Automated Inference of Upper Complexity Bounds for Java Programs

Florian Frohn[1] Marc Brockschmidt[2] Jürgen Giesl[1]

[1] RWTH Aachen University, Germany

[2] Microsoft Research, Cambridge, UK

September 6, 2016

Java and JBC

```
class List{
  int value; List next;
  List(int v, List n){...}
  boolean member(int n){...}
  int max(){...}

  List sort(){
    int n = 0;
    List r = null;
    while (this.max() >= n){
      if (this.member(n))
        r = new List(n,r);
      n++;
    }
    return r;
  }

}
```

Java and JBC

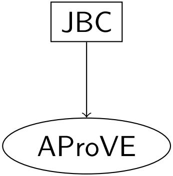
```java
class List{
  int value; List next;
  List(int v, List n){...}
  boolean member(int n){...}
  int max(){...}

  List sort(){
    int n = 0;
    List r = null;
    while (this.max() >= n){
      if (this.member(n))
        r = new List(n,r);
      n++;
    }
    return r;
  }

}
```

```
IntList sort();
  Code:
     0: iconst_0
     1: istore_1
     2: aconst_null
     3: astore_2
     4: aload_0
     5: invokevirtual #4
     8: iload_1
     9: if_icmplt 36
    12: aload_0
    13: iload_1
    14: invokevirtual #5
    17: ifeq 30
    20: new #6
    23: dup
    24: iload_1
    25: aload_2
    26: invokespecial #7
    29: astore_2
    30: iinc 1, 1
    33: goto 4
    36: aload_2
    37: areturn
```

Overview

JBC

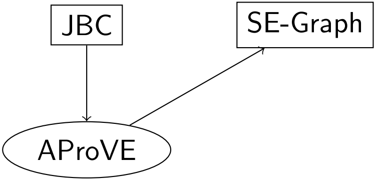
Overview



JBC

AProVE

Overview

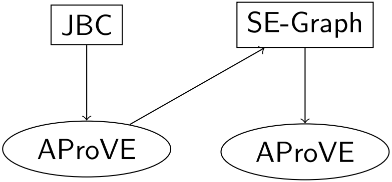


JBC

SE-Graph

AProVE

Overview

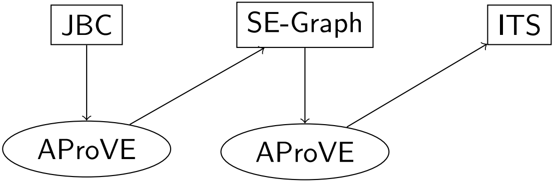

JBC

SE-Graph

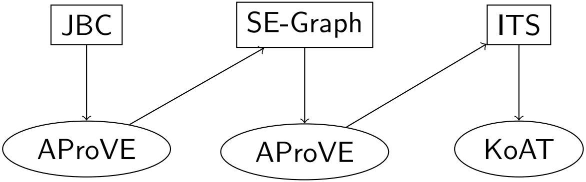
AProVE

AProVE

Overview



Overview


JBC

SE-Graph

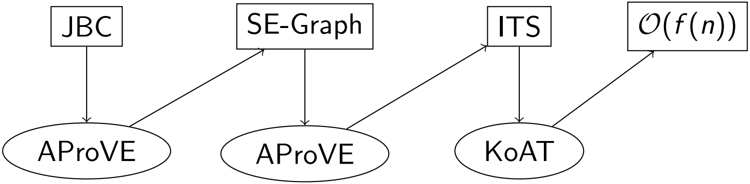
ITS

AProVE

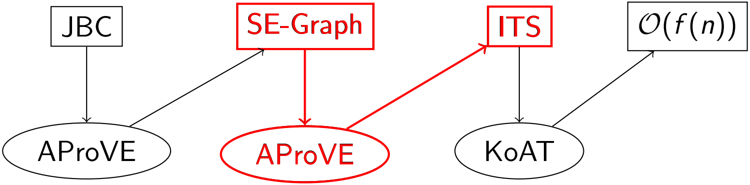
AProVE

KoAT

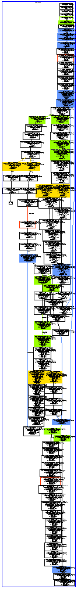
Overview



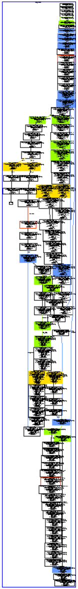
Overview



AProVE's Symbolic Evaluation Graphs

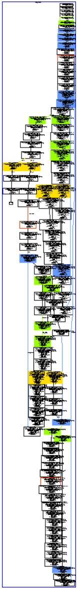


AProVE's Symbolic Evaluation Graphs



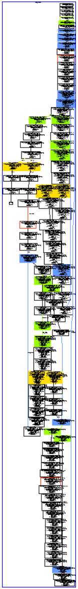
- developed for termination analysis

AProVE's Symbolic Evaluation Graphs



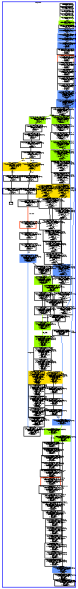
- developed for termination analysis
- intuition: CFG with invariants

AProVE's Symbolic Evaluation Graphs



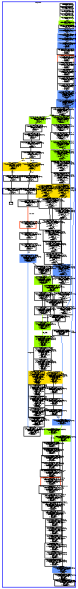
- developed for termination analysis
- intuition: CFG with invariants
  - node $\iff$ program location

AProVE's Symbolic Evaluation Graphs



- developed for termination analysis
- intuition: CFG with invariants
    - node $\iff$ program location
    - node-content $\iff$ invariant

AProVE's Symbolic Evaluation Graphs



- developed for termination analysis
- intuition: CFG with invariants
  - node $\Longleftrightarrow$ program location
  - node-content $\Longleftrightarrow$ invariant
- details: see ...
  - Otto et al. RTA '10
  - Brockschmidt et al., RTA '11
  - Brockschmidt et al., FoVeOOS '11
  - Brockschmidt et al., CAV '12
  - ...

