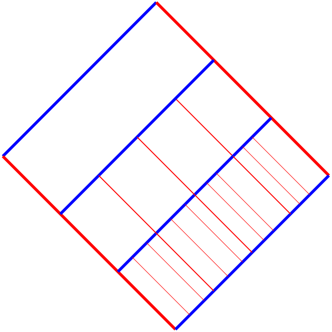

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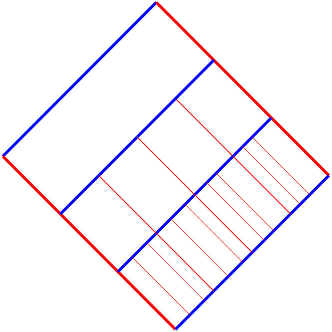

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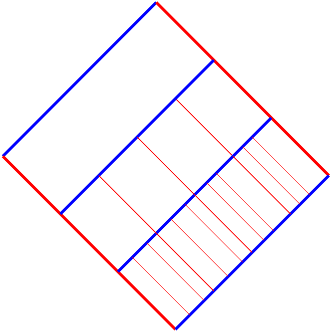


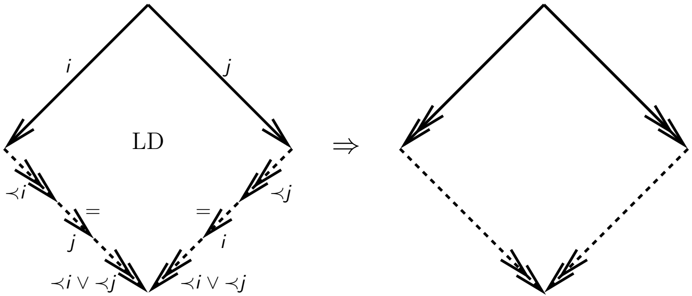
must stop: each ▶ stripe is eventualy filled □

Unify Newman/Pous with Hindley/uet?

Decreasing Diagrams (trough version)

Theorem (de Bruijn 1978,vO 1994)

locally decreasing implies confluence



$\rightarrow = \bigcup_{i \in I} \rightarrow_i$, $\prec$ *well-founded order on* I

Decreasing Diagrams (trough version)

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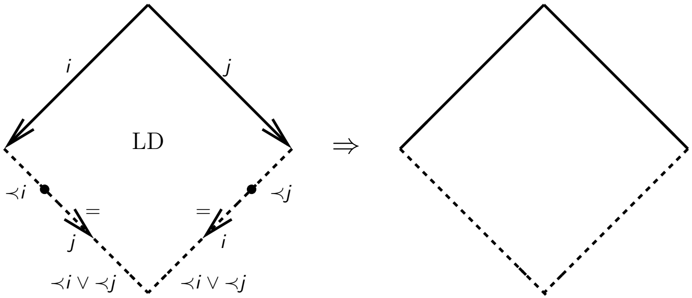

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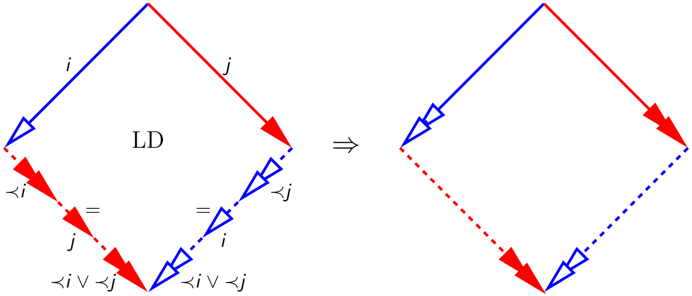
Decreasing Diagrams (trough version)

Theorem (vO 1994)

locally decreasing implies commutation



$\triangleright = \bigcup_{i \in I} \triangleright_i$, $\blacktriangleright = \bigcup_{j \in J} \blacktriangleright_j$, $\prec$ *well-founded order on* $I \cup J$

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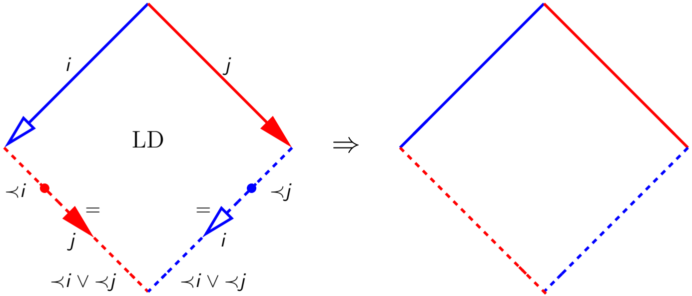

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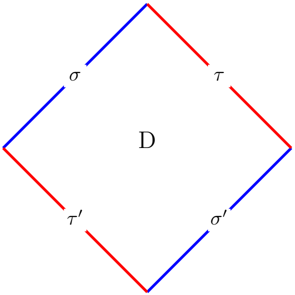
Decreasing Diagrams (trough version)

Proof.

by decreasingness



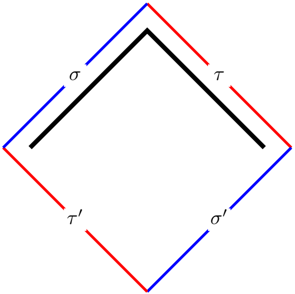
peak σ,τ as large as lhs $\sigma\tau'$ and rhs $\tau\sigma'$ after filtering

