Towards Complexity Analysis of Programs by Rewriting

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Example

```
public class Flatten {
  public static IntList flatten(TreeList list) {
    TreeList cur = list; IntList result = null;
    while (cur != null) {
      Tree tree = cur.value;
      if (tree != null) {
        IntList oldIntList = result;
        result = new IntList();
        result.value = tree.value;
        result.next = oldIntList;
        TreeList oldCur = cur;
        cur = new TreeList();
        cur.value = tree.left;
        cur.next = oldCur;
        oldCur.value = tree.right;
      } else cur = cur.next;
  } return result;
}
```

Howto Analyse the Complexity of `flatten` Automatically?

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- automated technique to prove termination of JBC programs
- transformation of JBC programs (via termination graphs) into TRSs

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Example

consider the TRS $\mathcal{R}_{\text{flatten}}$

$$\mathsf{f}(\mathsf{g}(0,x),\mathsf{g}(0,x),y) \to \mathsf{f}(\mathsf{g}(0,x),x,y)$$
$$\mathsf{f}(\mathsf{g}(\mathsf{h}(x,y,z),u),\mathsf{g}(\mathsf{h}(x,y,z),u),v) \to \mathsf{f}(\mathsf{g}(z,u),\mathsf{g}(y,\mathsf{g}(z,u)),\mathsf{i}(x,v))$$
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Result has been presented at RTA

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Example (cont'd)

re-consider $\mathcal{R}_{\mathsf{flatten}}$

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- call $\mathsf{T}_{\mathcal{C}}\mathsf{T}$ on $\mathcal{R}_{\mathsf{flatten}}$

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- call TCT on $\mathcal{R}_{\text{flatten}}$
- TCT answers YES$(?, O(n^1))$

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- linear upper-bound on complexity of `flatten` automatically verified

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\end{aligned}$$

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Assessment

so, let's see ...

- linear upper-bound on com[illegible]ten automatically verified
- but is this certificate really correct?

Challenge ①

is the runtime of `flatten` faithfully represented by the derivation height wrt its TRS representation?

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is the analysis extendable to TRSs with fresh variables?

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and what about automation?

is the analysis extendable to ITRSs with fresh variables?

Overview

- Preliminaries
- Termination Analysis of Java Programs
- Challenges
  - Challenge ①
  - Challenge ②
  - Challenge ③
  - Challenge ④
- Main Result

Complexity of Term Rewrite Systems

Definition

the derivation height of a term t wrt to T and $\to$

$$\mathrm{dh}(t, \to) = \max\{n \mid \exists u\ t \to^n u\}$$

$$\mathrm{dh}(n, T, \to) = \max\{\mathrm{dh}(t, \to) \mid \exists t \in T \text{ and } |t| \leqslant n\}$$

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Definition

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How To Analyse Complexity

$$t_1 \to_{\mathcal{R}} t_2 \to_{\mathcal{R}} t_3 \to_{\mathcal{R}} \dots$$

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1. $\exists$ termination technique such that
2. termination of $\mathcal{R}$ is certified

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termination techniques can be used to measure the derivation height

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WIDP + POP* + argument filtering induce polynomial innermost runtime complexity

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Exan

WIDP: restriction of DP for innermost rewriting

WIDP + POP* + argument filtering induce polynomial innermost runtime complexity

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POP*: restriction of MPO, ¬ restriction of EPO*

WIDP + POP* + argument filtering induce polynomial innermost runtime complexity

Termination Analysis of Java Programs

Example

```
public class Int {
int val;
public static void count(Int orig, Int limit)
{
  if (orig == null || limit == null) {return;}

  Int copy = orig;

  while (orig.val < limit.val) {copy.val++;}
}}
```