Nodes

$\texttt{New List} \mid \texttt{this} : o_1, \texttt{n} : i_1, \texttt{r} : o_2 \mid \varepsilon$

$o_1 : \texttt{List}, \; o_2 : \texttt{List}$

$i_1 \geq 0$

Nodes

$\texttt{New List} \mid \texttt{this} : o_1, \texttt{n} : i_1, \texttt{r} : o_2 \mid \varepsilon$
$o_1 : \texttt{List},\ o_2 : \texttt{List}$
$i_1 \geq 0$

Invariants:

- `this` is a DAG

Nodes

$\texttt{New List} \mid \texttt{this} : o_1, \texttt{n} : i_1, \texttt{r} : o_2 \mid \varepsilon$

$o_1 : \texttt{List}$, $o_2 : \texttt{List}$

$i_1 \geq 0$

Invariants:

- `this` is a DAG
  - otherwise: $o_1\circlearrowleft$

Nodes

$$\boxed{\begin{array}{l}\texttt{New List} \mid \texttt{this} : o_1, \texttt{n} : i_1, \texttt{r} : o_2 \mid \varepsilon \\ o_1 : \texttt{List},\ o_2 : \texttt{List} \\ i_1 \geq 0\end{array}}$$

Invariants:

- `this` is a DAG
  - otherwise: $o_1 \circlearrowleft$
- `this` and `r` don't share

Nodes

$$\boxed{\begin{array}{l} \texttt{New List} \mid \texttt{this} : o_1, \texttt{n} : i_1, \texttt{r} : o_2 \mid \varepsilon \\ o_1 : \texttt{List},\ o_2 : \texttt{List} \\ i_1 \geq 0 \end{array}}$$

Invariants:

- `this` is a DAG
  - otherwise: $o_1 \circlearrowleft$
- `this` and `r` don't share
  - otherwise: $o_1 \curlyvee o_2$

Nodes

$\texttt{New List} \mid \texttt{this} : o_1, \texttt{n} : i_1, \texttt{r} : o_2 \mid \varepsilon$
$o_1 : \texttt{List},\ o_2 : \texttt{List}$
$i_1 \geq 0$

Invariants:

- `this` is a DAG
  - otherwise: $o_1 \circlearrowleft$
- `this` and `r` don't share
  - otherwise: $o_1 \searrow\!\swarrow o_2$
- $\texttt{n} \geq 0$

Goal: Transform SE-Graph to Integer Transition Systems

```
start(o522', i190) ->
  sort_ConstantStackPush_1(o522', i190)
sort_ConstantStackPush_1(o1) ->
  sort_Load_573(o1, 0, o1, o3', i1') |
  -o1 < i1' && o1 > 0 && o3' >= 0 && i1' < o1 && o3' < o1
sort_EQ_744(o529, x, i147, o531, o530, i172) ->
  sort_Inc_750(o529, i147, o531, o530, i172) |
  0 <= i147 && o530 >= 0 && o531 > 0 && o529 > 0 && x = 0
member_NE_734(i193, x, o521, o507, o509, o522, o508, i172) ->
  sort_EQ_744(o507, 1, i193, o509, o508, i172) |
  o509 > 0 && 0 <= i193 && o522 >= 0 && o508 >= 0 && o507 > 0 && o521 > 0 && x = i193
member_NE_734(i193, i147, o521, o507, o509, o522, o508, i172) ->
  member_Load_720(i147, o522, o507, o509, o508, i172) |
  o509 > 0 && 0 <= i147 && o522 >= 0 && o508 >= 0 && o521 > 0 && o507 > 0 && ...
sort_EQ_744(o529, x, i147, o531, o530, i172) ->
  sort_Inc_750(o529, i147, o542'1, o530, i172) |
  0 <= i147 && 0 <= 1 && o530 >= 0 && o542'1 > 0 && o531 > 0 && o529 > 0 && ...
max_Load_653(o438, i188, o439, i147, o441, o440, i172) ->
  max_NULL_654(o438, i188, o439, i147, o441, o440, i172) |
  o440 >= 0 && o441 > 0 && o439 > 0 && 0 <= i188 && o438 >= 0 && 0 <= i147
max_NULL_654(x, i188, o439, i147, o441, o440, i172) ->
  member_Load_720(i147, o439, o439, o441, o440, i172) |
  i188 >= i147 && 0 <= i147 && o440 >= 0 && o439 >= 0 && 0 <= i188 && o439 > 0 && ...
max_FieldAccess_679(o453, i188, o439, i147, o441, o454, i190, o440, i172) ->
  max_Load_653(o454, i188, o439, i147, o441, o440, i172) |
  o453 > 0 && 0 <= i147 && o439 > 0 && 0 <= i188 && o441 > 0 && o440 >= 0 && o454 >= 0
max_NULL_654(o449, i188, o439, i147, o441, o440, i172) ->
  max_LE_668(i190', i188, o449, o439, i147, o441, o454', o440, i172) |
  -o449 < i190' && 0 <= i147 && o440 >= 0 && o449 > 0 && o441 > 0 && 0 <= i188 && ...
...
```

