Termination Tools in Ordered Completion

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Content

- Completion Inference Systems

  Bachmair, Dershowitz, Plaisted '89
  oKB

L. Bachmair, N. Dershowitz and D.A. Plaisted
Completion Without Failure

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- Completion Inference Systems

Bachmair, Dershowitz, Plaisted '89
oKB

↓

Kurihara & Kondo '99
oMKB

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Completion Without Failure

M. Kurihara and H. Kondo
Completion for Multiple Reduction Orderings

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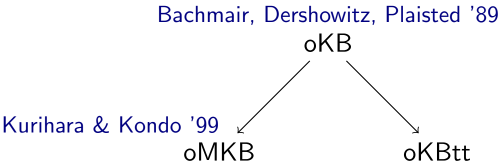

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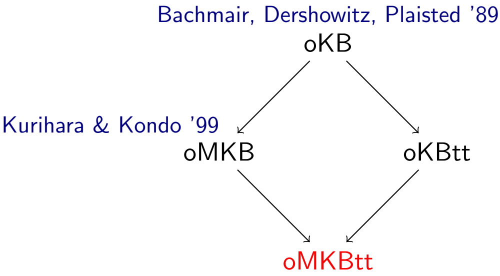

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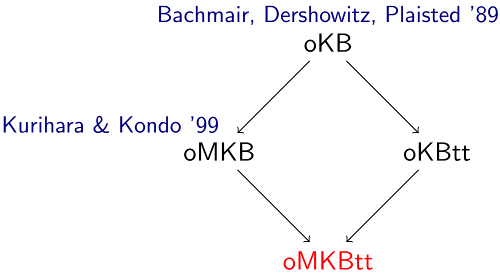

- Theorem Proving with oMKBtt

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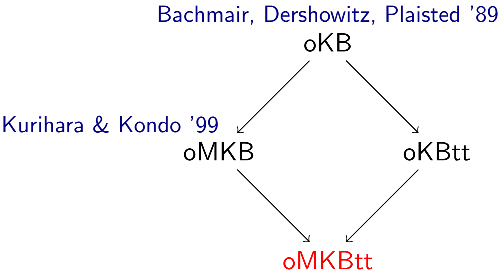

- Theorem Proving with oMKBtt
- Experiments and Conclusion

L. Bachmair, N. Dershowitz and D.A. Plaisted
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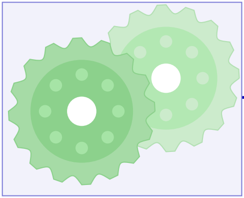
Completion for Multiple Reduction Orderings

Ordered Completion

$(xy)(zw) \approx (xz)(yw)$
$(xy)x \approx x$ $\longrightarrow$

$\succ$ $\longrightarrow$

$\mathcal{E}_0$



oKB

$\longrightarrow$

$(xy)x \to x$
$x(yz) \to xz$
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$\mathcal{E}, \mathcal{R}$

$\mathcal{E} \cup \mathcal{R}$ has same theory as $\mathcal{E}_0$ and

$\mathcal{E} \cup \mathcal{R}$ is ground-confluent wrt $>$ which is complete for $\mathcal{E}_0$ and extends $\succ$

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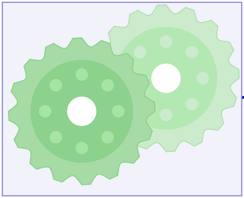

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Definition

- $>$ is complete for $\mathcal{E}_0$ if for ground $s \leftrightarrow^*_{\mathcal{E}_0} t$ with $s \neq t$ either $s > t$ or $t > s$ holds

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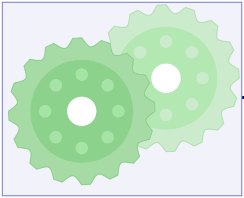

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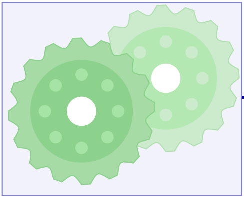

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$l\sigma \to r\sigma \in \mathcal{E}_>$ if
$l \approx r \in \mathcal{E}$ and $l\sigma > r\sigma$

Definition (Standard Completion KB)