□

Decreasing Diagrams (trough version)

Proof.

by decreasingness



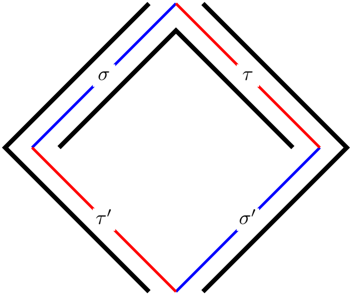
measure peak by multiset sum $|\sigma| \uplus |\tau|$

$|\_|$ filters smaller labels to right, $|32343| = [3, 3, 4]$ $\square$

Decreasing Diagrams (trough version)

Proof.

by decreasingness

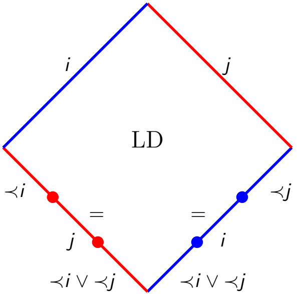


decreasing if $|\sigma| \uplus |\tau|$ as large as both $|\sigma\tau'|$ and $|\tau\sigma'|$ in multiset-extension of $\prec$ □

Decreasing Diagrams (trough version)

Proof.

(1) locally decreasing $\Rightarrow$ decreasing



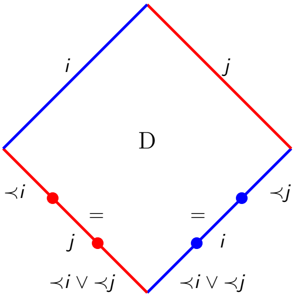
peak $|i| \uplus |j|$

lhs $|i(\prec i)^*(j+\varepsilon)(\prec i + \prec j)^*|$ □

Decreasing Diagrams (trough version)

Proof.

(1) locally decreasing $\Rightarrow$ decreasing



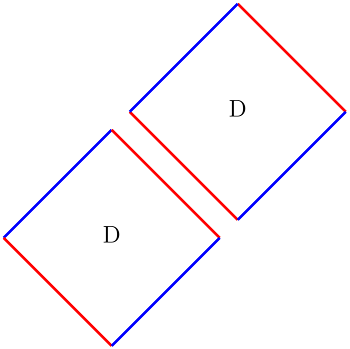
$|i| \uplus |j|$ is $[i] \uplus [j] = [i,j]$

$|i(\prec i)^*(j+\varepsilon)(\prec i + \prec j)^*|$ is $[i]$, $[i,j]$ or $[i, j_1, \ldots, j_n]$ □

Decreasing Diagrams (trough version)

Proof.

(2) decreasingness preserved under pasting

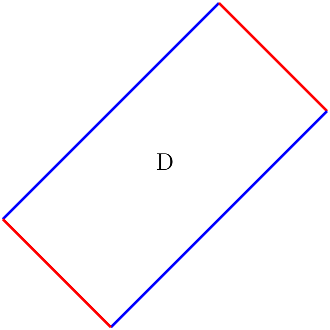


□

Decreasing Diagrams (trough version)

Proof.

(2) decreasingness preserved under pasting on left



□

Decreasing Diagrams (trough version)

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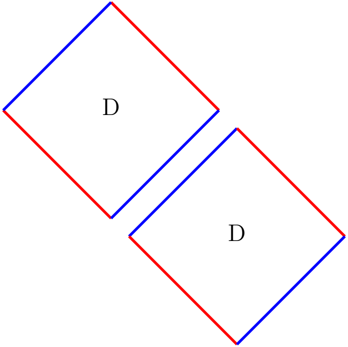
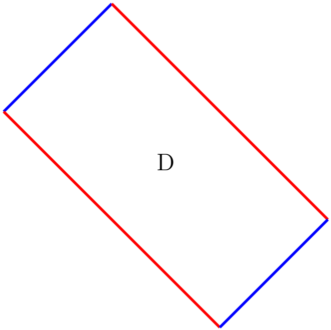


□

Decreasing Diagrams (trough version)

Proof.

(2) decreasingness preserved under pasting on right

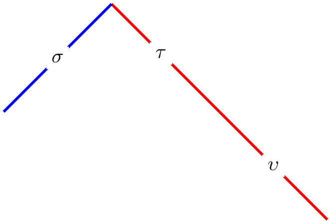


□

Decreasing Diagrams (trough version)

Proof.

(3) filling with decreasing diagram decreases measure



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Decreasing Diagrams (trough version)

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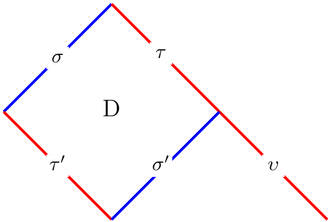

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Decreasing Diagrams (trough version)

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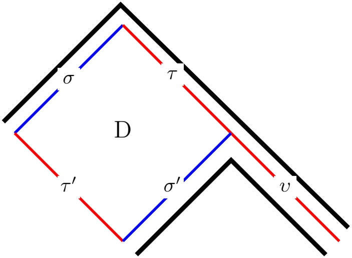
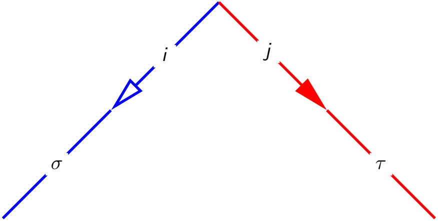


$|\sigma| \uplus |\tau\upsilon|$ greater than $|\sigma'| \uplus |\upsilon|$ □

Decreasing Diagrams (trough version)

Proof.