Integer Transition System

rule-based representation of Integer Programs

Integer Transition System

rule-based representation of Integer Programs

Example

$$\begin{array}{lcl} f_{\mathsf{start}}(x) & \rightarrow & f(x) \\ f(x) & \rightarrow & f(x-z) \quad \mid \quad x > 0 \wedge z > 0 \end{array}$$

Integer Transition System

rule-based representation of Integer Programs

Example

$$\begin{array}{lcll} f_{\mathsf{start}}(x) & \to & f(x) & \\ f(x) & \to & f(x-z) & \mid \quad x > 0 \wedge z > 0 \end{array}$$

$f_{\mathsf{start}}(3)$

Integer Transition System

rule-based representation of Integer Programs

Example

$$\begin{array}{lcll} f_{\text{start}}(x) & \rightarrow & f(x) & \\ f(x) & \rightarrow & f(x-z) & \mid \quad x > 0 \land z > 0 \end{array}$$

$f_{\text{start}}(3) \rightarrow f(3)$

Integer Transition System

rule-based representation of Integer Programs

Example

$$\begin{array}{lcll} f_{\mathsf{start}}(x) & \to & f(x) & \\ f(x) & \to & f(x-z) & \mid \quad x > 0 \land z > 0 \end{array}$$

$f_{\mathsf{start}}(3) \to f(3) \to f(1)$

Integer Transition System

rule-based representation of Integer Programs

Example

$$\begin{array}{lcll} f_{\text{start}}(x) & \to & f(x) & \\ f(x) & \to & f(x-z) & \mid \quad x > 0 \wedge z > 0 \end{array}$$

$f_{\text{start}}(3) \to f(3) \to f(1) \to f(-2)$

Integer Transition System

rule-based representation of Integer Programs

Example

$$f_{\text{start}}(x) \quad \to \quad f(x)$$

$f($

$f_{\text{start}}(3) \to f(3$

Why Not Term Rewriting?

Integer Transition System

rule-based representation of Integer Programs

Example

$$f_{\text{start}}(x) \rightarrow f(x)$$

$f($

$f_{\text{start}}(3) \rightarrow f(3$

Why Not Term Rewriting?

- no integers

Integer Transition System

rule-based representation of Integer Programs

Example

$$f_{\mathsf{start}}(x) \quad \to \quad f(x)$$

$f($

$f_{\mathsf{start}}(3) \to f(3$

Why Not Term Rewriting?

- no integers
- no start states

SE-Graph → Integer Transition System

- translate each edge to a rule

SE-Graph → Integer Transition System

- translate each edge to a rule
- challenges

SE-Graph → Integer Transition System

- translate each edge to a rule
- challenges
    - abstract objects to integers

SE-Graph → Integer Transition System

- translate each edge to a rule
- challenges
  - abstract objects to integers
  - encode semantics of JBC instructions

SE-Graph → Integer Transition System

- translate each edge to a rule
- challenges
  - **abstract objects to integers**
  - encode semantics of JBC instructions

$\texttt{New List} \mid \texttt{this} : o_1, \texttt{n} : i_1, \texttt{r} : o_2 \mid \varepsilon$
$o_1 : \texttt{List},\ o_2 : \texttt{List}$
$i_1 \geq 0$

$\curvearrowright \quad f(o_1,i_1,o_2)$

Size Abstraction

- terms are trees ↷ number of nodes

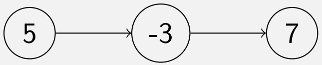
Size Abstraction

- terms are trees $\curvearrowright$ number of nodes
- objects are graphs $\curvearrowright$ number of nodes

Size Abstraction

- terms are trees $\curvearrowright$ number of nodes
- objects are graphs $\curvearrowright$ number of nodes

Example



Size Abstraction

- terms are trees $\curvearrowright$ number of nodes
- objects are graphs $\curvearrowright$ number of nodes

Example

```
...
while (this.max() >= n){
  if (this.member(n))
    r = new List(n,r);
  n++;
}
...
```

Size Abstraction

- terms are trees $\curvearrowright$ number of nodes
- objects are graphs $\curvearrowright$ number of nodes

Example

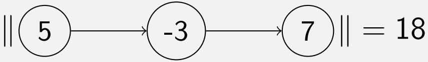
```
...
while (this.max() >= n){
  if (this.member(n))
    r = new List(n,r);
  n++;
}
...
```

$$\|o\| = \#\text{reachable objects} + \sum \text{absolute values of reachable integers}$$

Size Abstraction

- terms are trees $\curvearrowright$ number of nodes
- objects are graphs $\curvearrowright$ number of nodes

