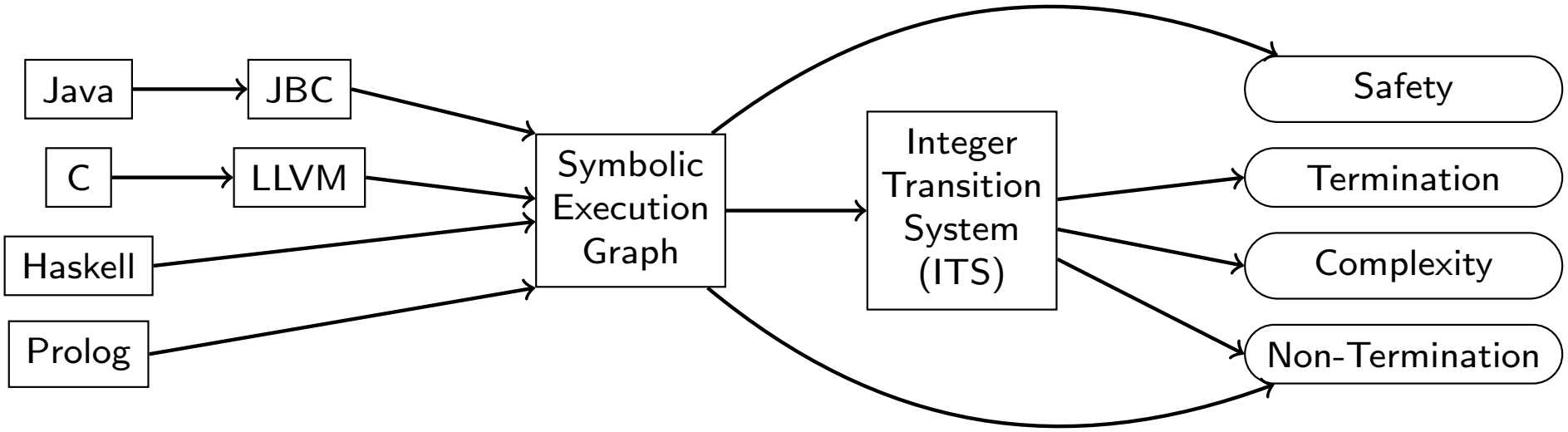
Termination Analysis of Programs with Bitvector Arithmetic by Symbolic Execution

Jürgen Giesl

LuFG Informatik 2, RWTH Aachen University, Germany

joint work with Jera Hensel, Florian Frohn, and Thomas Ströder

Termination Analysis in AProVE



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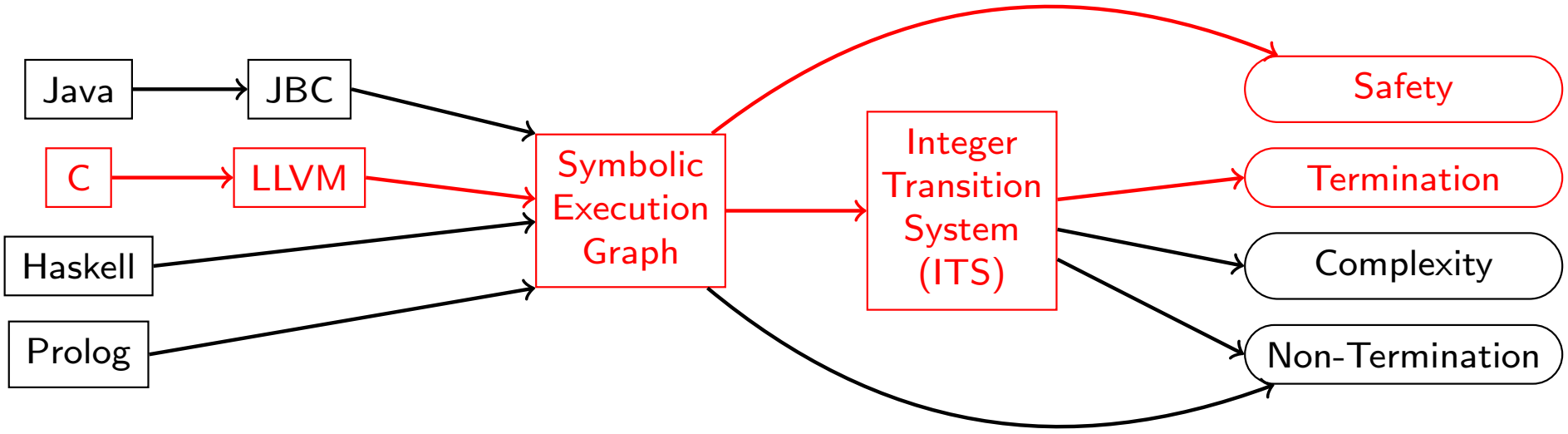

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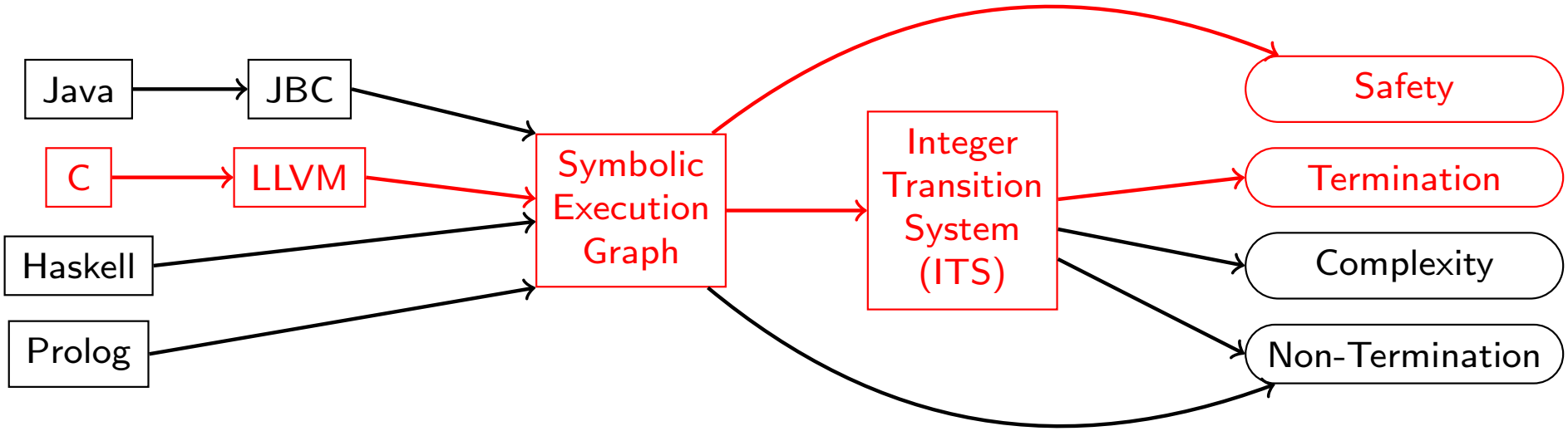

- Termination of C programs with explicit pointer arithmetic

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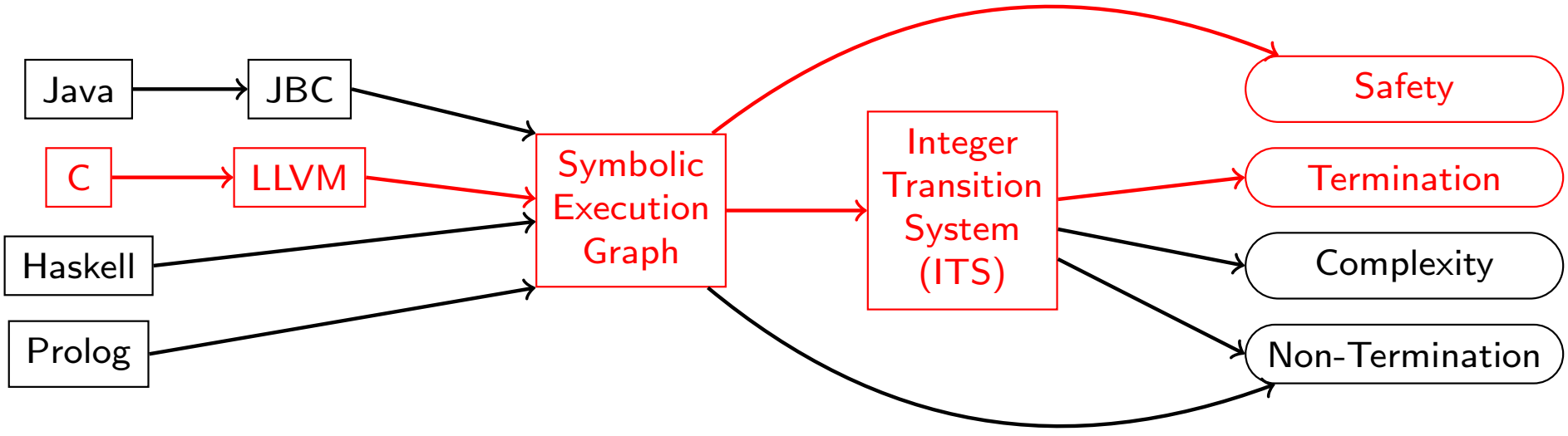

- Termination of C programs with explicit pointer arithmetic
- Winner of *SV-COMP 2015 & 2016* termination competition

