Modular Semantic Labeling and Unlabeling

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Overview

- Unlabeling as Termination Technique
- Unlabeling in the DP Framework

Termination Techniques

Definition

A termination technique is a function $TT : TRS \to TRS$.
TT is sound if

$$SN(TT(\mathcal{R})) \Rightarrow SN(\mathcal{R})$$

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- (or one applies DP-transformation and then continues handling DP problems)

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some termination techniques

- rule removal (KBO, RPO, polynomial interpretations, matrix-interpretations, arctic matrices, ...), matchbounds, semantic labeling and unlabeling, root-labeling, predictive labeling, string reversal, ...

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Semantic Labeling and Unlabeling

- semantic labeling: label symbols to distinguish them

Example

$$\mathsf{f}(\mathsf{f}(x)) \to \mathsf{f}(\mathsf{g}(\mathsf{f}(x))) \qquad \mathsf{g}(\dots) \to \dots \qquad \dots$$

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- counting f_1's allows to remove critical rules
- it remains to analyze large labeled system $\mathsf{g}_0(\dots) \to \dots, \mathsf{g}_1(\dots) \to \dots, \dots$

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- unlabel to continue with small system $\mathsf{g}(\dots) \to \dots \quad \dots$

Semantic Labeling and Unlabeling (Quasi-Models)

- to apply semantic labeling one needs a model of rule
- alternative: only require quasi-model, but add decreasing rules

$$\mathcal{D}ecr = \{f_\ell(x_1 \ldots x_n) \to f_{\ell'}(x_1 \ldots x_n) \mid \ell > \ell'\}$$

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- after some steps labeled system remains
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- after some steps labeled system remains $\mathsf{g}_0(\dots) \to \dots, \mathsf{g}_1(\dots) \to \dots, \mathsf{g}_1(x) \to \mathsf{g}_0(x), \dots$
- unlabeling yields system $\mathsf{g}(\dots) \to \dots, \mathsf{g}(x) \to \mathsf{g}(x), \quad \dots$

Unlabeling in Combination with Quasi-Models

in case of quasi models one needs to remove the decreasing rules before unlabeling

- from $\mathrm{g}_0(\dots) \to \dots, \mathrm{g}_1(\dots) \to \dots, \mathrm{g}_1(x) \to \mathrm{g}_0(x), \dots$
- first remove decreasing rule $\mathrm{g}_1(x) \to \mathrm{g}_0(x)$
- such that unlabeling yields system $\mathrm{g}(\dots) \to \dots, \quad \dots$

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problem: unlabeling should be a modular technique on its own

- no book-keeping of the order that is used in semantic labeling which determines the decreasing rules

$\Rightarrow$ need definition of the decreasing rules of a TRS: $\mathcal{D}(\mathcal{R})$

$$\mathcal{D}(\mathcal{R}) = \{\ell \to r \mid unlab(\ell) = unlab(r) \land \ell \neq r\}$$

$\Rightarrow$ define the unlabeling of a TRS as $\mathcal{U}(\mathcal{R}) = unlab(\mathcal{R} - \mathcal{D}(\mathcal{R}))$

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problems

- to remove $\mathcal{D}(\mathcal{R})$, $\mathcal{D}(\mathcal{R})$ must not be cause for nontermination

$\Rightarrow$ instead of proving $SN(\mathcal{R})$, prove $SN(\mathcal{D}(\mathcal{R})) \Rightarrow SN(\mathcal{R})$

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$\Rightarrow$ require stronger notion than soundness of termination techniques

$$\begin{array}{rcl} & & \{\mathsf{f}(\mathsf{a}) \to \mathsf{f}(\mathsf{f}(\mathsf{a}))\} \\ \text{sem. lab.} & \hookrightarrow & \{\mathsf{f}_1(\mathsf{a}) \to \mathsf{f}_0(\mathsf{f}_1(\mathsf{a})), \mathsf{f}_1(x) \to \mathsf{f}_0(x)\} \\ \text{sound TT} & \hookrightarrow & \{\mathsf{f}_0(x) \to \mathsf{f}_1(x), \mathsf{f}_1(x) \to \mathsf{f}_0(x)\} \\ \mathcal{U} & \hookrightarrow & \emptyset \end{array}$$

Unlabeling as Termination Technique

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$\Rightarrow$ to support recursive labeling, after unlabeling with $\mathcal{U}$ one still wants to be able to conclude $SN(\mathcal{D}(\mathcal{U}(\mathcal{R})))$ to apply next unlabeling

$\Rightarrow$ need stronger hypothesis: $n =$ nr of allowed unlabel steps

goal: prove $\mathcal{D}\text{-}SN(\mathcal{R}, n) = \forall m < n. SN(\mathcal{D}(\mathcal{U}^m(\mathcal{R}))) \Rightarrow SN(\mathcal{R})$

New framework for transforming termination problems

$$\mathcal{D}(\mathcal{R}) = \{\ell \to r \mid unlab(\ell) = unlab(r) \wedge \ell \neq r\}$$
$$\mathcal{U}(\mathcal{R}) = unlab(\mathcal{R} - \mathcal{D}(\mathcal{R}))$$
$$\mathcal{D}\text{-}SN(\mathcal{R}, n) = \forall m < n. SN(\mathcal{D}(\mathcal{U}^m(\mathcal{R}))) \Rightarrow SN(\mathcal{R})$$