Example



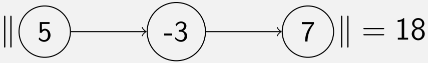
```
...
while (this.max() >= n){
  if (this.member(n))
    r = new List(n,r);
  n++;
}
...
```

$$\|o\| = \#\text{reachable objects} + \sum \text{absolute values of reachable integers}$$

Size Abstraction

- terms are trees $\curvearrowright$ number of nodes
- objects are graphs $\curvearrowright$ number of nodes

Example



```
...
while (this.max() >= n){
  if (this.member(n))
    r = new List(n,r);
  n++;
}
...
```

$\|o\|$ = #reachable objects + $\sum$ absolute values of reachable integers

$$\curvearrowright \mathcal{O}(\|\texttt{this}\|^2)$$

SE-Graph → Integer Transition System

- translate each edge to a rule
- challenges
  - **abstract objects to integers**
  - encode semantics of JBC instructions

$\texttt{New List} \mid \texttt{this} : o_1, \texttt{n} : i_1, \texttt{r} : o_2 \mid \varepsilon$

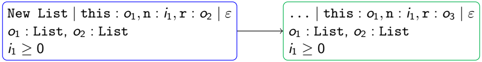
$o_1 : \texttt{List},\ o_2 : \texttt{List}$

$i_1 \geq 0$

$\curvearrowright \quad f(o_1,i_1,o_2)$

SE-Graph → Integer Transition System

- translate each edge to a rule
- challenges
  - abstract objects to integers
  - **encode semantics of JBC instructions**



`New List` | `this` : o_1, `n` : i_1, `r` : o_2 | ε
o_1 : `List`, o_2 : `List`
$i_1 \geq 0$

`...` | `this` : o_1, `n` : i_1, `r` : o_3 | ε
o_1 : `List`, o_2 : `List`
$i_1 \geq 0$

$\curvearrowright \quad f(o_1,i_1,o_2) \rightarrow g(o_1,i_1,o_2,o_3) | \varphi$

Encoding New

$\|o\| = \#$reachable objects $+ \sum$ absolute values of reachable integers

Encoding New

$\|o\| = \#$reachable objects $+ \sum$ absolute values of reachable integers

A Fresh List Instance

Encoding New

$\|o\| = \#$reachable objects $+ \sum$ absolute values of reachable integers

A Fresh List Instance

$$\left\| \textcircled{0} \longrightarrow \texttt{null} \right\| = 1$$

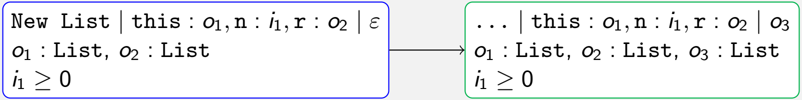
Encoding New

$$\|o\| = \#\text{reachable objects} + \sum \text{absolute values of reachable integers}$$

A Fresh List Instance

$$\left\| \textcircled{0} \longrightarrow \texttt{null} \right\| = 1$$

Example (Create New List Instance)



$$\boxed{\begin{array}{l} \texttt{New List} \mid \texttt{this} : o_1, \texttt{n} : i_1, \texttt{r} : o_2 \mid \varepsilon \\ o_1 : \texttt{List},\ o_2 : \texttt{List} \\ i_1 \geq 0 \end{array}} \longrightarrow \boxed{\begin{array}{l} \texttt{...} \mid \texttt{this} : o_1, \texttt{n} : i_1, \texttt{r} : o_2 \mid o_3 \\ o_1 : \texttt{List},\ o_2 : \texttt{List},\ o_3 : \texttt{List} \\ i_1 \geq 0 \end{array}}$$

Encoding New

$\|o\| = \#$reachable objects $+ \sum$ absolute values of reachable integers

A Fresh List Instance

$$\| \textcircled{0} \longrightarrow \texttt{null} \| = 1$$

Example (Create New List Instance)

$$\boxed{\begin{array}{l} \texttt{New List} \mid \texttt{this} : o_1, \texttt{n} : i_1, \texttt{r} : o_2 \mid \varepsilon \\ o_1 : \texttt{List},\ o_2 : \texttt{List} \\ i_1 \geq 0 \end{array}} \longrightarrow \boxed{\begin{array}{l} \texttt{...} \mid \texttt{this} : o_1, \texttt{n} : i_1, \texttt{r} : o_2 \mid o_3 \\ o_1 : \texttt{List},\ o_2 : \texttt{List},\ o_3 : \texttt{List} \\ i_1 \geq 0 \end{array}}$$

$$f(\|o_1\|, i_1, \|o_2\|) \to g(\|o_1\|, i_1, \|o_2\|, \|o_3\|) \mid \|o_1\| \geq 0 \wedge \|o_2\| \geq 0 \wedge i_1 \geq 0 \wedge \|o_3\| = 1$$

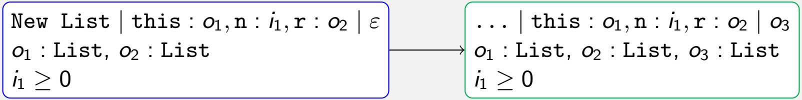
Encoding New

$\|o\| = \#$reachable objects $+ \sum$ absolute values of reachable integers

A Fresh List Instance

$$\left\| \text{\textcircled{0}} \longrightarrow \texttt{null} \right\| = 1$$

Example (Create New List Instance)