$\mathcal{E}$: set of equations $\mathcal{R}$: set of rewrite rules $\succ$: reduction order

inference system contains rules

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compose $$\frac{\mathcal{E}, \mathcal{R} \cup \{s \to t\}}{\mathcal{E}, \mathcal{R} \cup \{s \to u\}}$$ if $t \to_{\mathcal{R}} u$

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compose$_2$ $\dfrac{\mathcal{E}, \mathcal{R} \cup \{s \to t\}}{\mathcal{E}, \mathcal{R} \cup \{s \to u\}}$ if $t \to_{\mathcal{E}_{\succ}} u$

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Definition (Extended Critical Pairs)

If $t \xleftarrow{r_1\sigma \leftarrow l_1\sigma} u \xrightarrow{l_2\sigma \to r_2\sigma} s$ such that $l_i \approx r_i \in \mathcal{E} \cup \mathcal{R}$ and $r_i\sigma \not\succ l_i\sigma$

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if $t \to_{\mathcal{E}_{\succ}} u$

Definition (Extended Critical Pairs)

If $t \xleftarrow{r_1\sigma \leftarrow l_1\sigma} u \xrightarrow{l_2\sigma \to r_2\sigma} s$ such that $l_i \approx r_i \in \mathcal{E} \cup \mathcal{R}$ and $r_i\sigma \not\succ l_i\sigma$
then $s \approx t$ is in $\mathrm{CP}_{\succ}(\mathcal{E} \cup \mathcal{R})$

Definition

inference sequence

$$\mathcal{S}: \quad (\mathcal{E}_0, \mathcal{R}_0) \vdash (\mathcal{E}_1, \mathcal{R}_1) \vdash (\mathcal{E}_2, \mathcal{R}_2) \vdash \cdots$$

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- $\mathcal{S}$ is fair if $\mathrm{CP}_\succ(\mathcal{E}_\omega \cup \mathcal{R}_\omega) \subseteq \bigcup_i \mathcal{E}_i$

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Theorem (Correctness)

Assume fair oKB run $(\mathcal{E}_0, \varnothing) \vdash^* (\mathcal{E}_\omega, \mathcal{R}_\omega)$ using $\succ$.

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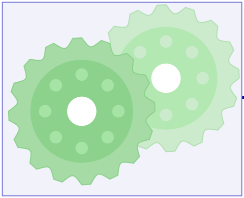
Theorem (Correctness)

Assume fair oKB run $(\mathcal{E}_0, \varnothing) \vdash^* (\mathcal{E}_\omega, \mathcal{R}_\omega)$ using $\succ$.
If $>$ is complete reduction order extending $\succ$ then $\mathcal{E}_\omega \cup \mathcal{R}_\omega$ has same theory as $\mathcal{E}_0$ and is ground confluent with respect to $>$.

Ordered Multi-Completion

$(xy)(zw) \approx (xz)(yw)$
$(xy)x \approx x$ $\longrightarrow$

$\succ_1, \succ_2, \ldots, \succ_n$ $\longrightarrow$



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$\mathcal{E}_0$ oMKB $\mathcal{E}, \mathcal{R}$

$\mathcal{E} \cup \mathcal{R}$ has same theory as $\mathcal{E}_0$

$\mathcal{E} \cup \mathcal{R}$ is ground-confluent wrt complete $>$ extending a specific $\succ_i$

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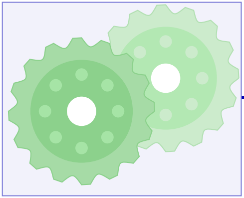

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Definition (oMKB node)

node is tuple $\langle s : t, R_0, R_1, E \rangle$

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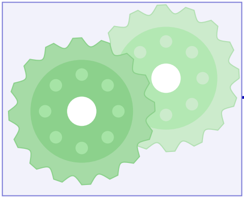

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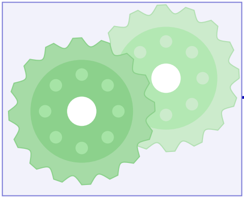
$\succ_1, \succ_2, \ldots, \succ_n$ →



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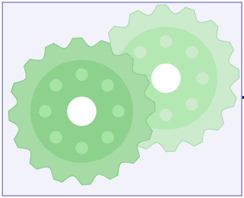

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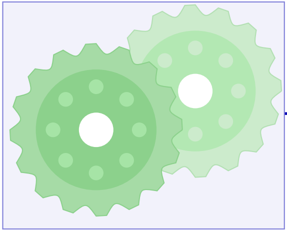
- oMKB specified by inference system on nodes