+Term Comp'16

C Category

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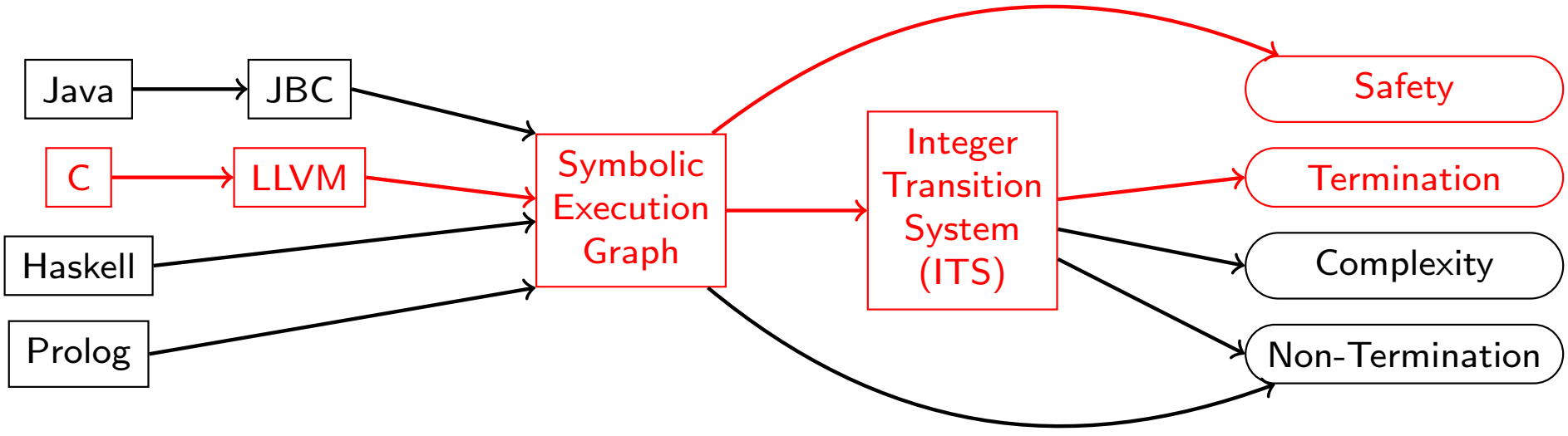

- Termination of C programs with explicit pointer arithmetic
- Winner of *SV-COMP 2015 & 2016* termination competition
- **Drawback:** assumes mathematical integers $\mathbb{Z}$ instead of bitvectors

Mathematical Integers $\mathbb{Z}$ vs. Bitvectors

```
void f(unsigned int x)  {
    unsigned int j = 0;
    while (j <= x) j++;  }
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void g(unsigned int j)  {
    while (j > 0) j++;   }
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- **Goal:** adapt termination analysis of C to bitvector arithmetic

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- **Solution:** express bitvector relations by relations on $\mathbb{Z}$

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  - standard SMT solving over $\mathbb{Z}$ for symbolic execution

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- **Goal:** adapt byte-accurate symbolic execution to bitvector arithmetic
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From C to LLVM

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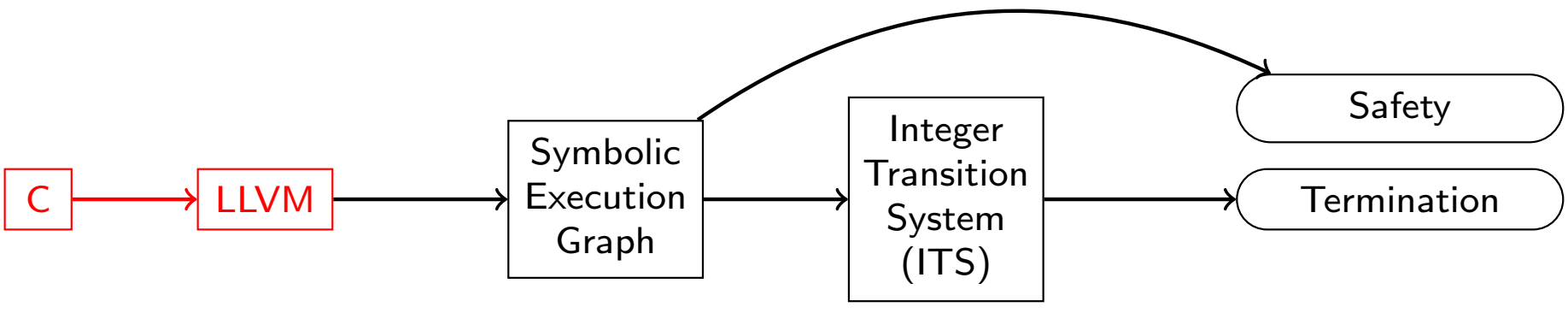

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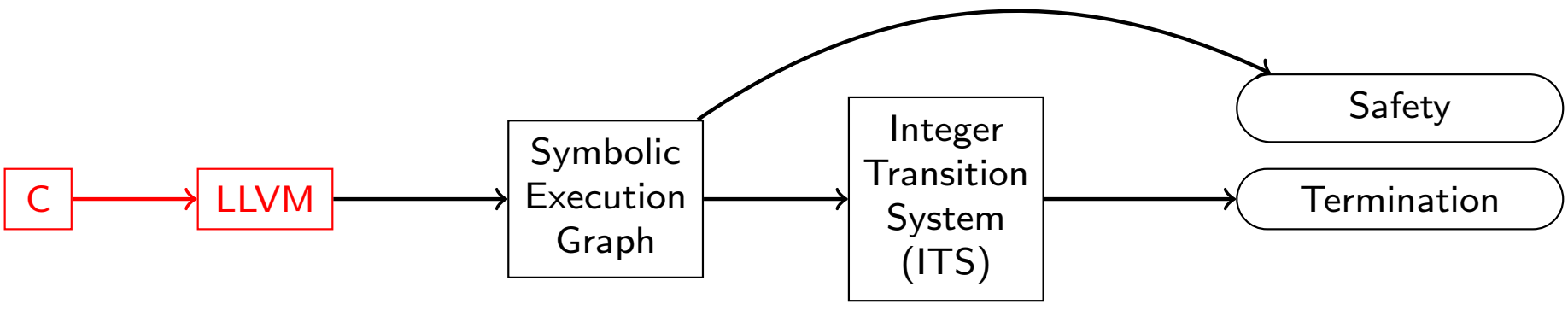

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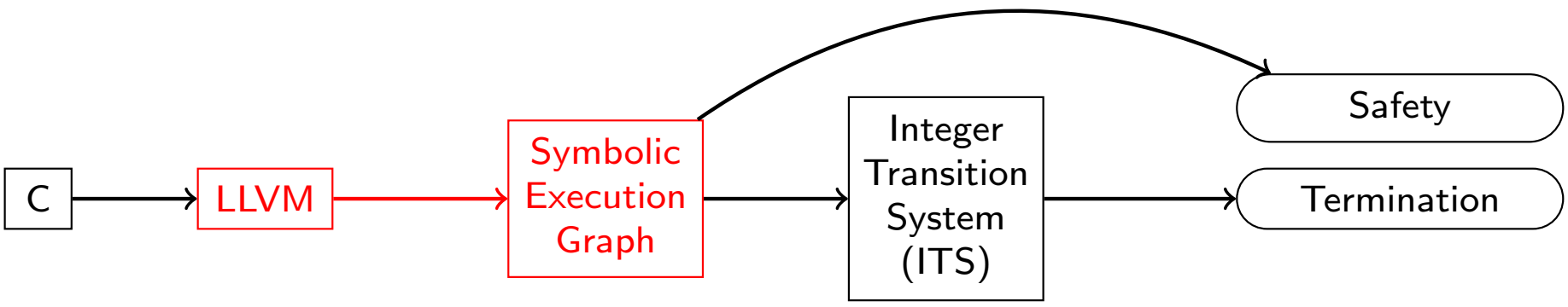

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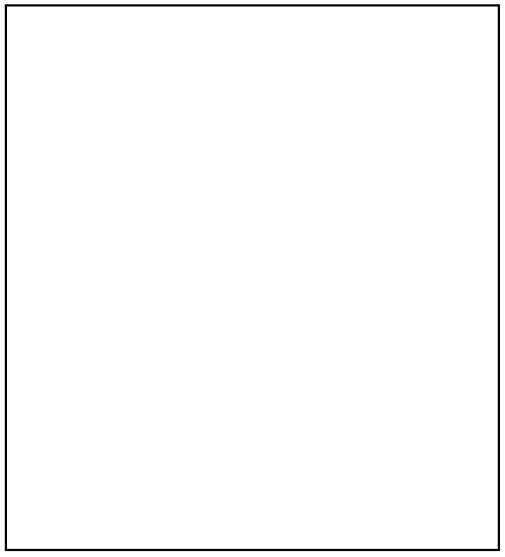

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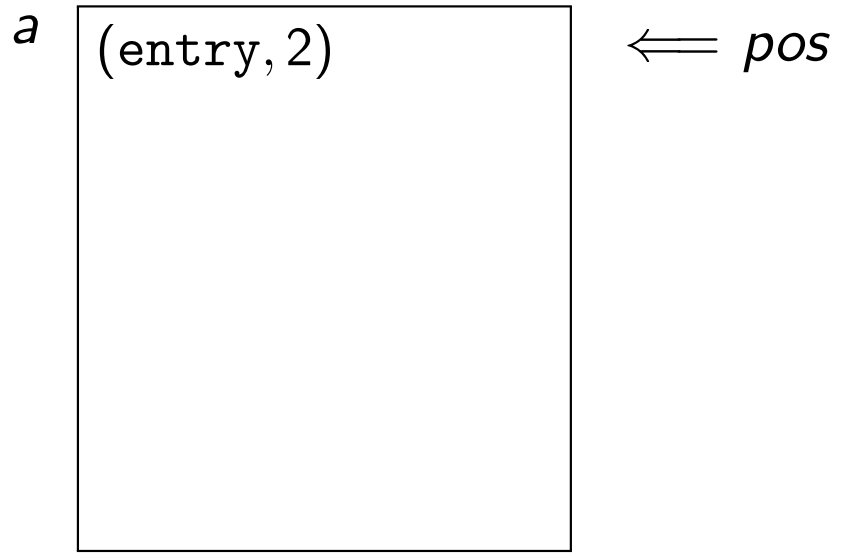

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$(\texttt{entry}, 2)$ $\Longleftarrow$ *pos*

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Abstract state *a*:

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Heuristic: partition program variables into $\mathcal{U} \uplus \mathcal{S}$

$\texttt{x} \in \mathcal{U}$: $PV(\texttt{x})$ represents value of $\texttt{x}$ as unsigned integer

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  - information on ranges of integers:

    $\mathtt{j} \in \mathcal{U}$ has type $\mathtt{i32} \quad \Rightarrow \quad 0 \leq \underbrace{PV(\mathtt{j})}_{v_{\mathtt{j}}} \leq \underbrace{\mathrm{umax}_{32}}_{2^{32}-1}$

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- $\langle a \rangle$: FO formula
- *a concrete*: $\forall$ symbolic variables v $\exists\, n \in \mathbb{Z}$ such that $\models \langle a \rangle \Rightarrow v = n$

Abstract States

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a

$$
\begin{array}{|l|l|}
\hline
(\texttt{entry}, 2) & \Longleftarrow \mathit{pos} \\
\{\mathtt{j} = v_{\mathtt{j}}, \mathtt{ad} = v_{\mathtt{ad}}\} & \Longleftarrow \mathit{PV} \\
\{[\![v_{\mathtt{ad}}, v_{\mathit{end}}]\!]\} & \Longleftarrow \mathit{AL} \\
\{v_{\mathit{end}} = v_{\mathtt{ad}} + 3\} & \Longleftarrow \mathit{KB} \\
\{v_{\mathtt{ad}} \hookrightarrow_{\mathtt{i32},u} v_{\mathtt{j}}\} & \Longleftarrow \mathit{PT} \\
\hline
\end{array}
$$

- $\langle a \rangle$: FO formula
- *a concrete*: $\forall$ symbolic variables v $\exists\, n \in \mathbb{Z}$ such that $\models \langle a \rangle \Rightarrow v = n$
- $\langle a \rangle_{SL}$: separation logic formula, extends $\langle a \rangle$ by details on memory

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```

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$(\texttt{entry}, 2)$	$\Longleftarrow pos$
$\{\texttt{j} = v_{\texttt{j}}, \texttt{ad} = v_{\texttt{ad}}\}$	$\Longleftarrow PV$
$\{[\![v_{\texttt{ad}}, v_{end}]\!]\}$	$\Longleftarrow AL$
$\{v_{end} = v_{\texttt{ad}} + 3\}$	$\Longleftarrow KB$
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- $\langle a \rangle_{SL}$: separation logic formula, extends $\langle a \rangle$ by details on memory
- abstract state a *represents* concrete state iff
  $\langle a \rangle_{SL}$ is satisfied by instantiation corresponding to concrete state

Symbolic Execution