`New List` | `this` : o_1, `n` : i_1, `r` : o_2 | ε
o_1 : `List`, o_2 : `List`
$i_1 \geq 0$

`...` | `this` : o_1, `n` : i_1, `r` : o_2 | o_3
o_1 : `List`, o_2 : `List`, o_3 : `List`
$i_1 \geq 0$

$$f(\ o_1\ , i_1,\ o_2\ ) \rightarrow g(\ o_1\ , i_1,\ o_2\ ,\ o_3\ ) \mid\ o_1\ \geq 0 \wedge\ o_2\ \geq 0 \wedge i_1 \geq 0 \wedge\ o_3\ = 1$$

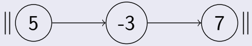
Encoding Write Accesses

$\|o\| = \#$reachable objects $+ \sum$ absolute values of reachable integers

Encoding Write Accesses

$\|o\| = \#\text{reachable objects} + \sum \text{absolute values of reachable integers}$

Write to Value



$= 18$

Encoding Write Accesses

$\|o\| = \#\text{reachable objects} + \sum \text{absolute values of reachable integers}$

Write to Value

Encoding Write Accesses

$\|o\| = \#\text{reachable objects} + \sum \text{absolute values of reachable integers}$

Write to Value

$\| 5 \longrightarrow -3 \longrightarrow 7 \| = 18 \curvearrowright \| 5 \longrightarrow -3 \longrightarrow -9 \| = 20$

Encoding Write Accesses

$\|o\| = \#$reachable objects $+ \sum$ absolute values of reachable integers

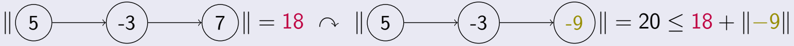
Write to Value

$$\|5 \longrightarrow -3 \longrightarrow 7\| = 18 \curvearrowright \|5 \longrightarrow -3 \longrightarrow -9\| = 20 \leq 18 + \|-9\|$$

Encoding Write Accesses

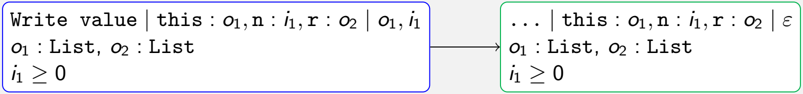
$$\|o\| = \#\text{reachable objects} + \sum \text{absolute values of reachable integers}$$

Write to Value



$$\|5 \rightarrow -3 \rightarrow 7\| = 18 \curvearrowright \|5 \rightarrow -3 \rightarrow -9\| = 20 \leq 18 + \|-9\|$$

Example (Write i_1 to o_1.value)



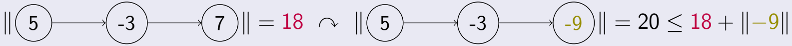
$\texttt{Write value} \mid \texttt{this} : o_1, \texttt{n} : i_1, \texttt{r} : o_2 \mid o_1, i_1$
$o_1 : \texttt{List},\ o_2 : \texttt{List}$
$i_1 \geq 0$

$\ldots \mid \texttt{this} : o_1, \texttt{n} : i_1, \texttt{r} : o_2 \mid \varepsilon$
$o_1 : \texttt{List},\ o_2 : \texttt{List}$
$i_1 \geq 0$

Encoding Write Accesses

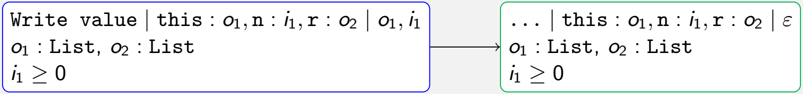
$\|o\| = \#$reachable objects $+ \sum$ absolute values of reachable integers

Write to Value



$\|5 \to \text{-}3 \to 7\| = 18 \curvearrowright \|5 \to \text{-}3 \to \text{-}9\| = 20 \leq 18 + \|-9\|$

Example (Write i_1 to o_1.value)



`Write value` | `this` : o_1, `n` : i_1, `r` : o_2 | o_1, i_1
o_1 : `List`, o_2 : `List`
$i_1 \geq 0$

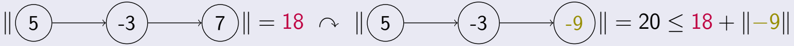
... | `this` : o_1, `n` : i_1, `r` : o_2 | ε
o_1 : `List`, o_2 : `List`
$i_1 \geq 0$

$$f(o_1, i_1, o_2) \quad \to \quad g(o_1', i_1, o_2) \quad | \quad \ldots \land i_1 \geq 0 \land o_1 + i_1 \geq o_1'$$

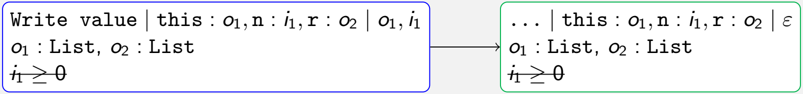
Encoding Write Accesses

$\|o\| = \#$reachable objects $+ \sum$ absolute values of reachable integers

Write to Value



Example (Write i_1 to o_1.value)



`Write value` | `this` : o_1, `n` : i_1, `r` : o_2 | o_1, i_1
o_1 : `List`, o_2 : `List`
$\cancel{i_1 \geq 0}$