Ordered Completion with Termination Tools

$(xy)(zw) \approx (xz)(yw)$
$(xy)x \approx x$

$\longrightarrow$



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where $\mathcal{C}$ is terminating rewrite system developed during deduction

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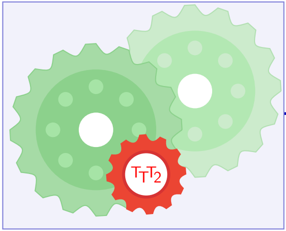

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Fact

If $\mathcal{C}$ terminates then $\to_{\mathcal{C}}^{+}$ is reduction order

Definition (oKBtt)

$\mathcal{E}$: set of equations $\mathcal{R}$: rewrite system $\mathcal{C}$: rewrite system

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- perform termination check in orient

orient $$\frac{\mathcal{E} \cup \{s \approx t\}, \mathcal{R}, \mathcal{C}}{}$$

if $\mathcal{C} \cup \{s \to t\}$ terminates

Definition (oKBtt)

$\mathcal{E}$: set of equations $\mathcal{R}$: rewrite system $\mathcal{C}$: rewrite system

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$\mathcal{E}$: set of equations $\mathcal{R}$: rewrite system $\mathcal{C}$: rewrite system

- perform termination check in orient, compose$_2$

orient

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if $\mathcal{C} \cup \{s \to t\}$ terminates

compose$_2$

$$\frac{\mathcal{E}, \mathcal{R} \cup \{s \to t\}, \mathcal{C}}{}$$

if $t \to_{\mathcal{E}} u$ using $l\sigma \to r\sigma$ and $\mathcal{C} \cup \{l\sigma \to r\sigma\}$ terminates

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- perform termination check in orient, compose$_2$

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Definition (oKBtt)

$\mathcal{E}$: set of equations $\quad$ $\mathcal{R}$: rewrite system $\quad$ $\mathcal{C}$: rewrite system

- perform termination check in orient, compose$_2$, collapse$_2$, simplify$_2$

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Lemma (Simulation Properties)

- *if* $(\mathcal{E}_0, \varnothing, \varnothing) \vdash^*_{oKBtt} (\mathcal{E}, \mathcal{R}, \mathcal{C})$ *then* $(\mathcal{E}_0, \varnothing) \vdash^*_{oKB} (\mathcal{E}, \mathcal{R})$

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- *if* $(\mathcal{E}_0, \varnothing) \vdash^*_{oKB} (\mathcal{E}, \mathcal{R})$ *using* $\succ$ *then* $(\mathcal{E}_0, \varnothing, \varnothing) \vdash^*_{oKBtt} (\mathcal{E}, \mathcal{R}, \mathcal{C})$

Obtaining Ground-Confluence

Theorem (Correctness)

For fair oKBtt run $(\mathcal{E}_0, \varnothing, \varnothing) \vdash^* (\mathcal{E}_\omega, \mathcal{R}_\omega, \mathcal{C}_\omega)$ *and complete reduction order* $>$ *extending* $\to^+_{\mathcal{C}_\omega}$

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Problem 1

Does $>$ exist?

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Problem 1

Does $>$ exist?

Problem 2

Fairness requires to deduce $\mathrm{CP}_{\to^+_{\mathcal{C}_\omega}}(\mathcal{E}_\omega \cup \mathcal{R}_\omega)$.

But reduction order $\to^+_{\mathcal{C}_\omega}$ is not known during run!

Problem 1: Does $>$ exist?

Example

$$\begin{array}{cc} \mathsf{f}(\mathsf{a}+\mathsf{c}) \approx \mathsf{f}(\mathsf{c}+\mathsf{a}) & \mathsf{a} \approx \mathsf{b} \\ \mathsf{g}(\mathsf{c}+\mathsf{b}) \approx \mathsf{g}(\mathsf{b}+\mathsf{c}) & x + y \approx y + x \end{array}$$

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as input for fair oKBtt run might produce

$$\begin{aligned} \mathcal{E} &= \phantom{\mathcal{R} \cup} \{ \; x+y \approx y+x \} \\ \mathcal{R} &= \phantom{\mathcal{R} \cup} \{ \mathsf{f}(\mathsf{b}+\mathsf{c}) \to \mathsf{f}(\mathsf{c}+\mathsf{b}) \quad \mathsf{a} \to \mathsf{b} \quad \mathsf{g}(\mathsf{c}+\mathsf{b}) \to \mathsf{g}(\mathsf{b}+\mathsf{c}) \} \\ \mathcal{C} &= \mathcal{R} \cup \{ \mathsf{f}(\mathsf{a}+\mathsf{c}) \to \mathsf{f}(\mathsf{c}+\mathsf{a}) \} \end{aligned}$$

Is $\mathcal{E} \cup \mathcal{R}$ is ground-confluent?