LD $\Rightarrow$ D by (1) assumption, (2) pasting, and (3) filling



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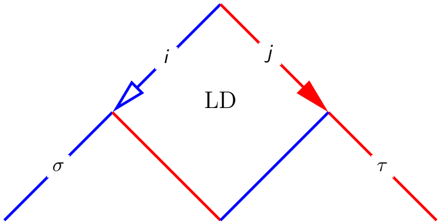
Decreasing Diagrams (trough version)

Proof.

LD ⇒ D by (1) assumption, (2) pasting, and (3) filling



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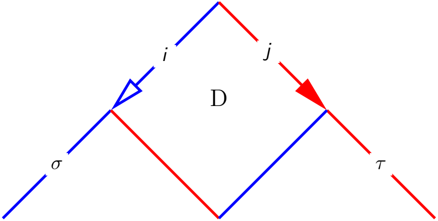

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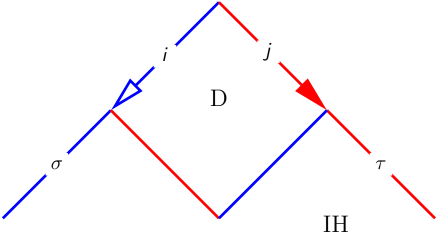
Proof.

LD $\Rightarrow$ D by (1) assumption, (2) pasting, and (3) filling

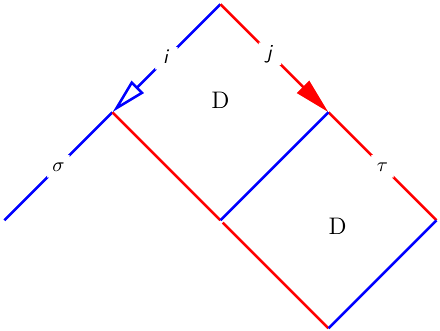


□

Decreasing Diagrams (trough version)

Proof.

LD ⇒ D by (1) assumption, (2) pasting, and (3) filling



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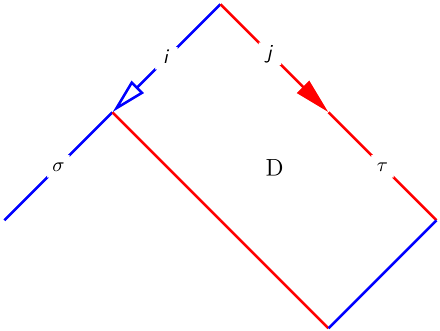

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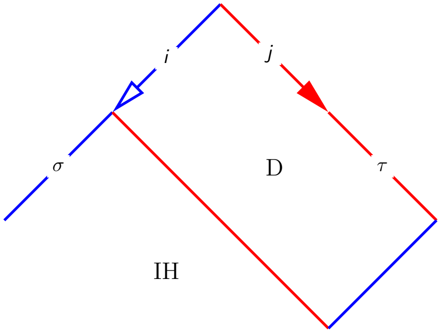

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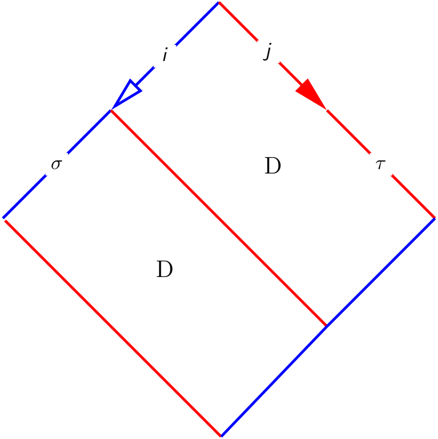

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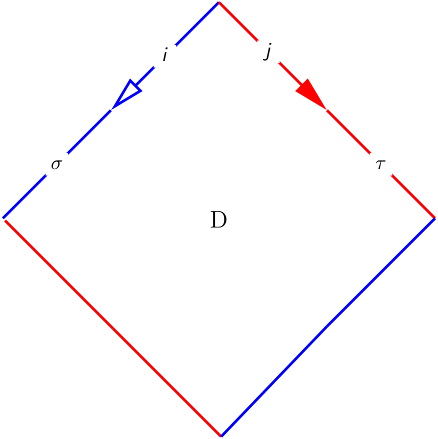


□

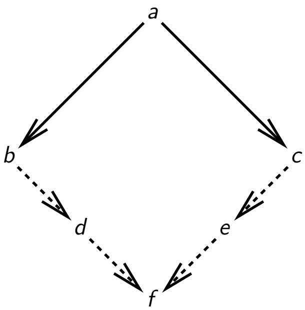
Unifies Newman/Pous with Hindley/uet!