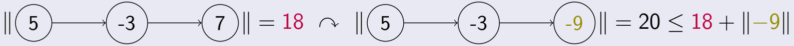
. . . | `this` : o_1, `n` : i_1, `r` : o_2 | ε
o_1 : `List`, o_2 : `List`
$\cancel{i_1 \geq 0}$

$$f(o_1, i_1, o_2) \;\rightarrow\; g(o_1', i_1, o_2) \;\mid\; \ldots \wedge i_1 \geq 0 \wedge o_1 + i_1 \geq o_1'$$

Encoding Write Accesses

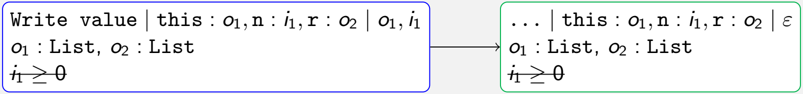
$\|o\| = \#$reachable objects $+ \sum$ absolute values of reachable integers

Write to Value



$\| 5 \to -3 \to 7 \| = 18 \curvearrowright \| 5 \to -3 \to -9 \| = 20 \leq 18 + \|-9\|$

Example (Write i_1 to o_1.`value`)



`Write value` | `this` : o_1, `n` : i_1, `r` : o_2 | o_1, i_1
o_1 : `List`, o_2 : `List`
~~$i_1 \geq 0$~~

... | `this` : o_1, `n` : i_1, `r` : o_2 | ε
o_1 : `List`, o_2 : `List`
~~$i_1 \geq 0$~~

$$\begin{array}{lcl|l} f(o_1, i_1, o_2) & \to & g(o_1', i_1, o_2) & \ldots \wedge i_1 \geq 0 \wedge o_1 + i_1 \geq o_1' \\ f(o_1, i_1, o_2) & \to & g(o_1', i_1, o_2) & \ldots \wedge i_1 < 0 \wedge o_1 - i_1 \geq o_1' \end{array}$$

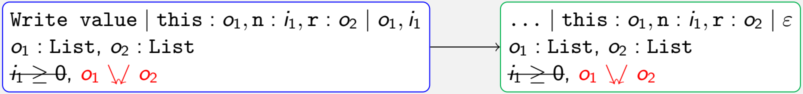
Encoding Write Accesses

$\|o\| = \#$reachable objects $+ \sum$ absolute values of reachable integers

Write to Value

$$\|5 \to -3 \to 7\| = 18 \curvearrowright \|5 \to -3 \to -9\| = 20 \leq 18 + \|-9\|$$

Example (Write i_1 to o_1.`value`)



`Write value` | `this` : o_1, `n` : i_1, `r` : o_2 | o_1, i_1
o_1 : `List`, o_2 : `List`
~~$i_1 \geq 0$~~, $o_1 \searrow\!\swarrow o_2$

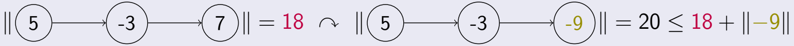
… | `this` : o_1, `n` : i_1, `r` : o_2 | ε
o_1 : `List`, o_2 : `List`
~~$i_1 \geq 0$~~, $o_1 \searrow\!\swarrow o_2$

$$\begin{array}{lclcl} f(o_1, i_1, o_2) & \to & g(o_1', i_1, o_2) & | & \ldots \land i_1 \geq 0 \land o_1 + i_1 \geq o_1' \\ f(o_1, i_1, o_2) & \to & g(o_1', i_1, o_2) & | & \ldots \land i_1 < 0 \land o_1 - i_1 \geq o_1' \end{array}$$

Encoding Write Accesses

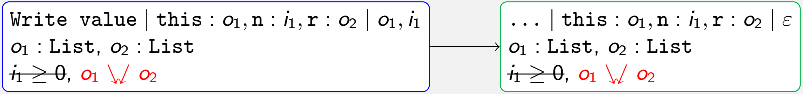
$$\|o\| = \#\text{reachable objects} + \sum \text{absolute values of reachable integers}$$

Write to Value



$$\|5 \to -3 \to 7\| = 18 \curvearrowright \|5 \to -3 \to -9\| = 20 \le 18 + \|-9\|$$

Example (Write i_1 to o_1.`value`)



`Write value` | `this` : o_1, `n` : i_1, `r` : o_2 | o_1, i_1
o_1 : `List`, o_2 : `List`
~~$i_1 \geq 0$~~, $o_1 \mathrel{\vee\!\!\!\vee} o_2$

... | `this` : o_1, `n` : i_1, `r` : o_2 | ε
o_1 : `List`, o_2 : `List`
~~$i_1 \geq 0$~~, $o_1 \mathrel{\vee\!\!\!\vee} o_2$

$$\begin{array}{lclcl}
f(o_1, i_1, o_2) & \to & g(o_1', i_1, o_2') & \mid & \ldots \wedge i_1 \geq 0 \wedge o_1 + i_1 \geq o_1' \wedge o_2 + i_1 \geq o_2' \\
f(o_1, i_1, o_2) & \to & g(o_1', i_1, o_2') & \mid & \ldots \wedge i_1 < 0 \wedge o_1 - i_1 \geq o_1' \wedge o_2 - i_1 \geq o_2'
\end{array}$$