Definition

$\mathcal{D}$-termination technique is mapping $DTT : TRS \times \mathbb{N} \to TRS \times \mathbb{N}$.
DTT is sound if

$$\mathcal{D}\text{-}SN(DTT(\mathcal{R}, n)) \Rightarrow \mathcal{D}\text{-}SN(\mathcal{R}, n)$$

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observations

- $SN(\mathcal{R}) = \mathcal{D}\text{-}SN(\mathcal{R}, 0)$, hence start with initial problem $(\mathcal{R}, 0)$
- $SN(\mathcal{R}) \Rightarrow \mathcal{D}\text{-}SN(\mathcal{R}, n)$, hence $\mathcal{D}\text{-}SN(\emptyset, n)$

$\Rightarrow$ transform $(\mathcal{R}, 0)$ by sound $\mathcal{D}$-termination techniques until $(\emptyset, n)$ is reached

Complexity of being sound

$$\mathcal{D}(\mathcal{R}) = \{\ell \to r \mid unlab(\ell) = unlab(r) \wedge \ell \neq r\}$$
$$\mathcal{U}(\mathcal{R}) = unlab(\mathcal{R} - \mathcal{D}(\mathcal{R}))$$
$$\mathcal{D}\text{-}SN(\mathcal{R}, n) = \forall m < n. SN(\mathcal{D}(\mathcal{U}^m(\mathcal{R}))) \Rightarrow SN(\mathcal{R})$$
$$DTT \text{ sound } \text{ if } \mathcal{D}\text{-}SN(DTT(\mathcal{R}, n)) \Rightarrow \mathcal{D}\text{-}SN(\mathcal{R}, n)$$

$\mathcal{D}$-termination techniques:

Complexity of being sound

$$\begin{aligned}
\mathcal{D}(\mathcal{R}) &= \{\ell \to r \mid \mathit{unlab}(\ell) = \mathit{unlab}(r) \wedge \ell \neq r\} \\
\mathcal{U}(\mathcal{R}) &= \mathit{unlab}(\mathcal{R} - \mathcal{D}(\mathcal{R})) \\
\mathcal{D}\text{-}\mathit{SN}(\mathcal{R}, n) &= \forall m < n. \mathit{SN}(\mathcal{D}(\mathcal{U}^m(\mathcal{R}))) \Rightarrow \mathit{SN}(\mathcal{R}) \\
\mathit{DTT} \text{ sound } & \text{ if } \mathcal{D}\text{-}\mathit{SN}(\mathit{DTT}(\mathcal{R}, n)) \Rightarrow \mathcal{D}\text{-}\mathit{SN}(\mathcal{R}, n)
\end{aligned}$$

$\mathcal{D}$-termination techniques:

- semantic labeling: $\mathit{DTT}(\mathcal{R}, n) = (\mathit{Lab}(\mathcal{R}) \cup \mathcal{D}\mathit{ecr}, n + 1)$

Complexity of being sound

$$\mathcal{D}(\mathcal{R}) = \{\ell \to r \mid unlab(\ell) = unlab(r) \wedge \ell \neq r\}$$
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$\mathcal{D}$-termination techniques:

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- unlabeling: $DTT(\mathcal{R}, n + 1) = (\mathcal{U}(\mathcal{R}), n)$,
  $DTT(\mathcal{R}, 0) = (unlab(\mathcal{R}), 0)$

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$\mathcal{D}$-termination techniques:

- semantic labeling: $DTT(\mathcal{R}, n) = (Lab(\mathcal{R}) \cup \mathcal{D}ecr, n+1)$
- unlabeling: $DTT(\mathcal{R}, n+1) = (\mathcal{U}(\mathcal{R}), n)$,
  $DTT(\mathcal{R}, 0) = (unlab(\mathcal{R}), 0)$
- sound termination technique TT, three alternatives:
  1. limited unlabeling afterwards: $DTT(\mathcal{R}, n) = (TT(\mathcal{R}), 0)$
  2. for special class of TT: $DTT(\mathcal{R}, n) = (TT(\mathcal{R}), n)$
  3. provide complex proof of soundness:
     $DTT(\mathcal{R}, n) = (TT(\mathcal{R}), n)$

1. Alternative: $DTT(\mathcal{R}, n) = (TT(\mathcal{R}), 0)$

- with this alternative, every sound termination technique can directly be integrated in the new framework
- setting the number n to 0 just forbids to remove the decreasing rules before unlabeling

$\Rightarrow$ try to avoid such a technique between labeling and unlabeling when using quasi-models

2. Alternative: $DTT(\mathcal{R}, n) = (TT(\mathcal{R}), n)$ for class of TTs

- these sound TTs can fully be used between labeling and unlabeling
- additional requirement to belong to class:

$$\forall \ell \to r \in TT(\mathcal{R}). \exists C. C[\ell] \to C[r] \in \mathcal{R}$$