```
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```

A

$(\texttt{entry}, 0)$
$\{\mathrm{j} = v_{\mathrm{j}}, \ldots\}$
$\varnothing$
$\{0 \leq v_{\mathrm{j}} \leq \mathrm{umax}, \ldots\}$
$\varnothing$

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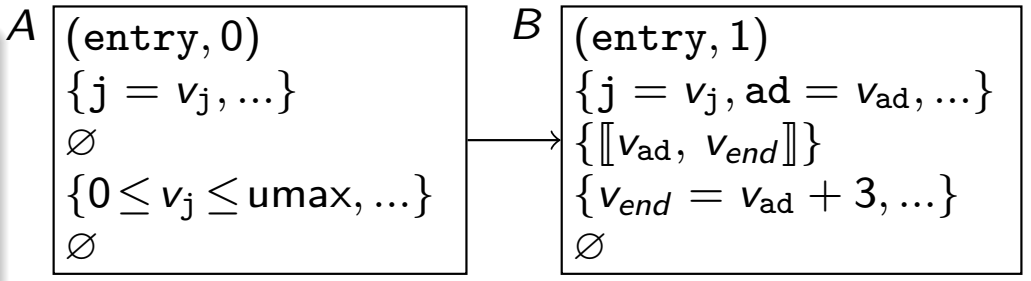

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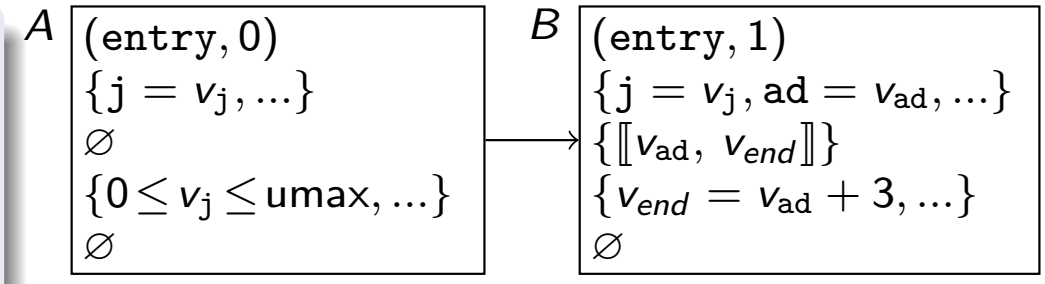

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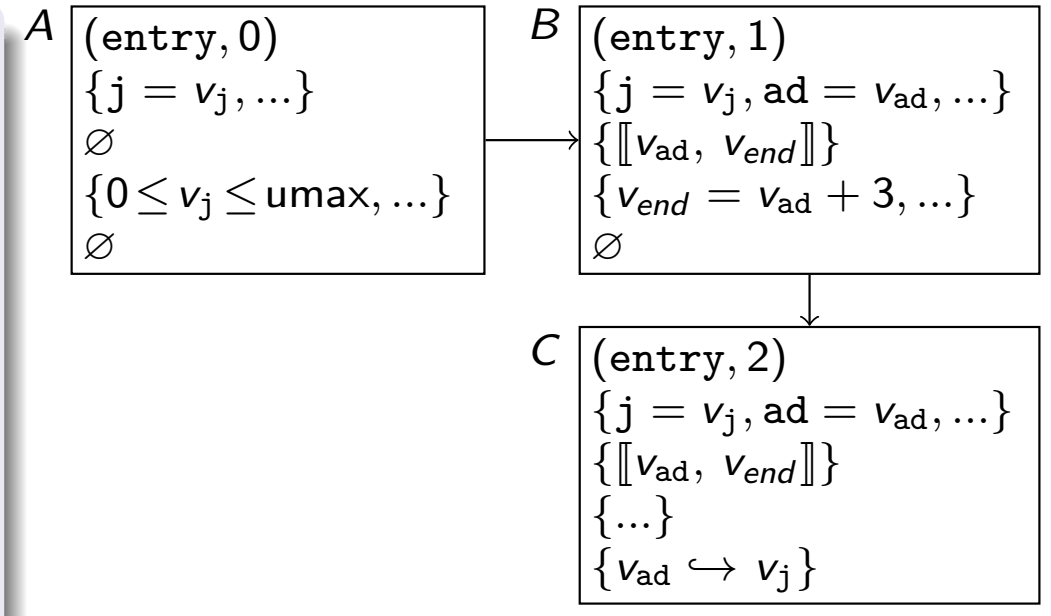

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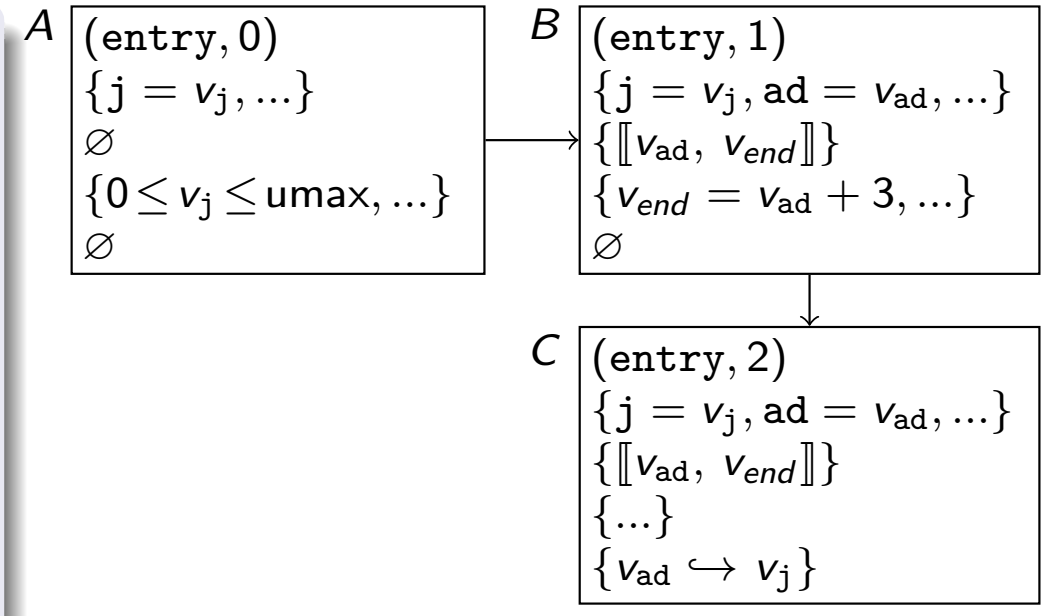