Lemma of Newman/Pous by decreasingness

Lemma of Newman/Pous by decreasingness

Proof.

local confluence $\Rightarrow$ confluence, if $\rightarrow$ terminating



label steps by their source, order labels by $\rightarrow^+$ $\square$

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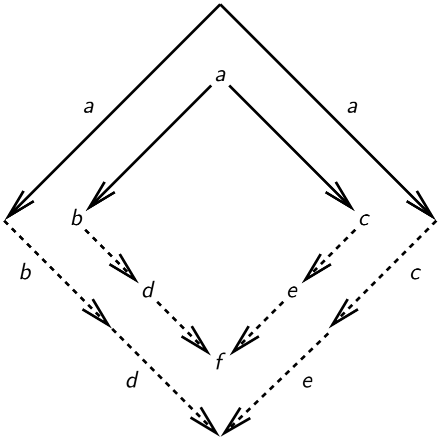

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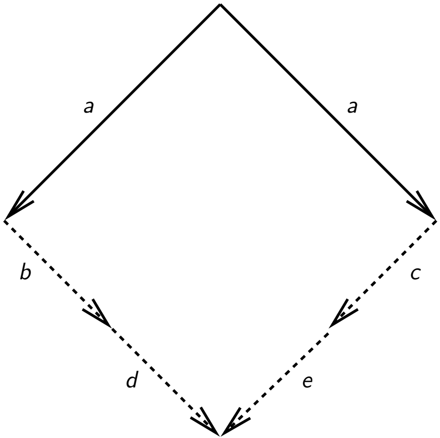

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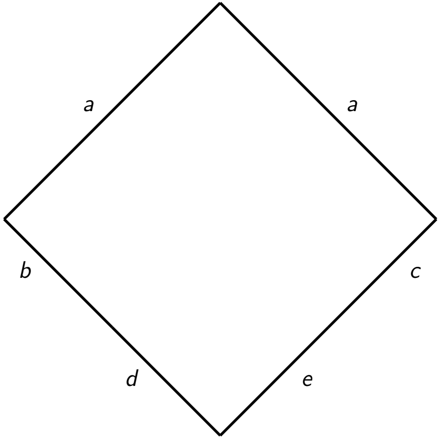

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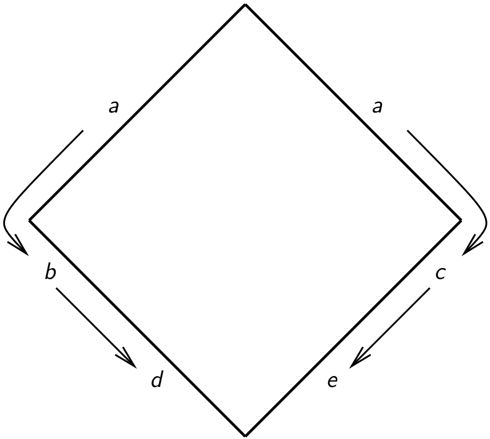

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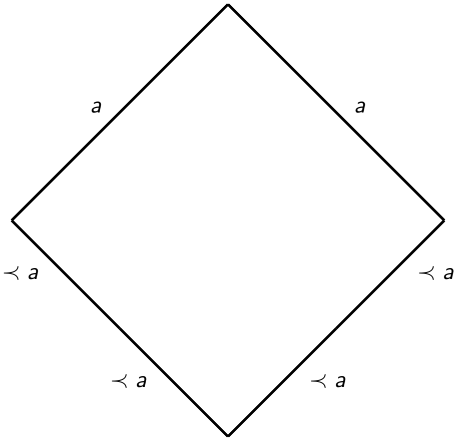

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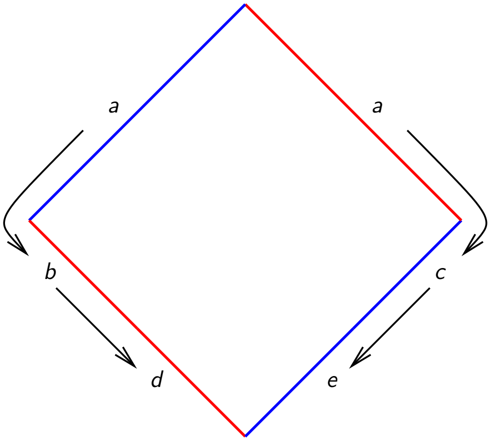

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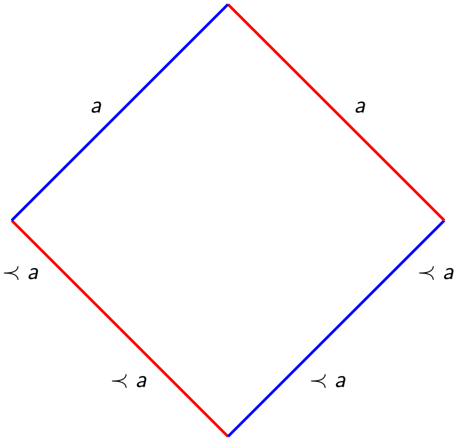