Problem 1: Does $>$ exist?

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$$\mathsf{f}(\mathsf{a}+\mathsf{c}) \approx \mathsf{f}(\mathsf{c}+\mathsf{a}) \qquad \mathsf{a} \approx \mathsf{b}$$
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\mathcal{E} &= \{\ x+y \approx y+x\} \\
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Is $\mathcal{E} \cup \mathcal{R}$ is ground-confluent?

If $>$ is complete and extends $\to_{\mathcal{C}}^{+}$,

- for any such $>$ must have $\mathsf{a}+\mathsf{c} > \mathsf{c}+\mathsf{a}$

Problem 1: Does $>$ exist?

Example

$$\begin{array}{cc} \mathsf{f(a+c)} \approx \mathsf{f(c+a)} & \mathsf{a} \approx \mathsf{b} \\ \mathsf{g(c+b)} \approx \mathsf{g(b+c)} & x+y \approx y+x \end{array}$$

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$$\begin{array}{ll} \mathcal{E} = & \{ \quad x+y \ \approx \ y+x \} \\ \mathcal{R} = & \{ \mathsf{f(b+c)} \to \mathsf{f(c+b)} \quad \mathsf{a} \to \mathsf{b} \quad \mathsf{g(c+b)} \to \mathsf{g(b+c)} \} \\ \mathcal{C} = \mathcal{R} \cup & \{ \mathsf{f(a+c)} \to \mathsf{f(c+a)} \} \end{array}$$

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Is $\mathcal{E} \cup \mathcal{R}$ is ground-confluent? No!

If $>$ is complete and extends $\to_{\mathcal{C}}^{+}$,

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- overlap not joinable

Problem 1: Does $>$ exist?

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Is $\mathcal{E} \cup \mathcal{R}$ is ground-confluent? No!

Definition

$\mathcal{R}$ is totally terminating if compatible with total reduction order on $\mathcal{T}(\mathcal{F})$

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$\text{oKBtt}_{\text{total}}$ restricts to termination techniques inducing total termination

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such as LPO, KBO, MPO or polynomial interpretations over $\mathbb{N}$

$\mathsf{oKBtt}_{\mathsf{total}}$ restricts to termination techniques inducing total termination

Problem 2: How to be fair?

Fact

If $\succ \subseteq >$ holds then $\mathrm{CP}_{>}(\mathcal{E}) \subseteq \mathrm{CP}_{\succ}(\mathcal{E})$

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Remark

Sufficiently fair oKBtt runs are fair

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Example

- oKBtt run is sufficiently fair if $\succ' = \varnothing$

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strict subterm relation

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- $\mathrm{oKBtt}_{\mathrm{total}}$ run is fair if $\succ' = \rhd$

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Example

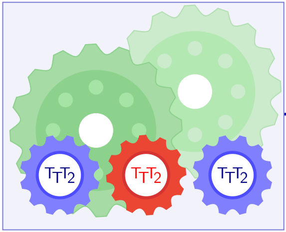
strict embedding relation

- oKBtt run is sufficiently fair if $\succ' = \varnothing$
- $\text{oKBtt}_{\text{total}}$ run is fair if $\succ' = \rhd$ or $\succ' = \rhd_{\text{emb}}$

Ordered Multi-Completion with Termination Tools

$(xy)(zw) \approx (xz)(yw)$
$(xy)x \approx x$

$\longrightarrow$



$\longrightarrow$

$(xy)x \to x$
$x(yz) \to xz$
$((xy)z)w \to xw$
$(xy)z \approx (xw)z$

$\mathcal{E}_0$ oMKBtt $\mathcal{E}, \mathcal{R}$

$\mathcal{E} \cup \mathcal{R}$ has same theory as $\mathcal{E}_0$

$\mathcal{E} \cup \mathcal{R}$ is ground-confluent wrt $>$ extending some $\to_{\mathcal{C}_p}^{+}$
where $\mathcal{C}_p$ is terminating rewrite system developed during deduction

