

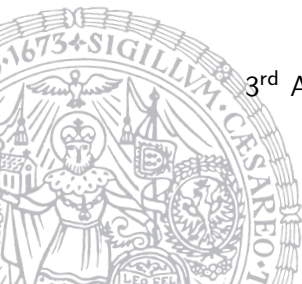
Relative Match-Bounds for Complexity Analysis

Martin Korp

Harald Zankl

Institute of Computer Science
University of Innsbruck

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Overview

Preliminaries

Match-Bounds

Modular Complexity Analysis

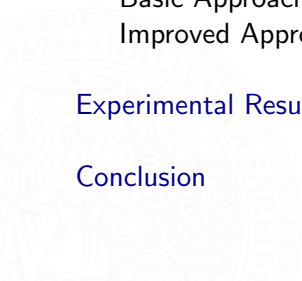
Relative Match-Bounds

Basic Approach

Improved Approach

Experimental Results

Conclusion



Definition

$$\rightarrow_{\mathcal{R}}^*(L) = \{t \mid \exists s \in L: s \rightarrow_{\mathcal{R}}^* t\}$$



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Definition

- ▶ $\text{lift}_c(f) = f_c$
- ▶ $\text{base}(f_c) = f$
- ▶ $\text{height}(f_c) = c$

Definition

- ▶ **relative** TRS \mathcal{R}/\mathcal{S} is pair of TRSs
- ▶ $\rightarrow_{\mathcal{R}/\mathcal{S}} = \rightarrow_{\mathcal{S}}^* \cdot \rightarrow_{\mathcal{R}} \cdot \rightarrow_{\mathcal{S}}^*$
- ▶ relative TRS is **terminating** if $\rightarrow_{\mathcal{R}/\mathcal{S}}$ is terminating



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Definition

- ▶ $dl_L(t, \rightarrow) = \max\{n \mid \exists u: t \rightarrow^n u, t \in L\}$
- ▶ $cp_L(n, \rightarrow) = \max\{dl_L(t, \rightarrow) \mid t \in L, |t| \leq n\}$

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\rightarrow has **linear complexity** on L if $cp_L(n, \rightarrow)$ is bounded by linear polynomial

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Idea

prove linear complexity of TRS \mathcal{R} using match-bounds

1. transform \mathcal{R} into an **enriched system** that simulates the original derivations



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prove linear complexity of TRS \mathcal{R} using match-bounds

1. transform \mathcal{R} into an enriched system that simulates the original derivations
2. search for an **upper bound** on the labels that can occur in derivations



Definition enrichment

$$\text{match}(\mathcal{R}) = \{l' \rightarrow \text{lift}_c(r) \mid \text{base}(l') \rightarrow r \in \mathcal{R}\}$$

where $c = 1 + \min\{\text{height}(l'(p)) \mid p \in \text{Pos}_{\mathcal{F}}(\text{base}(l'))\}$



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Example

aabb \rightarrow bbbaaa

$$\begin{aligned} \text{match}(\mathcal{R}) \quad a_0a_0b_0b_0 &\rightarrow b_1b_1b_1a_1a_1a_1 \\ a_0a_3b_0b_1 &\rightarrow b_1b_1b_1a_1a_1a_1 \\ a_1a_2b_5b_1 &\rightarrow b_2b_2b_2a_2a_2a_2 \\ &\vdots \end{aligned}$$

Definition

a TRS \mathcal{R} is called **match-bounded** for a language L if there exists a $c \in \mathbb{N}$ such that the maximal height of function symbols occurring in terms in $\rightarrow_{\text{match}(\mathcal{R})}^*(\text{lift}_0(L))$ is at most c



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Theorem (Geser *et al.* (RTA 2005))

if \mathcal{R} is linear and *match-bounded* for L then

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aabb \rightarrow bbaaaa

- ▶ GAT shows that \mathcal{R} is **match-bounded** by 5, by constructing a tree automaton with 185 transitions and 256 states



Example

aabb \rightarrow bbbaaa

- ▶ GAT shows that \mathcal{R} is match-bounded by 5, by constructing a tree automaton with 185 transitions and 256 states
- ▶ \mathcal{R} is **terminating** and $\rightarrow_{\mathcal{R}}$ has **linear** complexity



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compute complexity of TRS \mathcal{R} using different techniques

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3. **sum up** all intermediate results



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3. sum up all intermediate results