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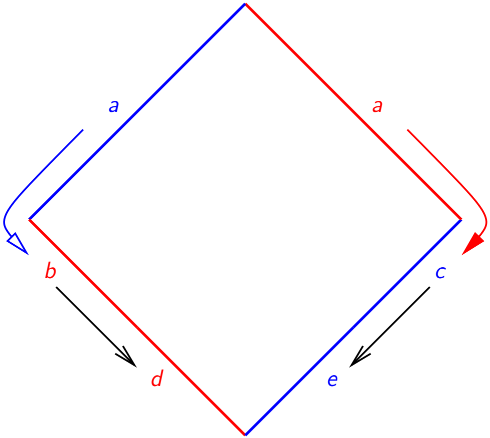

label by colored source, $x \succ y$ if $x\ (\triangleright \cup \blacktriangleright)^+\ y$ with $\triangleright$-step $\square$

Lemma of Newman/Pous by decreasingness

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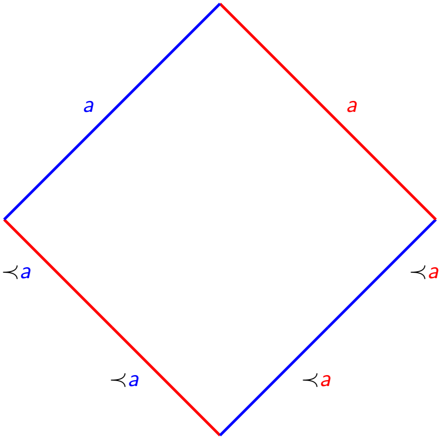

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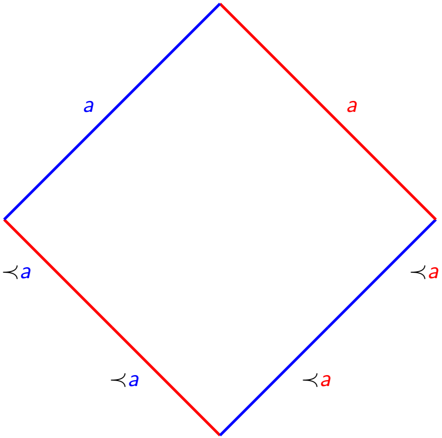

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Lemma of Hindley/uet by decreasingness

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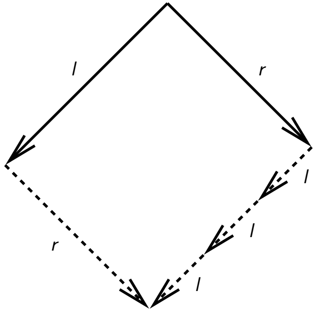
Proof.

strong confluence ⇒ confluence



label steps by their direction (l or r), order r above l □

Lemma of Hindley/uet by decreasingness

Proof.

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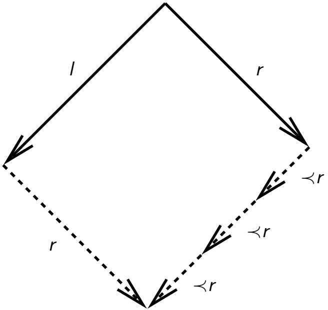

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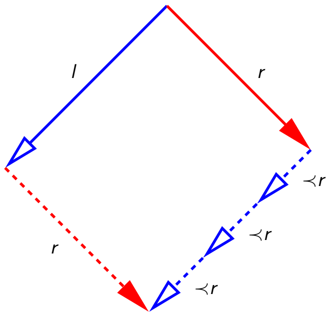

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More than Newman/Pous $\cup$ Hindley/uet...

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1. stronger properties (than confluence/commutation)

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2. troughs $\rightsquigarrow$ seascapes (more general local diagrams)

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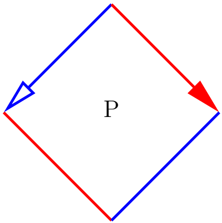
Today:

1. stronger properties (than confluence/commutation)
2. troughs $\rightsquigarrow$ seascapes (more general local diagrams)
3. heuristics (for constructing labels)

(1) Stronger properties

Theorem

If P holds locally and preserved by pasting, then holds for all D.



holds locally

(1) Stronger properties

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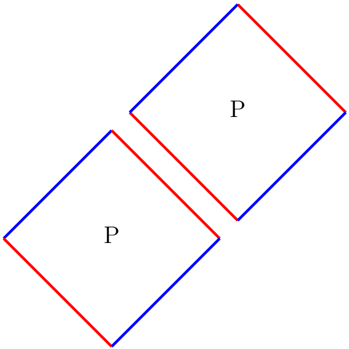

preserved by pasting on left

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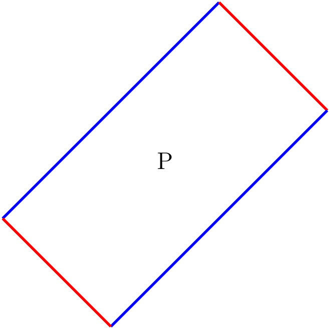

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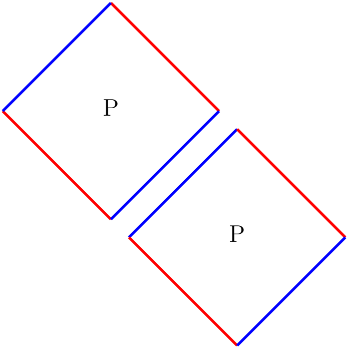

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Theorem

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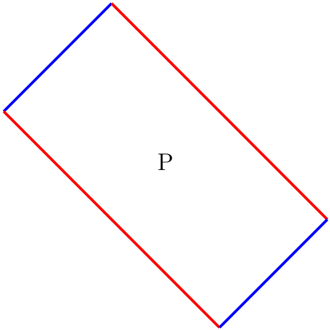

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(1) Stronger properties

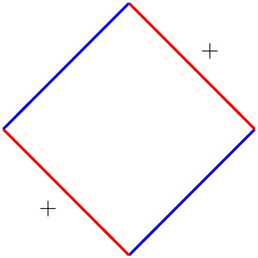
Proof.

trivial from decreasing diagrams proof. load induction with P. □

(1) Stronger properties

Example

If local diagrams are decreasing with non-empty ▶ $(+)$, then ▷ commutes with non-empty ▶ $(+)$.