- Use multi-completion to simulate multiple oKBtt processes but share inferences

Definition (oMKBtt node)

- processes are strings in $\mathcal{L}((0+1)^*)$

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rewrite rule $s \to t$
for process in R_0

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rewrite rule $t \to s$
for process in R_1

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equation $s \approx t$
for process in E

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constraint rule $s \to t$
for process in C_0

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- projection of node set $\mathcal{N}$ to process p yields equations $E_p(\mathcal{N})$, rules $R_p(\mathcal{N})$ and constraints $C_p(\mathcal{N})$
- initial node set for axioms $\mathcal{E}$ is $\mathcal{N}_{\mathcal{E}} = \{\langle s : t, \varnothing, \varnothing, \{\epsilon\}, \varnothing, \varnothing \rangle \mid s \approx t \in \mathcal{E}\}$

Definition (oMKBtt)

inference system oMKBtt consists of 5 rules

orient
$$\frac{\mathcal{N} \cup \{\langle s : t, R_0, R_1, E, C_0, C_1\rangle\}}{\quad}$$

if

- $E_{lr} \subseteq E$ such that $C_p(\mathcal{N}) \cup \{s \to t\}$ terminates for all $p \in E_{lr}$,

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- split set $S = E_{lr} \cap E_{rl}$,

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$$\frac{\mathcal{N} \cup \{\langle s : t, R_0, R_1, E, C_0, C_1\rangle\}}{\mathcal{N} \;\; \cup \{\langle s : t, R_0 \cup R_{lr}, \qquad\qquad C_0 \cup R_{lr}, \qquad\qquad \rangle\}}$$

if

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- split set $S = E_{lr} \cap E_{rl}$,
- $R_{lr} = (E_{lr} \setminus E_{rl}) \cup \{p0 \mid p \in S\}$ and

Definition (oMKBtt)

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orient

$$\frac{\mathcal{N} \cup \{\langle s : t, R_0, R_1, E, C_0, C_1 \rangle\}}{\mathcal{N} \cup \{\langle s : t, R_0 \cup R_{lr}, R_1 \cup R_{rl}, \quad C_0 \cup R_{lr}, C_1 \cup R_{rl} \rangle\}}$$

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- $E_{lr} \subseteq E$ such that $C_p(\mathcal{N}) \cup \{s \to t\}$ terminates for all $p \in E_{lr}$, $E_{rl} \subseteq E$ such that $C_p(\mathcal{N}) \cup \{t \to s\}$ terminates for all $p \in E_{rl}$
- split set $S = E_{lr} \cap E_{rl}$,
- $R_{lr} = (E_{lr} \setminus E_{rl}) \cup \{p0 \mid p \in S\}$ and $R_{rl} = (E_{rl} \setminus E_{lr}) \cup \{p1 \mid p \in S\}$,

Definition (oMKBtt)

inference system oMKBtt consists of 5 rules

orient
$$\frac{\mathcal{N} \cup \{\langle s : t, R_0, R_1, E, C_0, C_1\rangle\}}{\mathcal{N} \cup \{\langle s : t, R_0 \cup R_{lr}, R_1 \cup R_{rl}, E', C_0 \cup R_{lr}, C_1 \cup R_{rl}\rangle\}}$$

if

- $E_{lr} \subseteq E$ such that $C_p(\mathcal{N}) \cup \{s \to t\}$ terminates for all $p \in E_{lr}$,
  $E_{rl} \subseteq E$ such that $C_p(\mathcal{N}) \cup \{t \to s\}$ terminates for all $p \in E_{rl}$
- split set $S = E_{lr} \cap E_{rl}$,
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- $E' = E \setminus (E_{lr} \cup E_{rl})$

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$$\frac{\mathcal{N} \cup \{\langle s : t, R_0, R_1, E, C_0, C_1\rangle\}}{\mathsf{split}_S(\mathcal{N}) \cup \{\langle s : t, R_0 \cup R_{lr}, R_1 \cup R_{rl}, E', C_0 \cup R_{lr}, C_1 \cup R_{rl}\rangle\}}$$