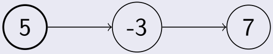
Encoding Read Accesses

$\|o\| = \#$reachable objects $+ \sum$ absolute values of reachable integers

Encoding Read Accesses

$\|o\| = \#$reachable objects $+ \sum$ absolute values of reachable integers

Read From Next

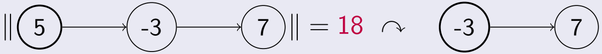
$\|$ 

$\| = 18$

Encoding Read Accesses

$\|o\| = \#$reachable objects $+ \sum$ absolute values of reachable integers

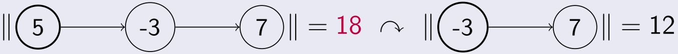
Read From Next



Encoding Read Accesses

$\|o\| = \#$reachable objects $+ \sum$ absolute values of reachable integers

Read From Next



Encoding Read Accesses

$$\|o\| = \#\text{reachable objects} + \sum \text{absolute values of reachable integers}$$

Read From Next

$$\left\| \text{5} \longrightarrow \text{-3} \longrightarrow \text{7} \right\| = 18 \;\curvearrowright\; \left\| \text{-3} \longrightarrow \text{7} \right\| = 12 < 18$$

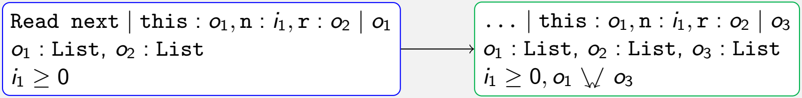
Encoding Read Accesses

$\|o\| = \#$reachable objects $+ \sum$ absolute values of reachable integers

Read From Next

$\|5 \to -3 \to 7\| = 18 \curvearrowright \|-3 \to 7\| = 12 < 18$

Example (Read o_1.`next`)



`Read next` | `this` : o_1, `n` : i_1, `r` : o_2 | o_1
o_1 : `List`, o_2 : `List`
$i_1 \geq 0$

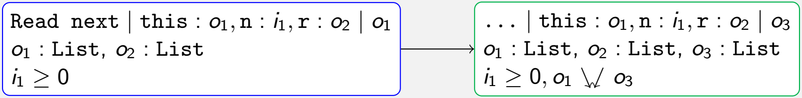
. . . | `this` : o_1, `n` : i_1, `r` : o_2 | o_3
o_1 : `List`, o_2 : `List`, o_3 : `List`
$i_1 \geq 0$, $o_1 \searrow\!\swarrow o_3$

Encoding Read Accesses

$\|o\| = \#$reachable objects $+ \sum$ absolute values of reachable integers

Read From Next

Example (Read o_1.`next`)



`Read next` | `this` : o_1, `n` : i_1, `r` : o_2 | o_1
o_1 : `List`, o_2 : `List`
$i_1 \geq 0$

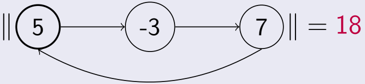
… | `this` : o_1, `n` : i_1, `r` : o_2 | o_3
o_1 : `List`, o_2 : `List`, o_3 : `List`
$i_1 \geq 0, o_1 \searrow\!\swarrow o_3$

$$f(o_1, i_1, o_2) \rightarrow g(o_1, i_1, o_2, o_3) \mid \ldots \wedge o_1 > o_3$$

Encoding Read Accesses

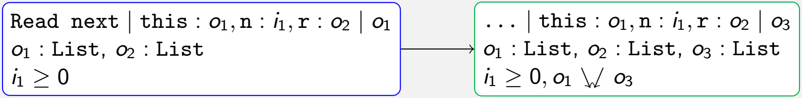
$\|o\| = \#$reachable objects $+ \sum$ absolute values of reachable integers

Read From Next



$\| \cdot \| = 18$

Example (Read o_1.next)



`Read next` | `this` : o_1, `n` : i_1, `r` : o_2 | o_1
o_1 : `List`, o_2 : `List`
$i_1 \geq 0$

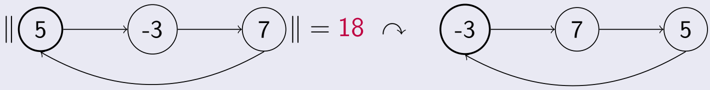
… | `this` : o_1, `n` : i_1, `r` : o_2 | o_3
o_1 : `List`, o_2 : `List`, o_3 : `List`
$i_1 \geq 0$, $o_1 \searrow\!\swarrow o_3$

$$f(o_1, i_1, o_2) \to g(o_1, i_1, o_2, o_3) \mid \ldots \wedge o_1 > o_3$$

Encoding Read Accesses

$\|o\| = \#$reachable objects $+ \sum$ absolute values of reachable integers

Read From Next



Example (Read o_1.next)

`Read next` | `this` : o_1, `n` : i_1, `r` : o_2 | o_1
o_1 : `List`, o_2 : `List`
$i_1 \geq 0$

→

... | `this` : o_1, `n` : i_1, `r` : o_2 | o_3
o_1 : `List`, o_2 : `List`, o_3 : `List`
$i_1 \geq 0, o_1 \searrow\!\!\swarrow o_3$

$$f(o_1, i_1, o_2) \to g(o_1, i_1, o_2, o_3) \mid \ldots \wedge o_1 > o_3$$

Encoding Read Accesses

$\|o\| = \#$reachable objects $+ \sum$ absolute values of reachable integers

Read From Next

Example (Read o_1.next)