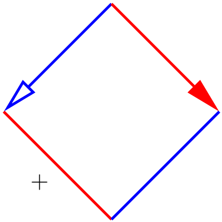


(1) Stronger properties

Example

If local diagrams are decreasing with non-empty $\blacktriangleright$ $(+)$, then $\triangleright$ commutes with non-empty $\blacktriangleright$ $(+)$.

checking $(+)$ locally suffices:

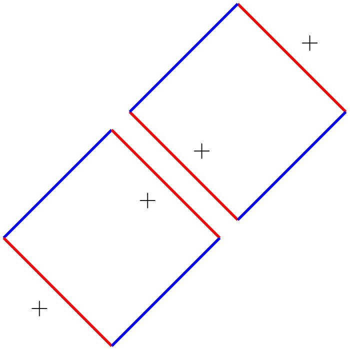


(1) Stronger properties

Example

If local diagrams are decreasing with non-empty $\blacktriangleright$ $(+)$, then $\triangleright$ commutes with non-empty $\blacktriangleright$ $(+)$.

$(+)$ preserved by pasting on left:



(1) Stronger properties

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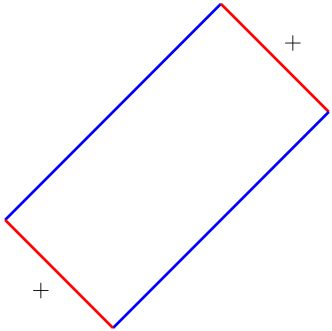

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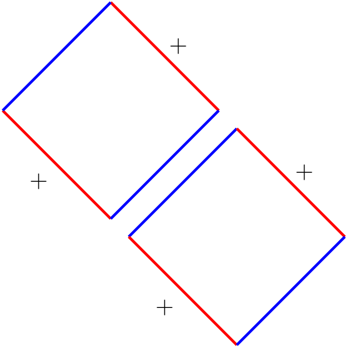

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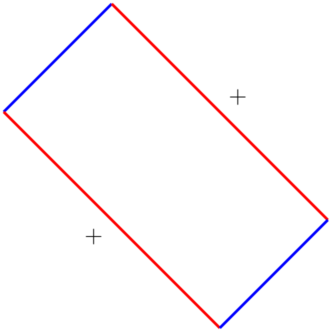

(2) Trough $\rightsquigarrow$ seascape

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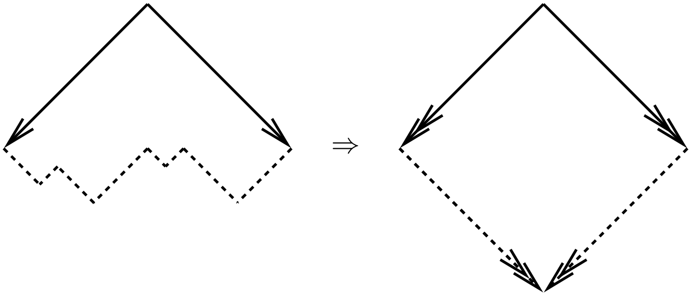
from Newman's Lemma (trough) to (seascape)

Theorem (Winkler & Buchberger 1983)

local confluence below $\Rightarrow$ *confluence, if* $\rightarrow$ *terminating*



Definition

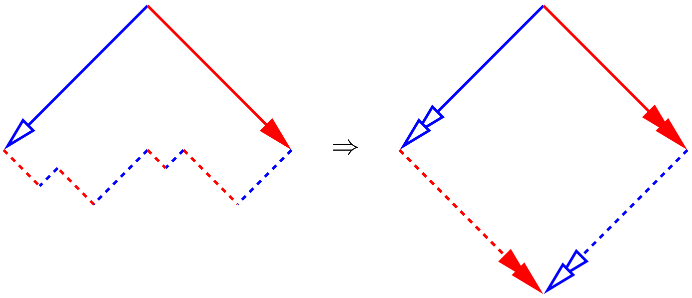
below: all objects in seascape $\rightarrow^+$-reachable from top

(2) Trough ⇝ seascape

from folklore lemma (trough) to (seascape)

Theorem

local commutation below ⇒ commutation, if $\triangleright \cup \blacktriangleright$ terminating



Definition

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(2) Trough $\rightsquigarrow$ seascape

from Pous' Lemma (trough) to (seascape)