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orewrite$_1$ $\dfrac{\mathcal{N} \cup \{\langle s : t, R_0, R_1, E, C_0, C_1 \rangle\}}{}$

if

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orewrite$_1$ $$\frac{\mathcal{N} \cup \{\langle s : t, R_0, R_1, E, C_0, C_1 \rangle\}}{}$$

if

- $\langle l : r, R_0', \ldots, E', \ldots \rangle \in \mathcal{N}$, $t \xrightarrow{l\sigma \to r\sigma} u$ and $t \doteq l$,

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orewrite$_1$

$$\frac{\mathcal{N} \cup \{\langle s : t, R_0, R_1, E, C_0, C_1 \rangle\}}{\quad}$$

if

- $\langle l : r, R_0', \dots, E', \dots \rangle \in \mathcal{N}$, $t \xrightarrow{l\sigma \to r\sigma} u$ and $t \doteq l$,
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Definition (oMKBtt)

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orewrite$_1$

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orewrite$_1$

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if

- $\langle l : r, R_0', \dots, E', \dots\rangle \in \mathcal{N}$, $t \xrightarrow{l\sigma \to r\sigma} u$ and $t \doteq l$,
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orewrite$_1$

$$\frac{\mathcal{N} \cup \{\langle s : t, R_0, R_1, E, C_0, C_1 \rangle\}}{\begin{array}{l}\mathcal{N} \cup \{\langle s : t, R_0 \setminus (R_0' \cup S), R_1, E \setminus R_0', C_0, C_1 \rangle \\ \quad \langle s : u, R_0 \cap (R_0' \cup S), \varnothing, E \cap R_0', \varnothing, \varnothing \rangle, \\ \quad \langle l\sigma : r\sigma, \varnothing, \varnothing, \varnothing, S, \varnothing \rangle\}\end{array}}$$

if

- $\langle l : r, R_0', \ldots, E', \ldots \rangle \in \mathcal{N}, \;\; t \xrightarrow{l\sigma \to r\sigma} u$ and $t \doteq l$,
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inference system oMKBtt consists of 5 rules

oduce

$$\frac{\mathcal{N}}{}$$

if

- $\langle l : r, R, \ldots, E, \ldots \rangle, \langle l' : r', R', \ldots, E' \ldots \rangle \in \mathcal{N},$

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oDeduce $\quad \dfrac{\mathcal{N}}{\quad}$

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- $\langle l : r, R, \ldots, E, \ldots \rangle, \langle l' : r', R', \ldots, E' \ldots \rangle \in \mathcal{N},$
- $s \leftarrow_{l \to r} u \to_{l' \to r'} t$

Definition (oMKBtt)

inference system oMKBtt consists of 5 rules

oDeduce
$$\frac{\mathcal{N}}{\mathcal{N} \cup \{\langle s : t, \varnothing, \varnothing, (R \cup E) \cap (R' \cup E'), \varnothing, \varnothing\rangle\}}$$

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- $\langle l : r, R, \ldots, E, \ldots\rangle, \langle l' : r', R', \ldots, E' \ldots\rangle \in \mathcal{N}$,
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Lemma (Simulation Properties)

$$\mathcal{N} \vdash_{oMKBtt} \mathcal{N}'$$

if and only if for every process p in $\mathcal{N}'$

$$(E_p(\mathcal{N}), R_p(\mathcal{N}), C_p(\mathcal{N})) \vdash^{=}_{oKBtt} (E_p(\mathcal{N}'), R_p(\mathcal{N}'), C_p(\mathcal{N}'))$$

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Theorem (Correctness)

Let $oMKBtt_{total}$ *run* $\mathcal{N}_{\mathcal{E}} \vdash^* \mathcal{N}$ *be sufficiently fair for p.*

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oMKBtt using total termination techniques

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if and only if for every process p in $\mathcal{N}'$

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Theorem (Correctness)

Let oMKBtt$_{total}$ *run* $\mathcal{N}_{\mathcal{E}} \vdash^* \mathcal{N}$ *be sufficiently fair for p.*

Then $E_p(\mathcal{N}) \cup R_p(\mathcal{N})$ *has same theory as* $\mathcal{E}$, *is ground-confluent for total reduction order* $>$ *extending* $\to^+_{\mathcal{C}}$, *where* $\mathcal{C} = C_p(\mathcal{N})$ *and such* $>$ *exists.*