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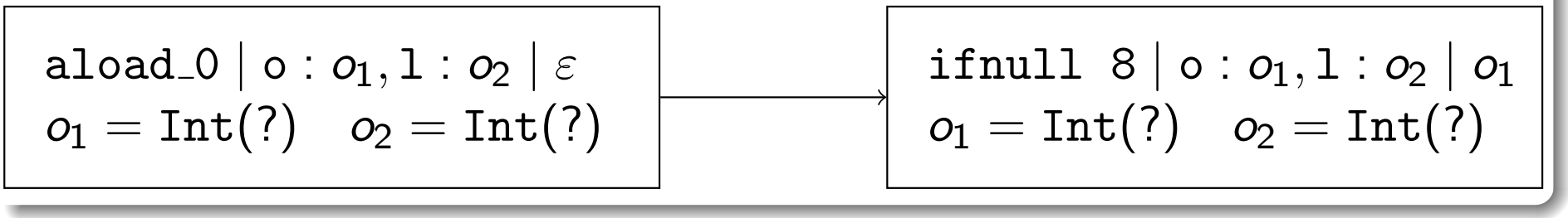
}}
```

Transformation to Java Bytecode

```
$javap -c Int > Int.bc
```

Transformation to Termination Graph



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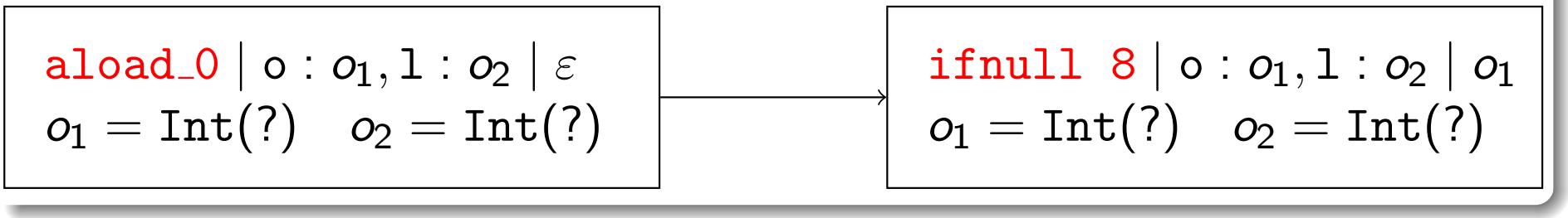

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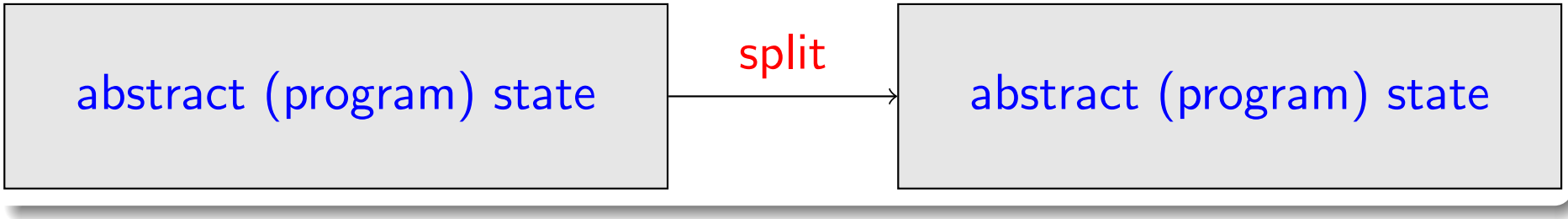

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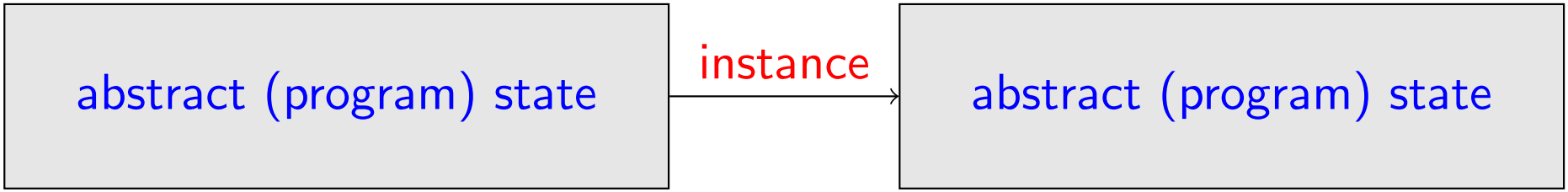

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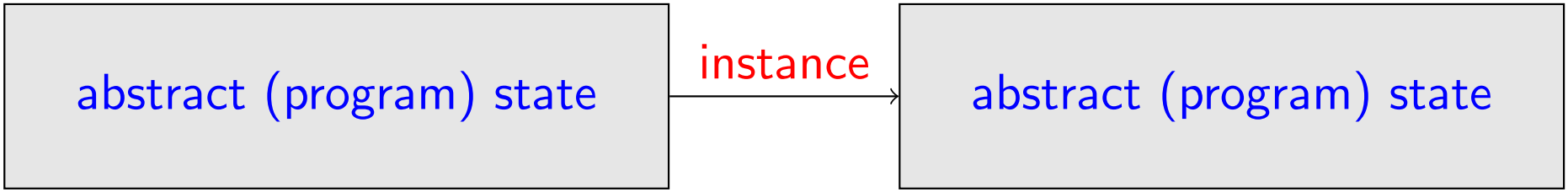
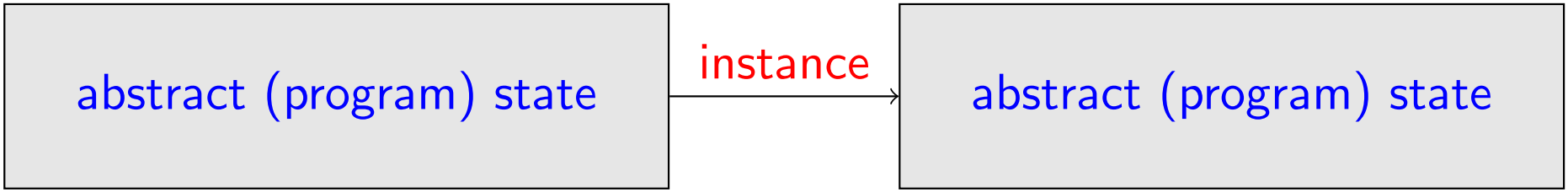


Transformation to TRS

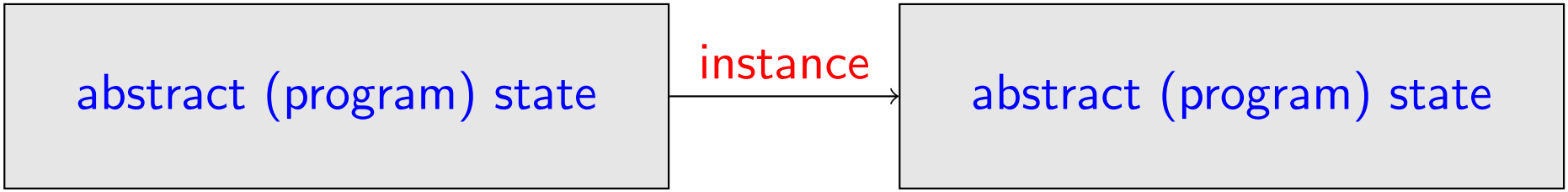
Transformation to Termination Graph



Transformation to TRS

$$\mathsf{f}_A(\mathsf{ts}(A)) \longrightarrow \mathsf{f}_B(\mathsf{ts}(B))$$

Transformation to Termination Graph



Transformation to TRS

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Example

consider ITRS $\mathcal{R}_{\mathsf{count}}$:

$$\mathsf{f}(x, y) \to \mathsf{fif}(x < y, x, y) \qquad \mathsf{fif}(\mathsf{true}, x, y) \to \mathsf{f}(x + 1, y)$$

Challenge ①

Question

is the runtime of any JBC program faithfully represented by the derivation height wrt its TRS representation?

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Theorem (Otto et al., 2010)

if the ITRS corresponding to a termination graph is terminating, then the termination graph is terminating as well

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Theorem (Otto et al., 2010)

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Corollary

if the ITRS corresponding to a termination graph is terminating, then the original JBC program is terminating for all concrete states t that are instances of some abstract state s in the graph

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Lemma

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$$\mathrm{f}_s(\text{"term representation of } s'\text{"}) \rightarrow^+ \mathrm{f}_t(\text{"term representation of } t'\text{"})$$

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covers all steps in the transformation graph, hence all JBC evaluation steps lead to at least one rewrite step

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Answer to Challenge ①

Yes

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- *transformation in Otto et al. restricts to "tree-shaped" objects*
- *the JBC class* `Flatten` *doesn't*

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Example