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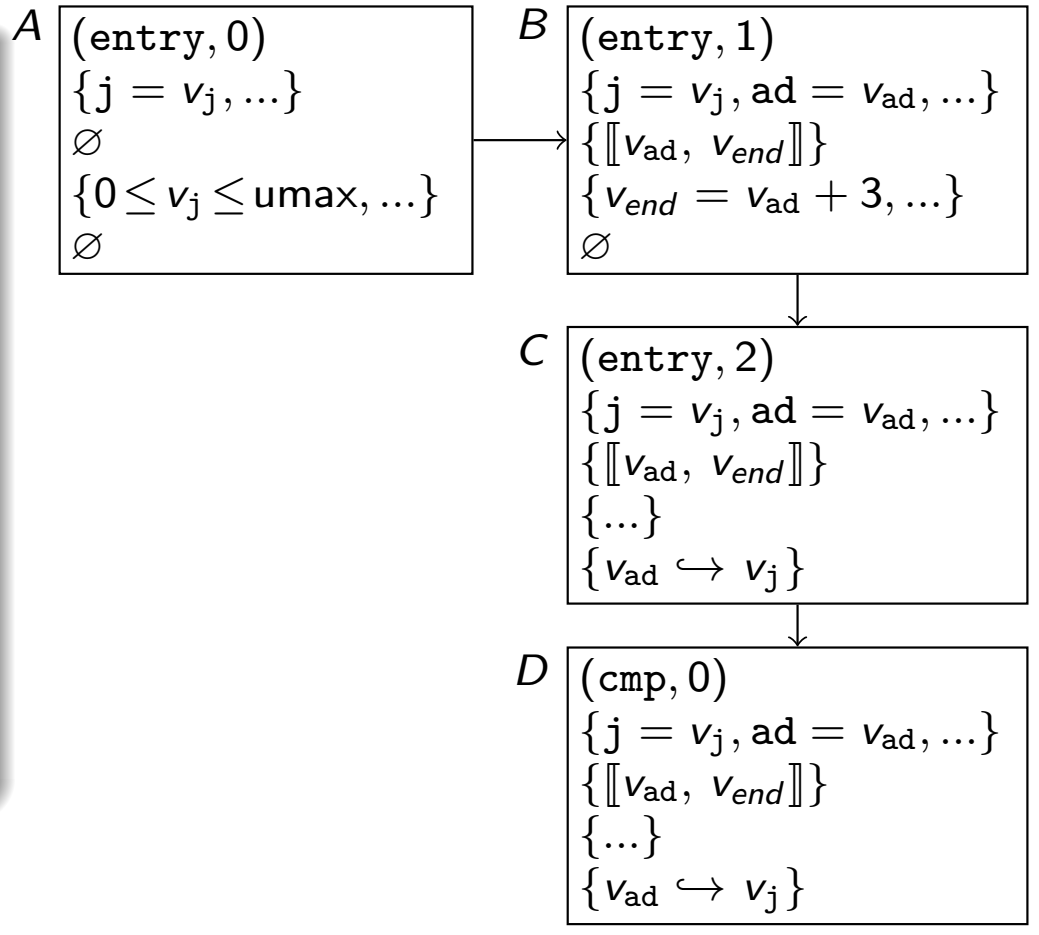

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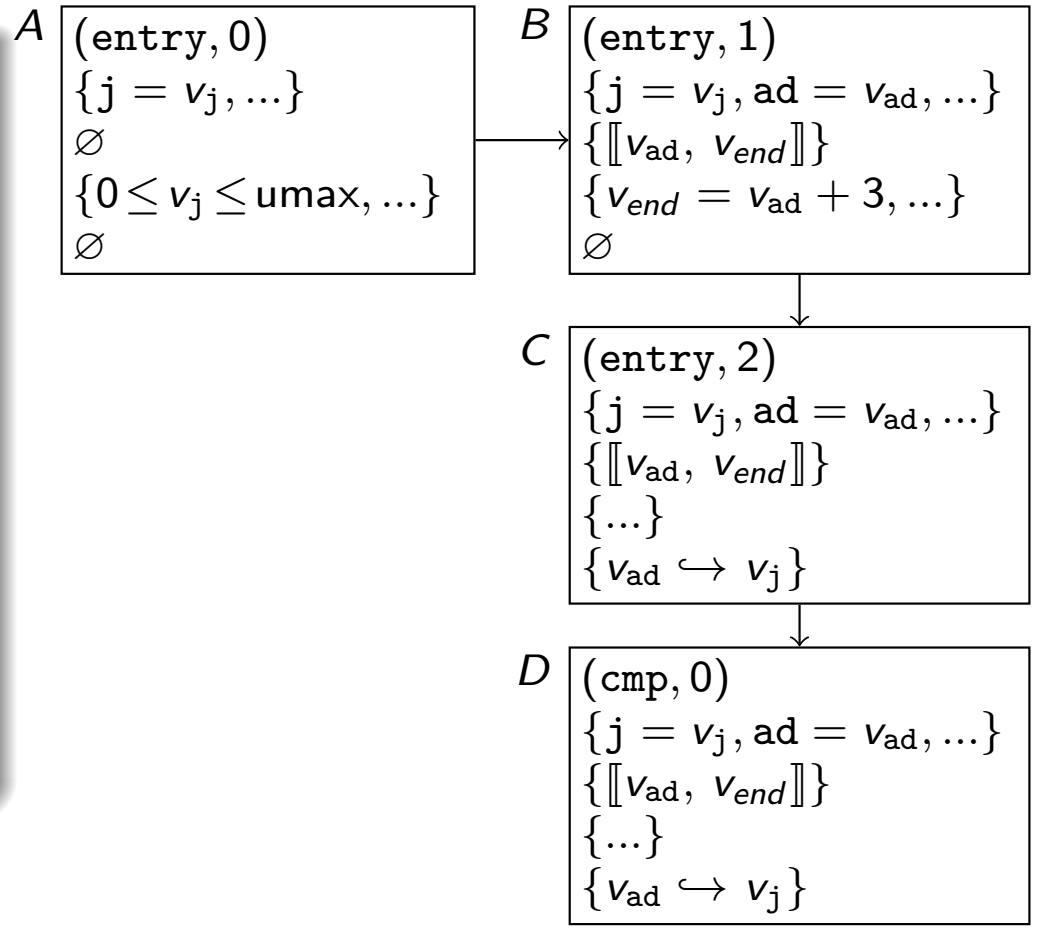

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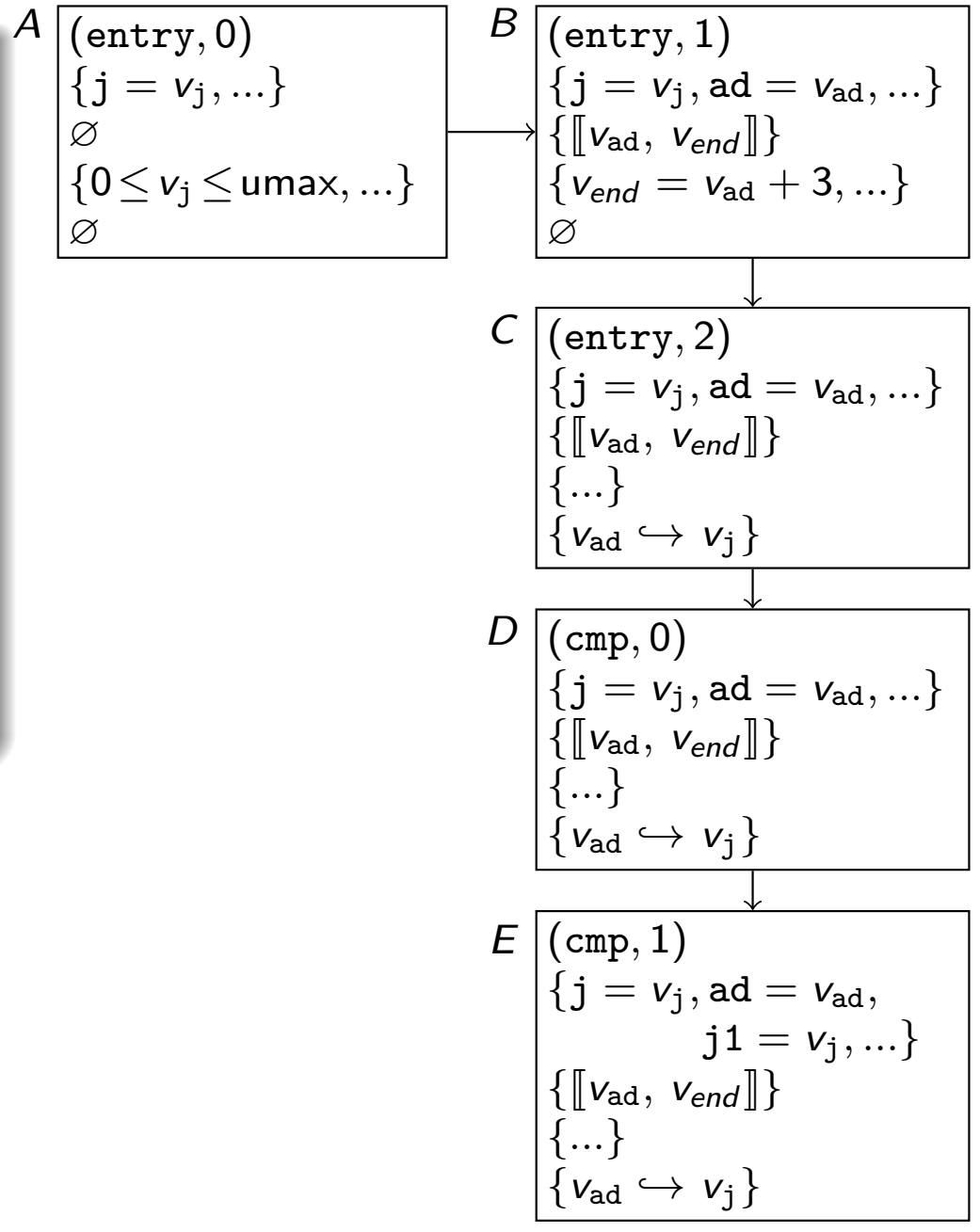