Example

oMKBtt run on

$$\mathcal{N}_0 = \begin{cases} \langle \mathsf{g}(\mathsf{f}(x, \mathsf{b})) : \mathsf{a}, \varnothing, \varnothing, \{\epsilon\}, \varnothing, \varnothing \rangle \\ \langle \mathsf{f}(\mathsf{g}(x), y) : \mathsf{f}(x, \mathsf{g}(y)), \varnothing, \varnothing, \{\epsilon\}, \varnothing, \varnothing \rangle \end{cases}$$

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$$\mathcal{N}_0 = \begin{cases} \langle \mathsf{g}(\mathsf{f}(x,\mathsf{b})) : \mathsf{a}, \varnothing, \varnothing, \{\epsilon\}, \varnothing, \varnothing \rangle \\ \langle \mathsf{f}(\mathsf{g}(x), y) : \mathsf{f}(x, \mathsf{g}(y)), \varnothing, \varnothing, \{\epsilon\}, \varnothing, \varnothing \rangle \end{cases}$$

where termination checks use polynomial interpretation

$$[\mathsf{f}](x,y) = x + 2y + 1,\ [\mathsf{g}](x) = x + 1 \text{ and } [\mathsf{a}] = [\mathsf{b}] = [\mathsf{c}] = 0$$

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oMKBtt run on

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succeeds with

$$\mathcal{E} \cup \mathcal{R} = \begin{cases} \mathsf{f}(\mathsf{f}(x,\mathsf{b}),\mathsf{a}) \approx \mathsf{f}(\mathsf{c},\mathsf{f}(y,\mathsf{b})) & \mathsf{g}(\mathsf{f}(x,\mathsf{b})) \to \mathsf{a} \\ \mathsf{f}(\mathsf{f}(x,\mathsf{b}),\mathsf{a}) \approx \mathsf{f}(\mathsf{f}(y,\mathsf{b}),\mathsf{a}) & \mathsf{f}(x,\mathsf{g}(y)) \to \mathsf{f}(\mathsf{g}(x),y) \\ \mathsf{f}(\mathsf{c},\mathsf{f}(x,\mathsf{b})) \approx \mathsf{f}(\mathsf{c},\mathsf{f}(y,\mathsf{b})) & \mathsf{f}(\mathsf{g}(x),\mathsf{f}(y,\mathsf{b})) \to \mathsf{f}(x,\mathsf{c}) \end{cases}$$

Example

oMKBtt run on

$$\mathcal{N}_0 = \left\{ \begin{array}{l} \langle \mathsf{g}(\mathsf{f}(x,\mathsf{b})) : \mathsf{a}, \varnothing, \varnothing, \{\epsilon\}, \varnothing, \varnothing \rangle \\ \langle \mathsf{f}(\mathsf{g}(x), y) : \mathsf{f}(x, \mathsf{g}(y)), \varnothing, \varnothing, \{\epsilon\}, \varnothing, \varnothing \rangle \end{array} \right.$$

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- no finite completion using LPO or KBO
  as orientation $\mathsf{f}(\mathsf{g}(x),y) \to \mathsf{f}(x,\mathsf{g}(y))$ leads to divergence

Refutational Theorem Proving with oMKBtt

Definition

Given ground conjecture $s \approx t$ and axioms $\mathcal{E}$, initial node set is

$$\begin{aligned} \mathcal{N}_{\mathcal{E}}^{s \approx t} = \mathcal{N}_{\mathcal{E}} &\cup \{\langle \mathsf{eq}(x, x) : \mathsf{true}, \varnothing, \varnothing, \{\epsilon\}, \ldots \rangle\} \\ &\cup \{\langle \mathsf{eq}(s, t) : \mathsf{false}, \varnothing, \varnothing, \{\epsilon\}, \ldots \rangle\} \end{aligned}$$

for fresh symbols eq, true and false

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for fresh symbols eq, true and false

Lemma

- *If* $\mathcal{N}_{\mathcal{E}}^{s\approx t} \vdash^* \mathcal{N} \cup \{\langle \mathsf{true} : \mathsf{false}, \ldots\rangle\}$ *then* $s \approx t \in \leftrightarrow_{\mathcal{E}}^*$

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for fresh symbols eq, true and false