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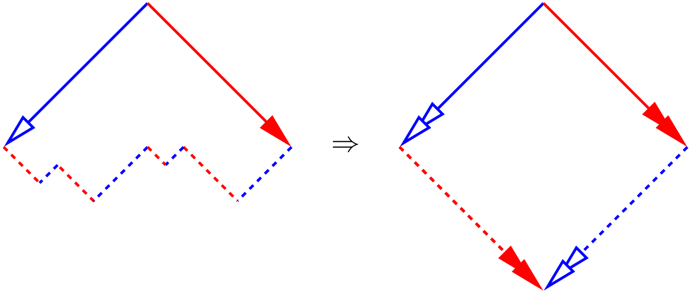

Definition

below: if $a \triangleright b$ in seascape, a $(\triangleright \cup \blacktriangleright)^+$-reachable from top with $\blacktriangleright$

(2) Trough $\rightsquigarrow$ seascape

from Pous' Lemma (trough) to (seascape)

Theorem

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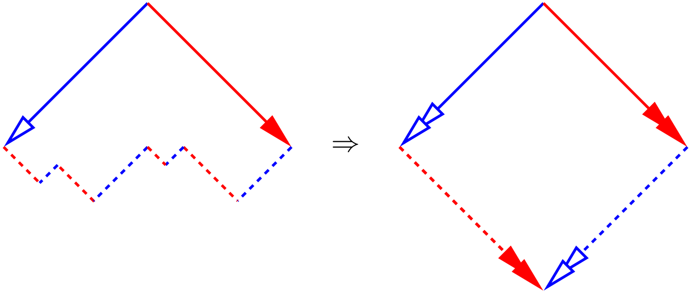


Definition

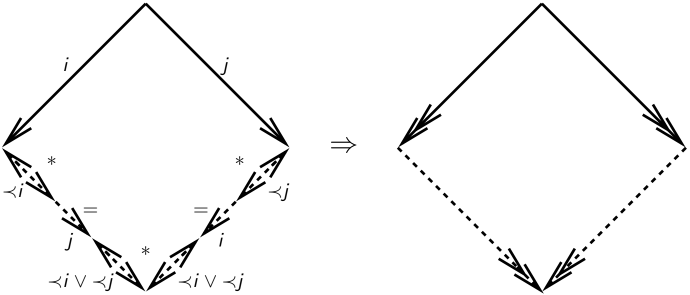
below: if $a \blacktriangleright b$ in seascape, a $(\triangleright \cup \blacktriangleright)^+$-reachable from top with $\triangleright$

(2) Trough $\rightsquigarrow$ seascape

from decreasing diagrams (trough) to (seascape)

Theorem

locally decreasing seascape $\Rightarrow$ confluence



$\to = \bigcup_{i \in I} \to_i$, $\prec$ *well-founded order on I*

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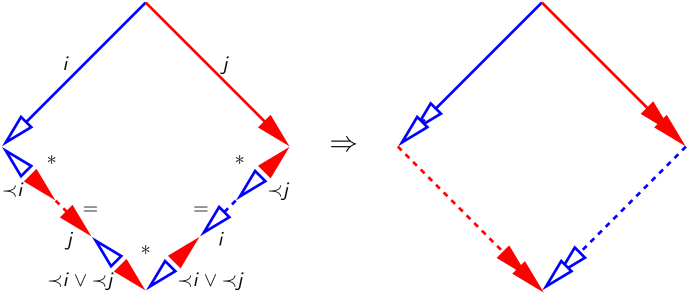
from decreasing diagrams (trough) to (seascape)

Theorem

locally decreasing seascape $\Rightarrow$ commutation

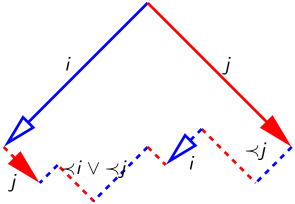


$\triangleright = \bigcup_{i \in I} \triangleright_i$, $\blacktriangleright = \bigcup_{j \in J} \blacktriangleright_j$, $\prec$ *well-founded order on* $I \cup J$

(2) Trough $\rightsquigarrow$ seascape

Proof.

same measure of peaks, but local peak may not be base case

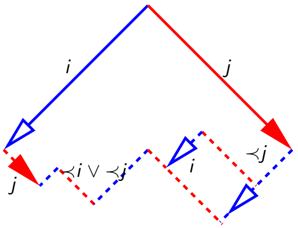


□

(2) Trough $\rightsquigarrow$ seascape

Proof.

but its peaks can be filled in by induction



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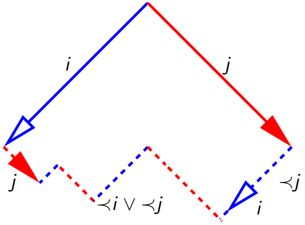

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(2) Trough ⇝ seascape

Proof.