Theorem (Zankl and Korp (RTA 2010))

if a term t is terminating with respect to \mathcal{R} then

$$\max\{\mathcal{O}(\text{dl}_L(t, \rightarrow_{\{s_i \rightarrow t_i\}/\mathcal{S}_{s_i \rightarrow t_i}}))\} = \mathcal{O}(\text{dl}_L(t, \rightarrow_{\mathcal{R}}))$$

where $\mathcal{R} = \{s_1 \rightarrow t_1, \dots, s_n \rightarrow t_n\}$

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1. transform \mathcal{R} into an enriched system that simulates the original derivations
2. increase heights in case of an \mathcal{R} -step, keep heights in case of an \mathcal{S} -step
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Definition

enrichment

$$\text{match}^c(\mathcal{S}) = \{l' \rightarrow \text{lift}_d(r) \mid \text{base}(l') \rightarrow r \in \mathcal{S}\}$$

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enrichment $\text{match}^c(\mathcal{R}/\mathcal{S}) = \text{match}(\mathcal{R})/\text{match}^c(\mathcal{S})$

Example

$$\begin{array}{l} \mathcal{R} \quad \text{rev}(x) \rightarrow \text{rev}'(x, \text{nil}) \quad \text{rev}'(\text{nil}, y) \rightarrow y \\ \mathcal{S} \quad \text{rev}'(\text{cons}(x, y), z) \rightarrow \text{rev}'(y, \text{cons}(x, z)) \end{array}$$

$$\begin{array}{l} \text{match}(\mathcal{R}) \quad \text{rev}_0(x) \rightarrow \text{rev}'_1(x, \text{nil}_1) \quad \text{rev}'_0(\text{nil}_0, y) \rightarrow y \\ \quad \text{rev}_1(x) \rightarrow \text{rev}'_2(x, \text{nil}_2) \quad \text{rev}'_2(\text{nil}_4, y) \rightarrow y \\ \quad \text{rev}_2(x) \rightarrow \text{rev}'_3(x, \text{nil}_3) \quad \dots \end{array}$$

$$\begin{array}{l} \text{match}^1(\mathcal{S}) \quad \text{rev}'_0(\text{cons}_0(x, y), z) \rightarrow \text{rev}'_1(y, \text{cons}_1(x, z)) \\ \quad \text{rev}'_1(\text{cons}_0(x, y), z) \rightarrow \text{rev}'_1(y, \text{cons}_1(x, z)) \\ \quad \text{rev}'_3(\text{cons}_5(x, y), z) \rightarrow \text{rev}'_1(y, \text{cons}_1(x, z)) \\ \quad \dots \end{array}$$

$$\text{match}^1(\mathcal{R}/\mathcal{S}) = \text{match}(\mathcal{R})/\text{match}^1(\mathcal{S})$$

Definition

a relative TRS \mathcal{R}/\mathcal{S} is called **relative match-bounded** for a language L if there exists a $c \in \mathbb{N}$ such that the maximal height of function symbols occurring in terms in $\rightarrow_{\text{match}^c(\mathcal{R}/\mathcal{S})}^*(\text{lift}_0(L))$ is at most c



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- ▶ \mathcal{R}/\mathcal{S} is **relative match-bounded** by 1; can be easily shown by constructing a tree automaton with 7 transitions and 1 state



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- ▶ \mathcal{R}/\mathcal{S} is relative match-bounded by 1; can be easily shown by constructing a tree automaton with 7 transitions and 1 state
- ▶ \mathcal{R}/\mathcal{S} is terminating and $\rightarrow_{\mathcal{R}/\mathcal{S}}$ has linear complexity
- ▶ $\mathcal{R} \cup \mathcal{S}$ is **not match-bounded** because $\rightarrow_{\mathcal{R} \cup \mathcal{S}}$ has **quadratic** complexity

Definition enrichment

$$\text{match-RT}^c(\mathcal{S}) = \{l' \rightarrow \text{lift}_d(r) \mid \text{base}(l') \rightarrow r \in \mathcal{R}\}$$

where

- ▶ $d = \min\{c, \text{height}(l'(\epsilon))\}$ if $\|\text{base}(l')\| \geq \|r\|$ and
 $\text{lift}_{\text{height}(l'(\epsilon))}(\text{base}(l')) = l'$
- ▶ $d = \min\{c, 1 + \text{height}(l'(p)) \mid p \in \mathcal{Pos}_{\mathcal{F}}(\text{base}(l'))\}$ otherwise