Integer Comparison

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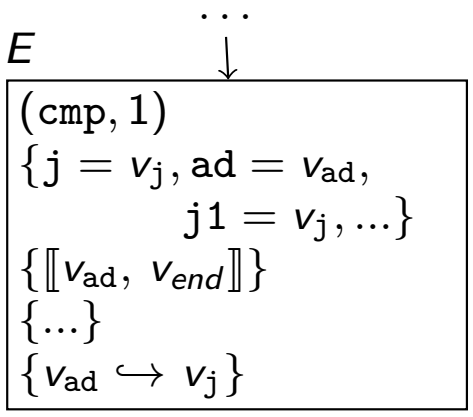

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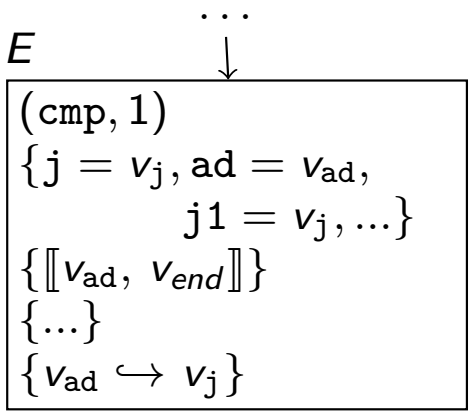

Symbolic execution rule for `x = icmp ugt i32` t_1, t_2

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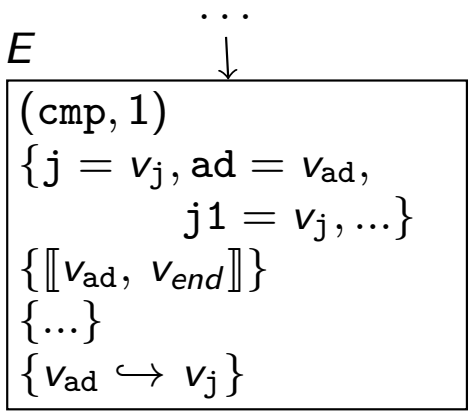

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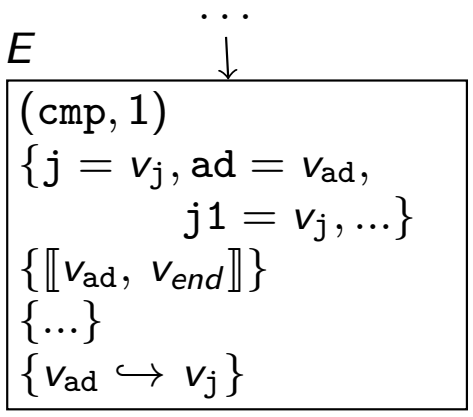

Symbolic execution rule for `x = icmp ugt i32` t_1, t_2

- set `x` to 1 if $\models \langle a \rangle \implies (PV_u(t_1) > PV_u(t_2))$
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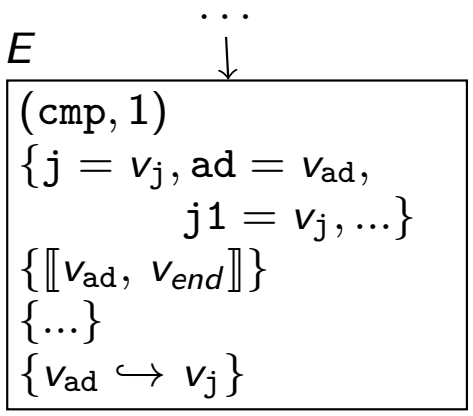

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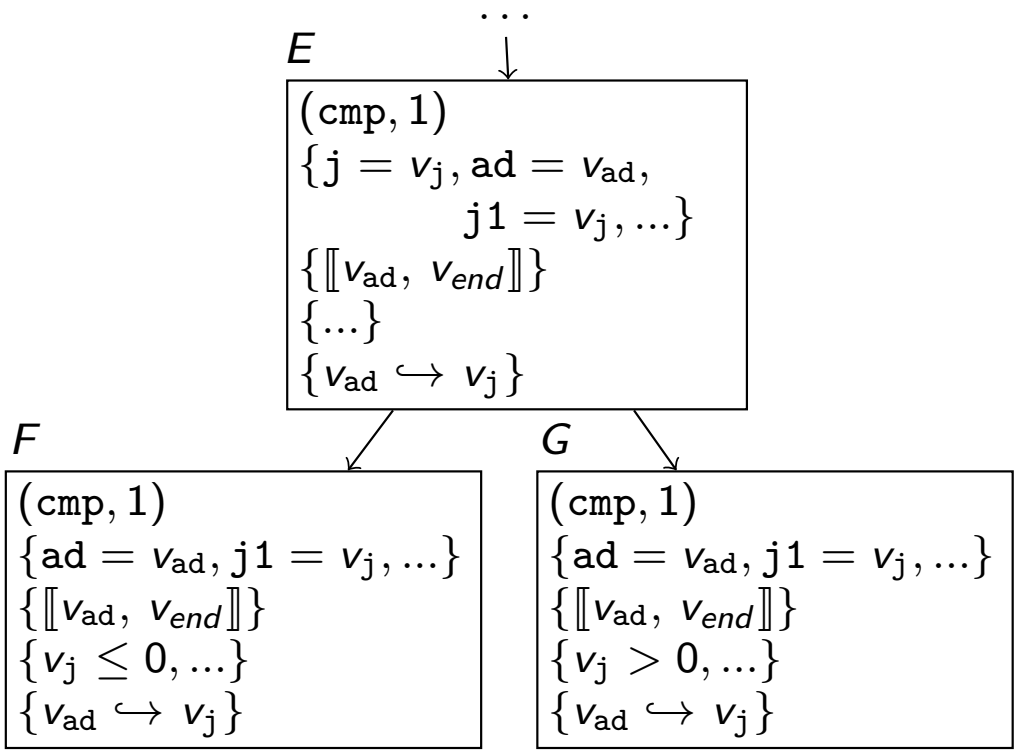

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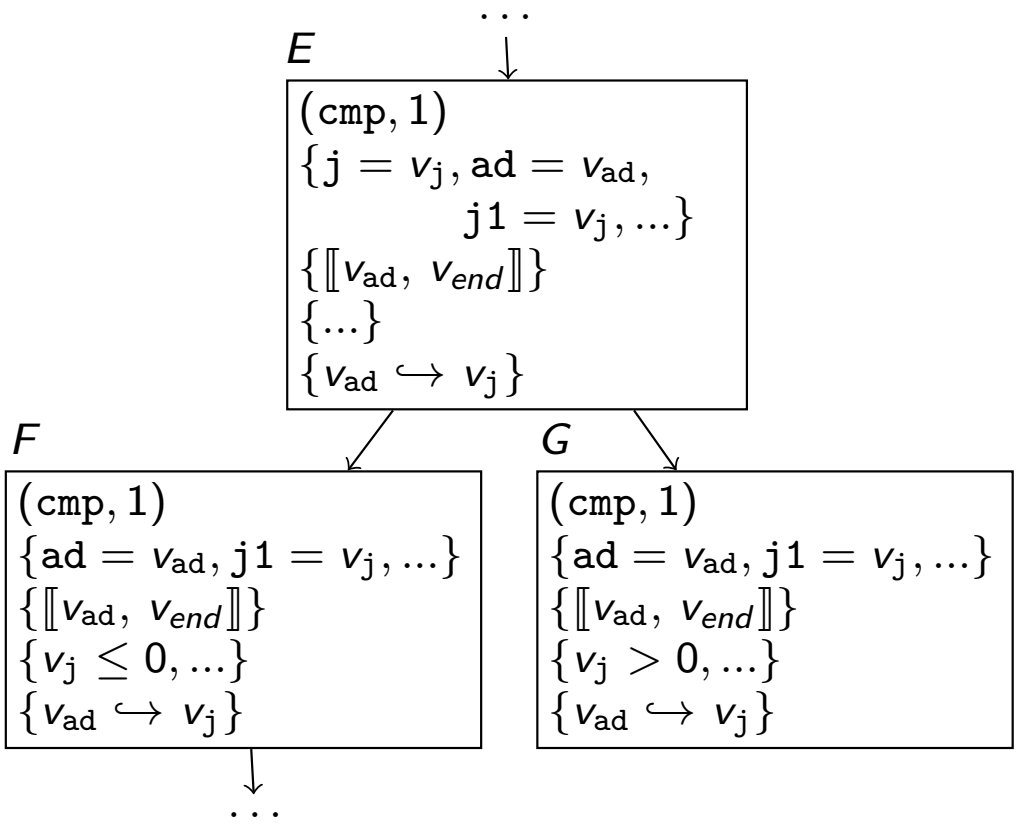