Lemma

- *If* $\mathcal{N}_{\mathcal{E}}^{s \approx t} \vdash^{*} \mathcal{N} \cup \{\langle \mathsf{true} : \mathsf{false}, \ldots \rangle\}$ *then* $s \approx t \in \leftrightarrow_{\mathcal{E}}^{*}$
- *If* $s \approx t \in \leftrightarrow_{\mathcal{E}}^{*}$ *then sufficiently fair* $\mathit{oMKBtt}_{\mathit{total}}$ *run generates* $\langle \mathsf{true} : \mathsf{false}, \ldots \rangle$

Experiments

Ordered Completion

- 767 theories of TPTP UEQ systems

oMKBtt interfacing T_TT_2 for termination checks

oMKBtt

kbo		lpo		mpo		poly		ttt_2total	
93	*20*	47	*90*	83	*19*	79	*21*	82	*23*

(1) # successes (2) *average execution time for success in seconds*

Experiments

Ordered Completion

- 767 theories of TPTP UEQ systems

T_TT_2 combining multiple total termination techniques

oMKBtt

kbo		lpo		mpo		poly		ttt_2total	
93	*20*	47	*90*	83	*19*	79	*21*	82	*23*

(1) # successes (2) *average execution time for success in seconds*

Experiments

Ordered Completion

- 767 theories of TPTP UEQ systems

oMKBtt										E	
kbo		lpo		mpo		poly		ttt_2total		auto	
93	*20*	47	*90*	83	*19*	79	*21*	82	*23*	45	*<1*

(1) # successes (2) *average execution time for success in seconds*

Experiments

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oMKBtt										E	
kbo		lpo		mpo		poly		ttt_2total		auto	
93	*20*	47	*90*	83	*19*	79	*21*	82	*23*	45	*<1*

Theorem Proving

- TPTP UEQ systems

	oMKBtt							
	kbo		lpo		poly		ttt_2fast	
easy (215)	197	*17*	164	*27*	143	*59*	138	*50*
difficult (565)	179	*64*	152	*50*	109	*96*	121	*55*

(1) # successes (2) *average execution time for success in seconds*

Experiments

Ordered Completion

- 767 theories of TPTP UEQ systems

oMKBtt										E	
kbo		lpo		mpo		poly		ttt_2total		auto	
93	*20*	47	*90*	83	*19*	79	*21*	82	*23*	45	*<1*

Theorem Proving

- TPTP UEQ systems

$\mathsf{T_TT_2}$ using DPs, DG and LPO

	oMKBtt							
	kbo		lpo		poly		ttt_2fast	
easy (215)	197	*17*	164	*27*	143	*59*	138	*50*
difficult (565)	179	*64*	152	*50*	109	*96*	121	*55*

(1) # successes (2) *average execution time for success in seconds*

Experiments

Ordered Completion

- 767 theories of TPTP UEQ systems

oMKBtt										E	
kbo		lpo		mpo		poly		ttt_2total		auto	
93	*20*	47	*90*	83	*19*	79	*21*	82	*23*	45	*<1*

Theorem Proving

- TPTP UEQ systems

	oMKBtt								Waldmeister	
	kbo		lpo		poly		ttt_2fast		auto	
easy (215)	197	*17*	164	*27*	143	*59*	138	*50*	199	*<2*
difficult (565)	179	*64*	152	*50*	109	*96*	121	*55*	>400	*<5*

(1) # successes (2) *average execution time for success in seconds*

Experiments

Ordered Completion

- 767 theories of TPTP UEQ systems

oMKBtt										E	
kbo		lpo		mpo		poly		ttt_2total		auto	
93	*20*	47	*90*	83	*19*	79	*21*	82	*23*	45	*<1*

Theorem Proving

- TPTP UEQ systems

	oMKBtt								Waldmeister	
	kbo		lpo		poly		ttt_2fast		auto	
easy (215)	197	*17*	164	*27*	143	*59*	138	*50*	199	*<2*
difficult (565)	179	*64*	152	*50*	109	*96*	121	*55*	>400	*<5*
CASC-J5 (100)	9	*47*							95	*13*

(1) # successes (2) *average execution time for success in seconds*

Conclusion

- oMKBtt is ordered completion tool + equational theorem prover not requiring explicit reduction order as input
- oMKBtt combines termination tools with multi-completion approach
- ground-confluence only with restriction on termination techniques

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Future Work

- check applicability to other variants of completion
- performance of implementation
- new competition: (ordered) completion?