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Definition

enrichment $\text{match-RT}^c(\mathcal{R}, s \rightarrow t, \mathcal{S})$ is defined as the relative TRS $\text{match}(s \rightarrow t) / \text{match-RT}^c(\mathcal{S}_{s \rightarrow t})$ where $\mathcal{S}_{s \rightarrow t} = (\mathcal{R} \cup \mathcal{S}) \setminus \{s \rightarrow t\}$

Example

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$$\text{match-RT}^1(\mathcal{R}, s \rightarrow t, \mathcal{S}) = \text{match}(s \rightarrow t) / \text{match-RT}^1(\mathcal{S}_{s \rightarrow t})$$

Definition

a relative TRS \mathcal{R}/\mathcal{S} is called **match-RT-bounded** for a rewrite rule $s \rightarrow t$ and a language L if there exists a $c \in \mathbb{N}$ such that the maximal height of function symbols occurring in terms belonging to $\rightarrow_{\text{match-RT}^c(\mathcal{R}, s \rightarrow t, \mathcal{S})}^*(\text{lift}_0(L))$ is at most c



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a relative TRS \mathcal{R}/\mathcal{S} is called *match-RT-bounded* for a rewrite rule $s \rightarrow t$ and a language L if there exists a $c \in \mathbb{N}$ such that the maximal height of function symbols occurring in terms belonging to $\rightarrow_{\text{match-RT}^c(\mathcal{R}, s \rightarrow t, \mathcal{S})}^*(\text{lift}_0(L))$ is at most c

Theorem

if \mathcal{R}/\mathcal{S} is linear and *match-RT-bounded* for $s \rightarrow t$ and L then

- ▶ $\{s \rightarrow t\}/\mathcal{S}_{s \rightarrow t}$ is *terminating* on L

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- ▶ \mathcal{R}/\mathcal{S} is **match-RT-bounded** for $s \rightarrow t = \text{rev}(x) \rightarrow \text{rev}'(x, \text{nil})$ by 1; can be easily shown by constructing a tree automaton with 7 transitions and 1 state



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- ▶ $\{s \rightarrow t\}/\mathcal{S}_{s \rightarrow t}$ is terminating and $\rightarrow_{\{s \rightarrow t\}/\mathcal{S}_{s \rightarrow t}}$ has linear complexity
- ▶ **match-RT-boundedness** for $s' \rightarrow t' = \text{rev}'(\text{nil}, y) \rightarrow y$ yields that \mathcal{R}/\mathcal{S} is **terminating** and $\rightarrow_{\mathcal{R}/\mathcal{S}}$ has **linear** complexity:

$$\max\{\mathcal{O}(\rightarrow_{\{s \rightarrow t\}/\mathcal{S}_{s \rightarrow t}}), \mathcal{O}(\rightarrow_{\{s' \rightarrow t'\}/\mathcal{S}_{s' \rightarrow t'}})\} = \mathcal{O}(\rightarrow_{\mathcal{R}/\mathcal{S}})$$

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Experimental Results with $\mathcal{G}\mathcal{T}$ (TPDB 7.2)

	derivational		
	pm	bpm	bprm
# successes	170	270	277
# linear	38	171	179
# quadratic	110	85	84
# cubic	22	14	14
average time (ms)	519	2349	4206

- b** match-bounds
- p** strongly linear interpretations
- m** matrix interpretations of dimension two and three
- r** match-RT-bounds

Experimental Results with $\mathcal{G}\mathcal{T}$ (TPDB 7.2)

	pm	derivational		runtime	
		bpm	bprm	bpm	bprm
# successes	170	270	277	921	928
# linear	38	171	179	907	916
# quadratic	110	85	84	13	11
# cubic	22	14	14	1	1
average time (ms)	519	2349	4206	284	397

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- ▶ original version uses a **upper bound** on the heights induced by \mathcal{S} -steps



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- ▶ new version uses **upper bounds** and a slightly modified **enrichment** to treat length-preserving and length-decreasing rewrite rules



Conclusion

- ▶ different versions of relative match-bounds
- ▶ original version uses a upper bound on the heights induced by \mathcal{S} -steps
- ▶ new version uses upper bounds and a slightly modified enrichment to treat length-preserving and length-decreasing rewrite rules
- ▶ latter one is especially adapted to prove complexity bounds of **single** rewrite rules