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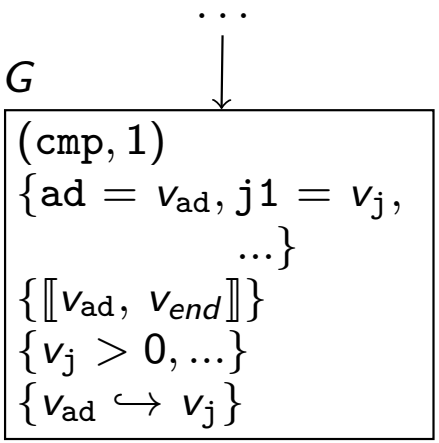

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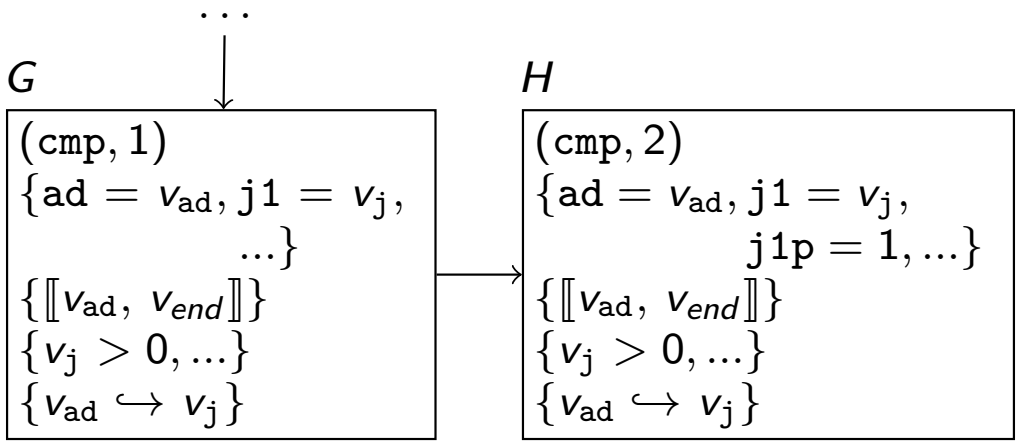

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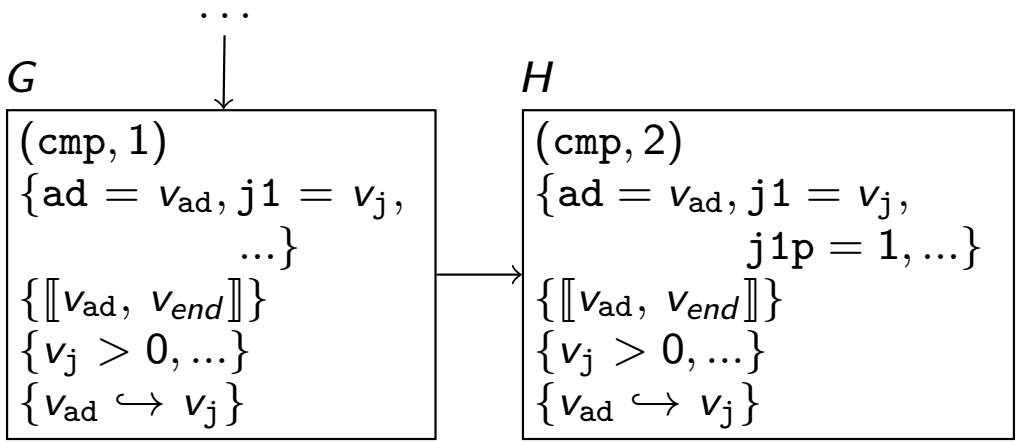

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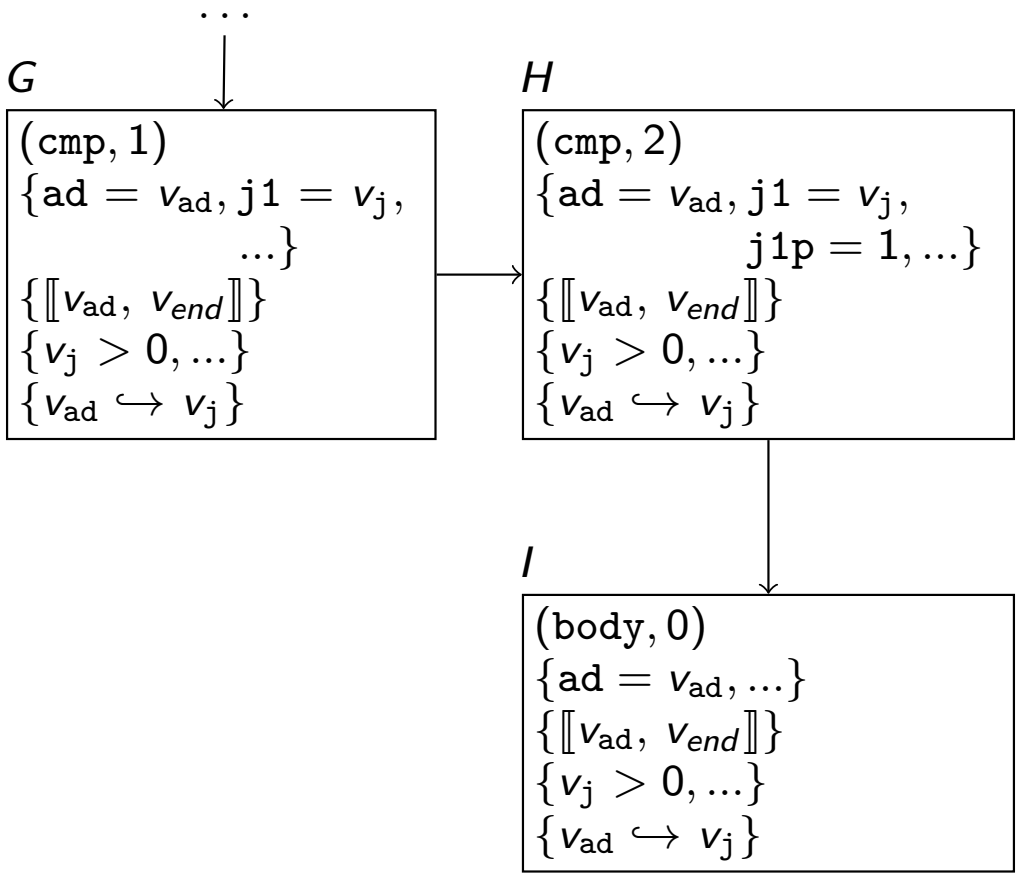

Symbolic Execution

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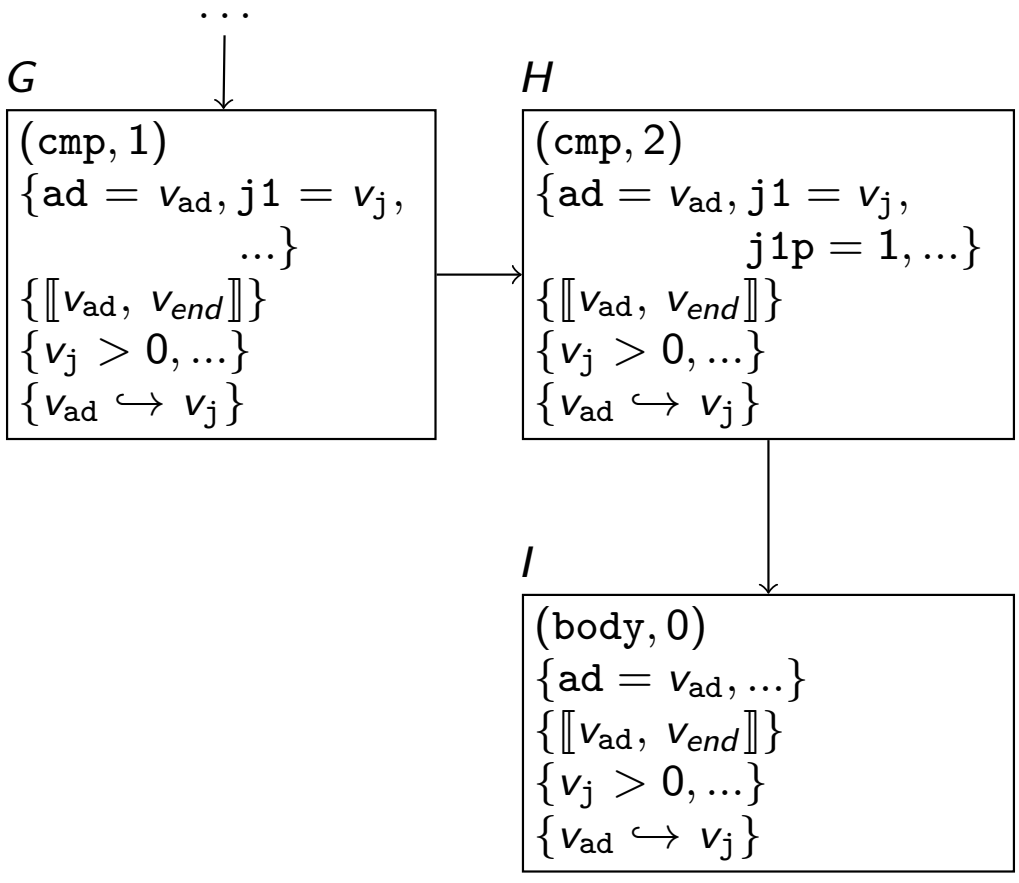