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$$\forall \ell \to r \in TT(\mathcal{R}). \exists C. C[\ell] \to C[r] \in \mathcal{R}$$

$\Rightarrow$ rule-removal belongs to this class
(then $TT(\mathcal{R}) \subseteq \mathcal{R}$ and C is always the empty context)

$\Rightarrow$ match-bounds belongs to this class (again $TT(\mathcal{R}) \subseteq \mathcal{R}$)

$\Rightarrow$ flat-context-closure belongs to this class

3. Alternative: provide explicit soundness proof for $DTT(\mathcal{R}, n) = (TT(\mathcal{R}), n)$

- these sound TTs can fully be used between labeling and unlabeling
- soundness proof more complex than standard soundness proof
- example: string-reversal, proof uses commutation of reversal and unlabeling

Example

$\{\ldots\}, 0$

Example

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\begin{array}{lcl}
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\mathsf{SL} & \hookrightarrow & \{\dots, \mathsf{f}_1(x) \to \mathsf{f}_0(x)\}, 1
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\mathsf{SR} & \hookrightarrow & \{\ldots, \mathsf{f}_{1_{\mathsf{a}_2}}(\mathsf{g}_{1_{\mathsf{f}_1}}(x)) \to \mathsf{f}_{0_{\mathsf{a}_2}}(\mathsf{g}_{1_{\mathsf{f}_0}}(x))\}, 2 \\
\mathsf{RR} & \hookrightarrow & \{\ldots, \mathsf{f}_{1_{\mathsf{a}_2}}(\mathsf{g}_{1_{\mathsf{f}_1}}(x)) \to \mathsf{f}_{0_{\mathsf{a}_2}}(\mathsf{g}_{1_{\mathsf{f}_0}}(x))\}, 2 \\
\mathsf{UL} & \hookrightarrow & \{\ldots, \mathsf{f}_1(\mathsf{g}_1(x)) \to \mathsf{f}_0(\mathsf{g}_1(x))\}, 1 \\
\mathsf{FCC} & \hookrightarrow & \{\ldots, \mathsf{h}_0(\mathsf{f}_1(\mathsf{g}_1(x))) \to \mathsf{h}_0(\mathsf{f}_0(\mathsf{g}_1(x)))\}, 1 \\
\mathsf{RL} & \hookrightarrow & \{\ldots, \mathsf{h}_{0_{\mathsf{f}_1}}(\mathsf{f}_{1_{\mathsf{g}_1}}(\mathsf{g}_{1_{\mathsf{b}_4}}(x))) \to \mathsf{h}_{0_{\mathsf{f}_0}}(\mathsf{f}_{0_{\mathsf{g}_1}}(\mathsf{g}_{1_{\mathsf{b}_4}}(x)))\}, 2 \\
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\end{array}
$$

Example

$$
\begin{array}{rcl}
 & & \{\ldots\}, 0 \\
\mathsf{SL} & \hookrightarrow & \{\ldots, \mathsf{f}_1(x) \to \mathsf{f}_0(x)\}, 1 \\
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$$

Overview

- Unlabeling as Termination Technique
- Unlabeling in the DP Framework

Unlabeling in the DP framework

recall situation for TTs

- unlabeling for models poses no problem with standard notion of soundness
- complex DTT-soundness to support unlabeling in combination with decreasing rules

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⇒ require complex notion of soundness similar to $\mathcal{D}$-SN to support modular unlabeling in combination with minimal chains

$$finite(\mathcal{P}, \mathcal{R}, n) = \neg \exists s_i, t_i, \sigma_i. \text{``}s_i \rightarrow t_i, \sigma_i \text{ is } (\mathcal{P}, \mathcal{R})\text{-chain''} \wedge \forall m \leq n. SN_{\mathcal{U}^m(\mathcal{R})}(unlab^m(t_i \sigma_i))$$

Support for existing processors

again: three alternatives to integrate existing sound processors

- all processors based on reduction pairs and the dependency graph can be used between labeling and unlabeling (as they belong to a special class of sound processor)
- flat-context-closure-processor is integrated in the easy way: forbid unlabeling afterwards
- open task: provide complex soundness proof for the flat-context-closure-processor

Summary

- unlabeling is desired to get rid of blow-up of semantic labeling
- unlabeling with quasi-models requires special treatment of decreasing rules

⇒ new framework to support unlabeling in a modular way

  - no book-keeping of decreasing rules, compute $\mathcal{D}(\mathcal{R})$
  - integration of several existing TTs in new framework

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- unlabeling is desired to get rid of blow-up of semantic labeling
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- extension to DP-framework also possible
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Summary

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⇒ new framework to support unlabeling in a modular way
  - no book-keeping of decreasing rules, compute $\mathcal{D}(\mathcal{R})$
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- extension to DP-framework also possible
  - no book-keeping of decreasing rules, compute $\mathcal{D}(\mathcal{R})$
  - integration of several existing processors in new framework
  - flat-context-closure-processor not fully supported by now
- all these proofs have been formally certified in our library Isabelle Formalization of Rewriting
- new framework forms basis of our proof checker CeTA