$\texttt{Read next} \mid \texttt{this} : o_1, \texttt{n} : i_1, \texttt{r} : o_2 \mid o_1$
$o_1 : \texttt{List},\ o_2 : \texttt{List}$
$i_1 \geq 0$

→

$\ldots \mid \texttt{this} : o_1, \texttt{n} : i_1, \texttt{r} : o_2 \mid o_3$
$o_1 : \texttt{List},\ o_2 : \texttt{List},\ o_3 : \texttt{List}$
$i_1 \geq 0, o_1 \curlyvee o_3$

$$f(o_1, i_1, o_2) \to g(o_1, i_1, o_2, o_3) \mid \ldots \land o_1 > o_3$$

Encoding Read Accesses

$$\|o\| = \#\text{reachable objects} + \sum \text{absolute values of reachable integers}$$

Read From Next

Example (Read o_1.`next`)

Read `next` | `this` : o_1, `n` : i_1, `r` : o_2 | o_1
o_1 : `List`, o_2 : `List`
$i_1 \geq 0$, $o_1\circlearrowleft$

→

... | `this` : o_1, `n` : i_1, `r` : o_2 | o_3
o_1 : `List`, o_2 : `List`, o_3 : `List`
$i_1 \geq 0$, $o_1 \searrow\!\!\swarrow o_3$, $o_1\circlearrowleft$, $o_3\circlearrowleft$

$$f(o_1, i_1, o_2) \to g(o_1, i_1, o_2, o_3) \mid \ldots \wedge o_1 > o_3$$

Encoding Read Accesses

$$\|o\| = \#\text{reachable objects} + \sum \text{absolute values of reachable integers}$$

Read From Next

Example (Read o_1.next)

Read next | this : o_1, n : i_1, r : o_2 | o_1
o_1 : List, o_2 : List
$i_1 \geq 0$, $o_1\circlearrowleft$

$\longrightarrow$

... | this : o_1, n : i_1, r : o_2 | o_3
o_1 : List, o_2 : List, o_3 : List
$i_1 \geq 0$, $o_1 \searrow\!\swarrow o_3$, $o_1\circlearrowleft$, $o_3\circlearrowleft$

$$f(o_1, i_1, o_2) \to g(o_1, i_1, o_2, o_3) \mid \ldots \wedge o_1 \geq o_3$$

Beyond Time Complexity

- attach costs to rules

Beyond Time Complexity

- attach costs to rules
- model network traffic, IO, memory consumption, ...

Beyond Time Complexity

- attach costs to rules
- model network traffic, IO, memory consumption, ...

Example

`new`: cost $= 1$

Beyond Time Complexity

- attach <u>costs</u> to rules
- model network traffic, IO, memory consumption, ...

Example

`new`: cost $= 1$

`anewarray`: cost $=$ size of the new array

Beyond Time Complexity

- attach costs to rules
- model network traffic, IO, memory consumption, ...

Example

`new`: cost $= 1$

`anewarray`: cost $=$ size of the new array

all other instructions: cost $= 0$

Beyond Time Complexity

- attach costs to rules
- model network traffic, IO, memory consumption, ...

Example

`new`: cost $= 1$

`anewarray`: cost $=$ size of the new array

all other instructions: cost $= 0$

$\curvearrowright$ models memory consumption

Conclusion And Experiments

- novel approach for complexity analysis of JBC

Conclusion And Experiments

- novel approach for complexity analysis of JBC
    - AProVE's termination technique lifted to complexity

Conclusion And Experiments

- novel approach for complexity analysis of JBC
    - AProVE's termination technique lifted to complexity
    - transformation to standard format

Conclusion And Experiments

- novel approach for complexity analysis of JBC
    - AProVE's termination technique lifted to complexity
    - transformation to standard format
    - succeeds in some cases where termination analysis fails

Conclusion And Experiments

- novel approach for complexity analysis of JBC
  - AProVE's termination technique lifted to complexity
  - transformation to standard format
  - succeeds in some cases where termination analysis fails
- experiments (TPDB, JBC non-recursive)

Conclusion And Experiments

- novel approach for complexity analysis of JBC
    - AProVE's termination technique lifted to complexity
    - transformation to standard format
    - succeeds in some cases where termination analysis fails
- experiments (TPDB, JBC non-recursive)
    - 300 examples

Conclusion And Experiments

- novel approach for complexity analysis of JBC
    - AProVE's termination technique lifted to complexity
    - transformation to standard format
    - succeeds in some cases where termination analysis fails
- experiments (TPDB, JBC non-recursive)
    - 300 examples
    - at least 83 non-terminating

Conclusion And Experiments

- novel approach for complexity analysis of JBC
  - AProVE's termination technique lifted to complexity
  - transformation to standard format
  - succeeds in some cases where termination analysis fails
- experiments (TPDB, JBC non-recursive)
  - 300 examples
  - at least 83 non-terminating
  - 151 polynomial bounds (termination: $\sim$180)

Conclusion And Experiments

- novel approach for complexity analysis of JBC
    - AProVE's termination technique lifted to complexity
    - transformation to standard format
    - succeeds in some cases where termination analysis fails
- experiments (TPDB, JBC non-recursive)
    - 300 examples
    - at least 83 non-terminating
    - 151 polynomial bounds (termination: $\sim$180)
    - success rate: 70%

Thank You!

Questions?