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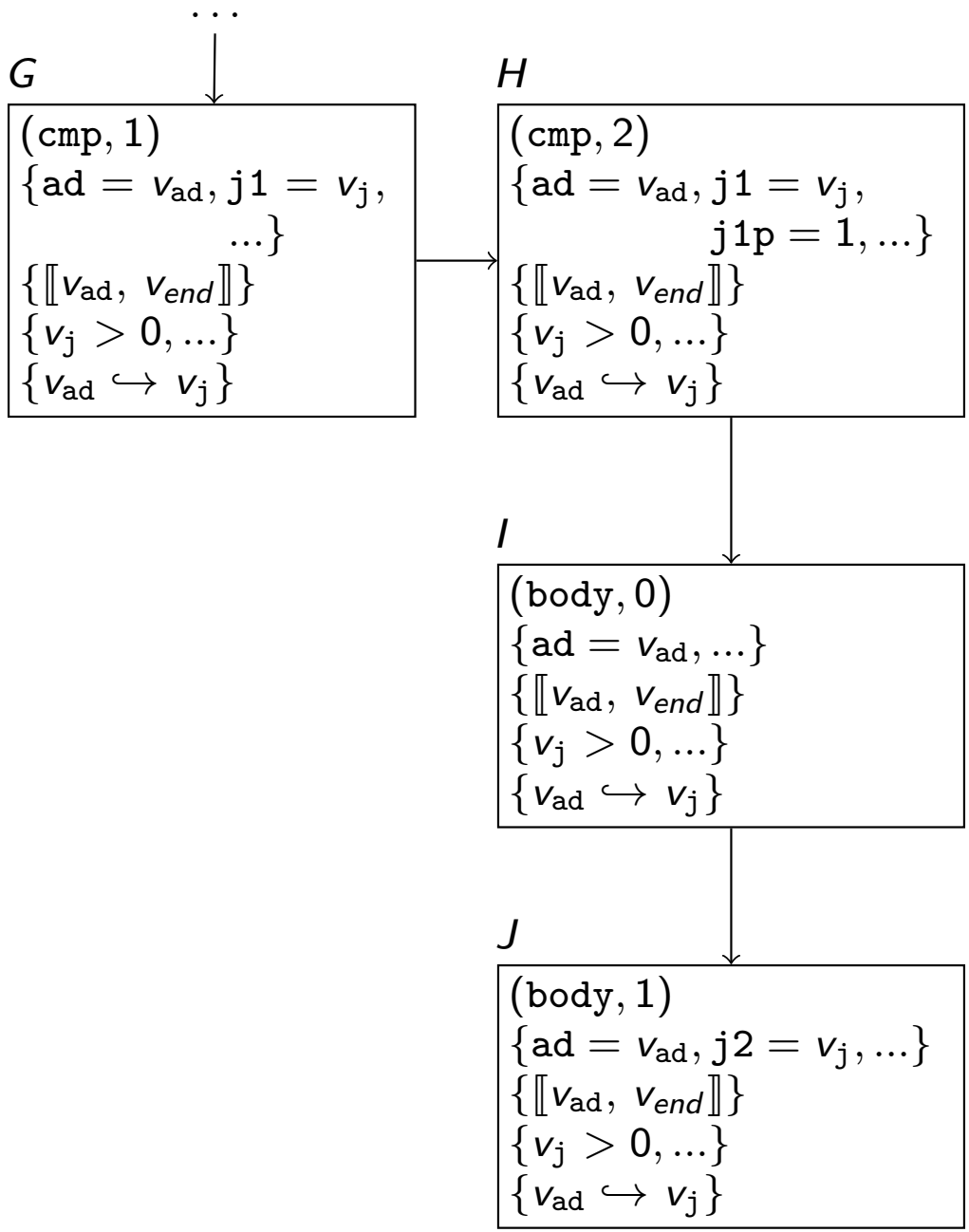

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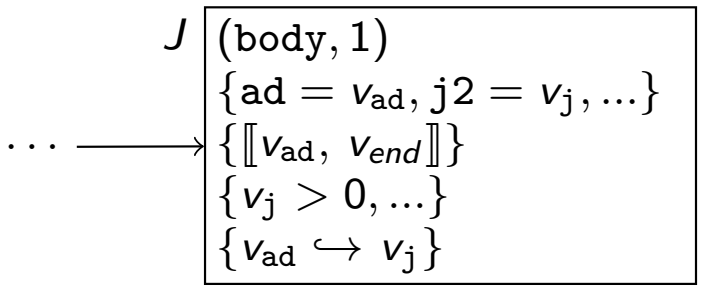

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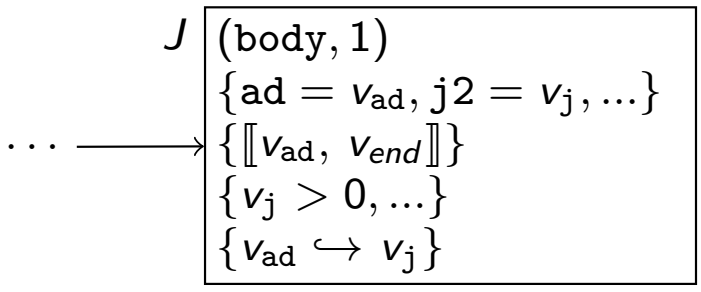

Symbolic execution rule for `x = add i32` t_1, t_2

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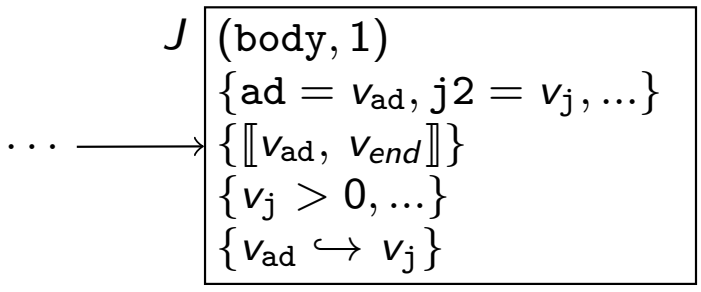

Symbolic execution rule for `x = add i32` t_1, t_2 where $\mathtt{x} \in \mathcal{U}$

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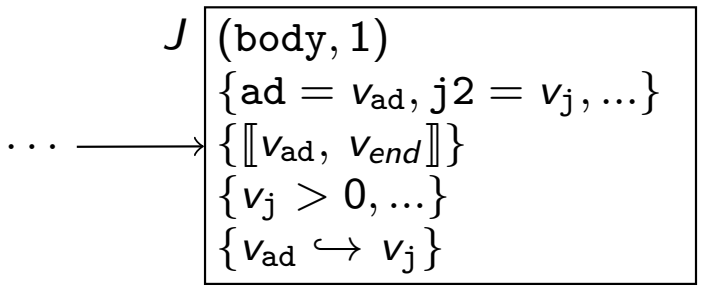

Symbolic execution rule for `x = add i32` t_1, t_2 where `x` $\in \mathcal{U}$

- set `x` to $PV_u(t_1) + PV_u(t_2)$ if $\models \langle a \rangle \implies PV_u(t_1) + PV_u(t_2) \leq \mathsf{umax}_{32}$

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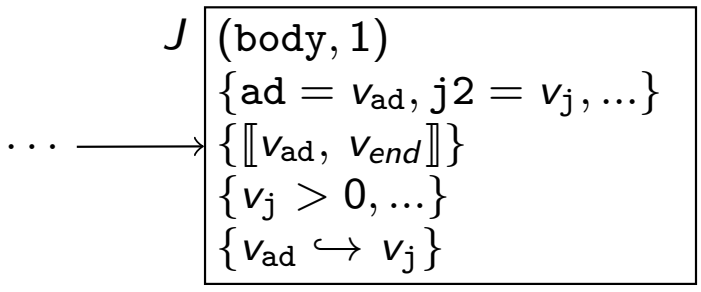