```
public class Tree {
  int value;
  Tree left;
  Tree right;
  }
```

Challenge ②

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Example

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public class Tree {
  int value;
  Tree left;
  Tree right;
  }
```

`flatten` revisited

```
// snip
cur = new TreeList();
cur.value = tree.left;
cur.next = oldCur;
oldCur.value = tree.right;
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solution: assume tree-shapes

Challenge ③

Question

is the runtime complexity of any JBC program faithfully represented by the runtime complexity wrt $\mathcal{R}_{\mathsf{flatten}}$, i.e, are the refinements complexity preserving?

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1. *merging of rules, for examples, rewriting of right-hand sides*
2. *create only rules for cycles in the termination graph*

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Answer to Challenge ③

Yes, merging of rules is not problematic and the cycles contain all necessary information

Challenge ④

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is the analysis extendable to ITRSs with fresh variables?

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(Simplified) Definition

a integer TRS (ITRS) $\mathcal{R}$ is a TRS over $\mathcal{F} \cup \mathcal{G}$ such that

1. $\mathcal{G} = \{\mathsf{true}, \mathsf{false}\} \cup \mathbb{Z} \cup \mathcal{F}_{\mathsf{arith}} \cup \mathcal{F}_{\mathsf{rel}} \cup \mathcal{F}_{\mathsf{bool}}$, where
   - $\mathcal{F}_{\mathsf{arith}} = \{+, -, \times\}$
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$$s \xrightarrow{\mathsf{itrs}}_{\mathcal{R}} t \iff s \xrightarrow{\mathsf{i}}_{\mathcal{R} \cup \mathcal{PD}} t$$

where $\mathcal{PD}$ collects all predefined rules for $\mathcal{F}_{\mathsf{arith}} \cup \mathcal{F}_{\mathsf{rel}} \cup \mathcal{F}_{\mathsf{bool}}$

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Example

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public class Exp{
  public static int exp (int x) {
    int y; int result = 1;
    while (x != 0) {
      y = result; result = 0;
      while (y != 0) {
        result += 2;
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polynomial complexity of $\mathcal{R}_{\text{count}}$ is shown by the WIDP method employing

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Answer to Challenge ④

Yes

Main Result

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"Lemma"

$\forall$ termination graph $\mathcal{G}$, the length of any path starting in state s is bounded by $\mathsf{dh}(t, \xrightarrow{\text{itrs}}_{\mathcal{R}})$*, where $\mathcal{R}$, t are the ITRS, term representation respectively*

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$\forall$ *ITRS* $\mathcal{R}$*,* $\mathsf{dh}(t, \xrightarrow{\mathsf{itrs}}_{\mathcal{R}})$ *is polynomial in* $|t|$*, if termination of* $\mathcal{R}$ *can be shown by the WIDP method in conjunction with (safe) reduction pairs based on max-polynomials*

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"Theo

calls to built-in functions are constant

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Really?

Example

consider ITRS $\mathcal{R}_{\mathsf{count}}$:

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Really, but not yet automated

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Thank You for Your Attention!