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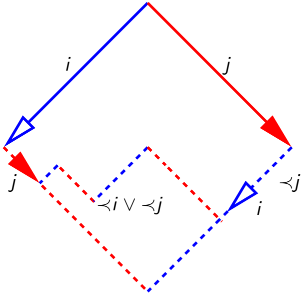
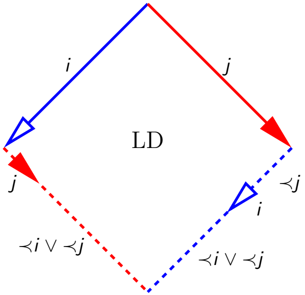


□

(2) Trough $\rightsquigarrow$ seascape

Proof.

giving in the end a (trough) locally decreasing diagram



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(2) Trough $\rightsquigarrow$ seascape

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- Newman $\Leftrightarrow$ Winkler & Buchberger (idem for variants)
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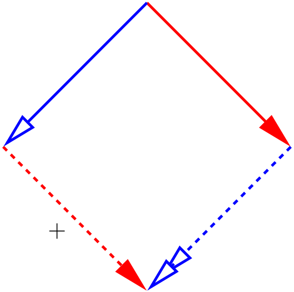
- covers all 'local $\ldots \Rightarrow$ confluence' results in Terese Chapter 1
- handy heuristic: self-labelling (label steps by themselves) allows to transfer wfo on objects to wfo on steps

(2) Trough $\rightsquigarrow$ seascape

trough version:

Theorem (Geser)

commutation holds, if ▶ *terminating and*

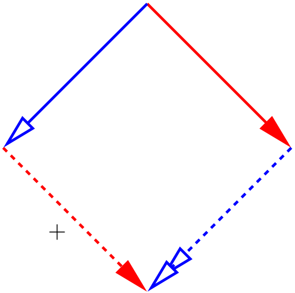


(2) Trough $\rightsquigarrow$ seascape

equivalent to (Bachmair & Dershowitz):

Theorem (Geser)

commutation holds, if $\blacktriangleright / \triangleleft \, (= \triangleleft\!\!\triangleleft \, ; \blacktriangleright \, ; \triangleleft\!\!\triangleleft)$ *terminating and*



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seascape version:

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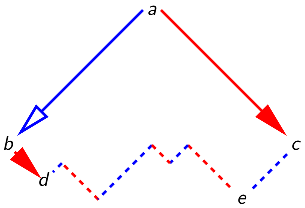

(2) Trough ⇝ seascape

Theorem (Geser)

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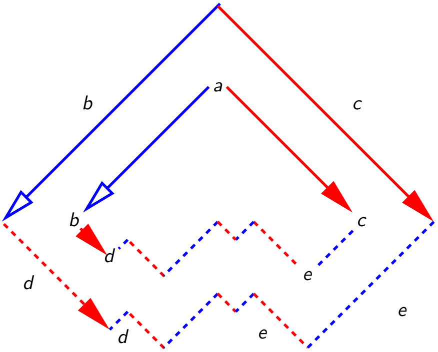
Proof.



label steps by target (heuristic), order by ▶/◁. □

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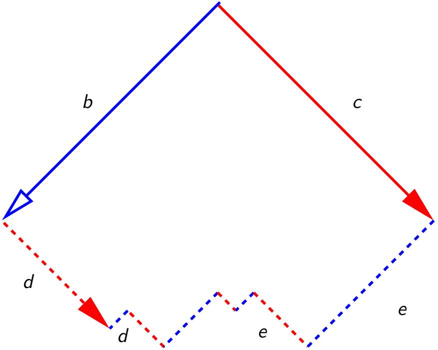

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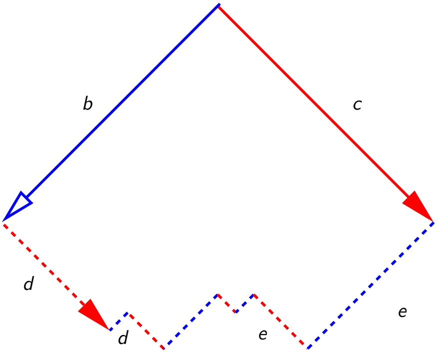

all labels in seascape $\blacktriangleright$/$\triangleleft$-reachable from (label of) $\triangleright$-step. $\square$

(1) Stronger properties

from (trough) to (seascape)

Theorem

If P holds locally and preserved by pasting, then holds for all D with now pasting also inside seascapes!

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- will yield: all maximal reductions have same length

(3) Heuristics

- label step by rule-name in a term rewriting system

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Theorem

Linear TRS is confluent, if critical peaks are locally decreasing.

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Example (Gramlich & Lucas)

1. $\mathsf{nats} \to 0 : \mathsf{inc}(\mathsf{nats})$
2. $\mathsf{inc}(x : y) \to \mathsf{s}(x) : \mathsf{inc}(y)$
3. $\mathsf{hd}(x : y) \to x$
4. $\mathsf{tl}(x : y) \to y$
5. $\mathsf{inc}(\mathsf{tl}(\mathsf{nats})) \to \mathsf{tl}(\mathsf{inc}(\mathsf{nats}))$

one critical peak

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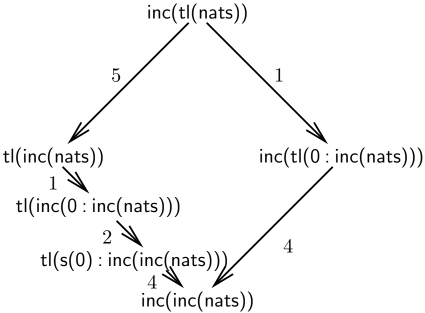

easy to order rule-symbols for decreasingness (like for RPO)

Concluding remarks

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- do other proofs of confluence by decreasing diagrams generalise (naturally)?