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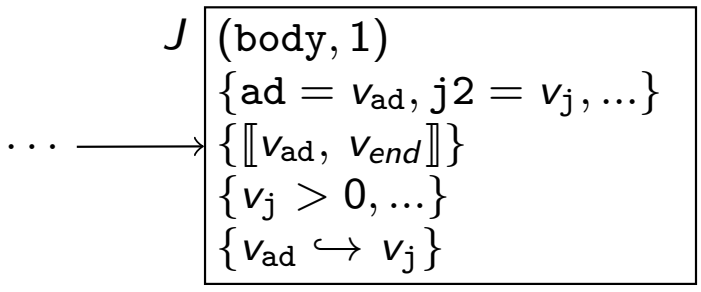

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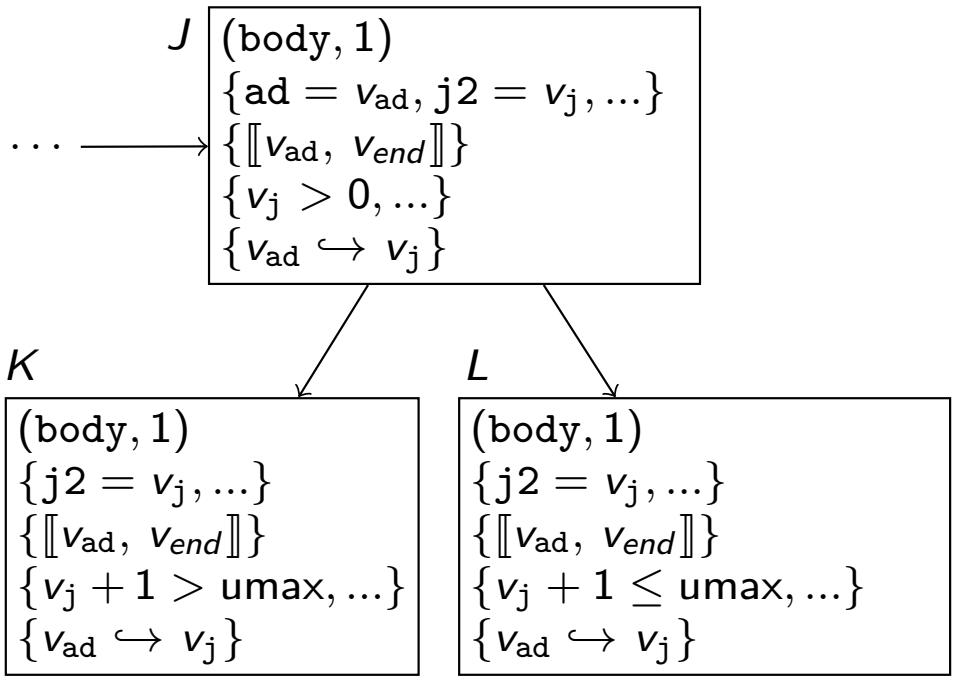

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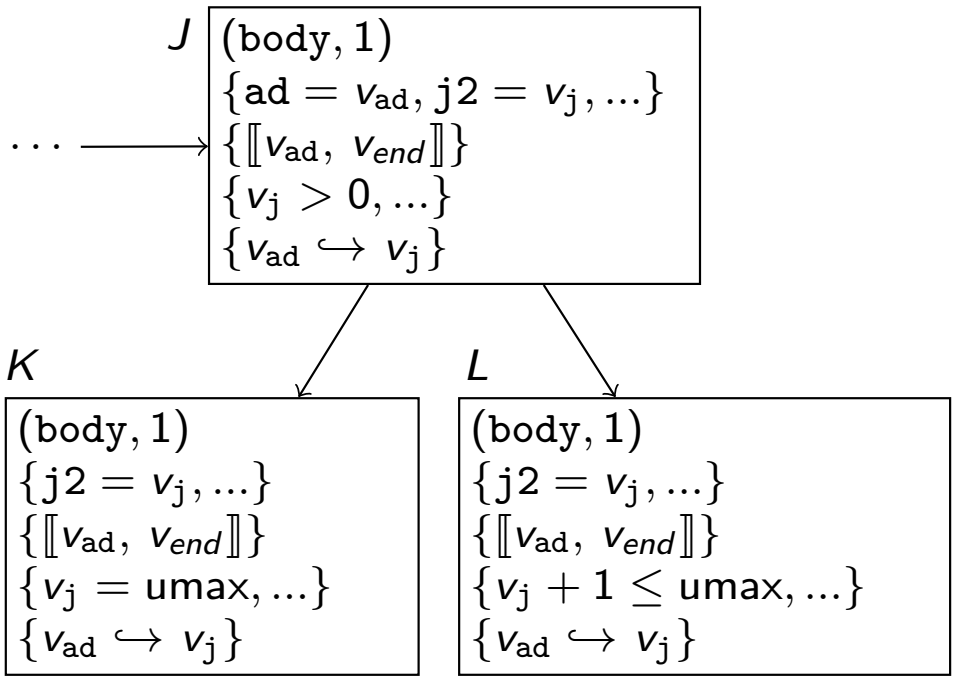

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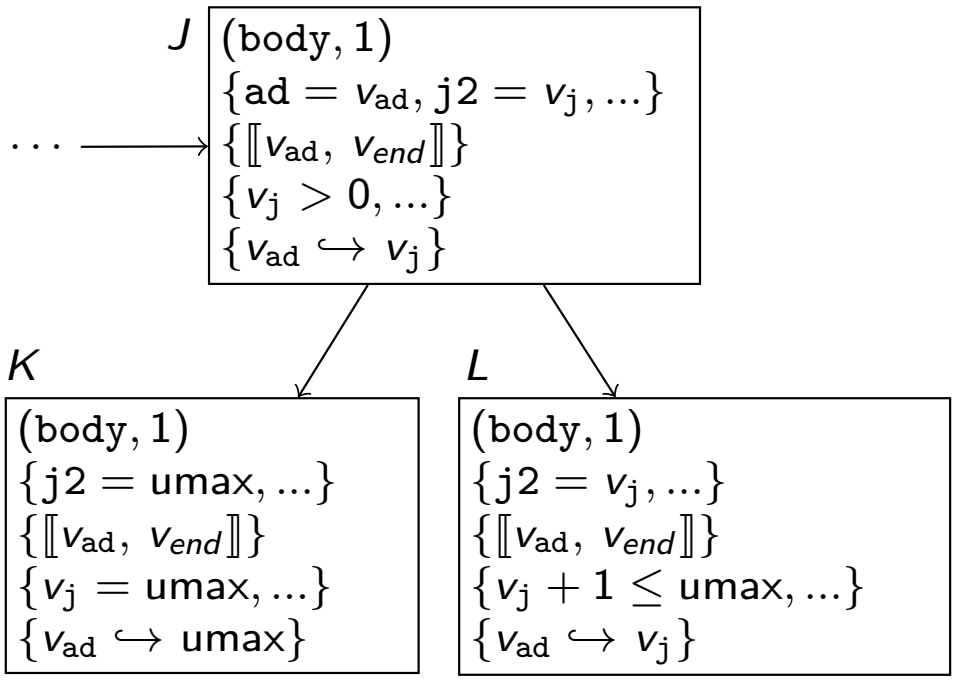

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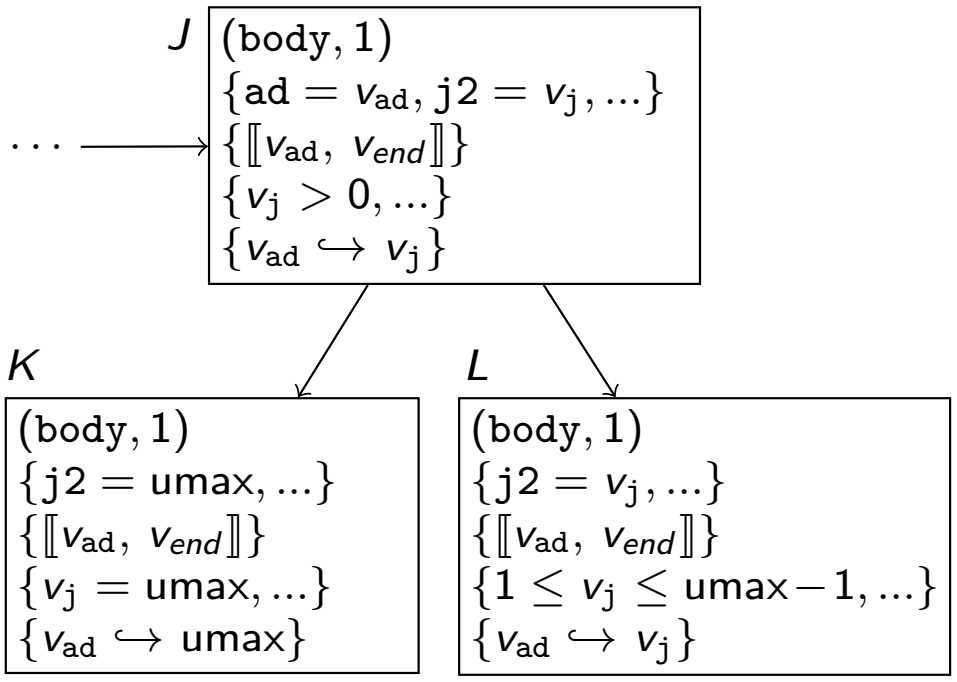

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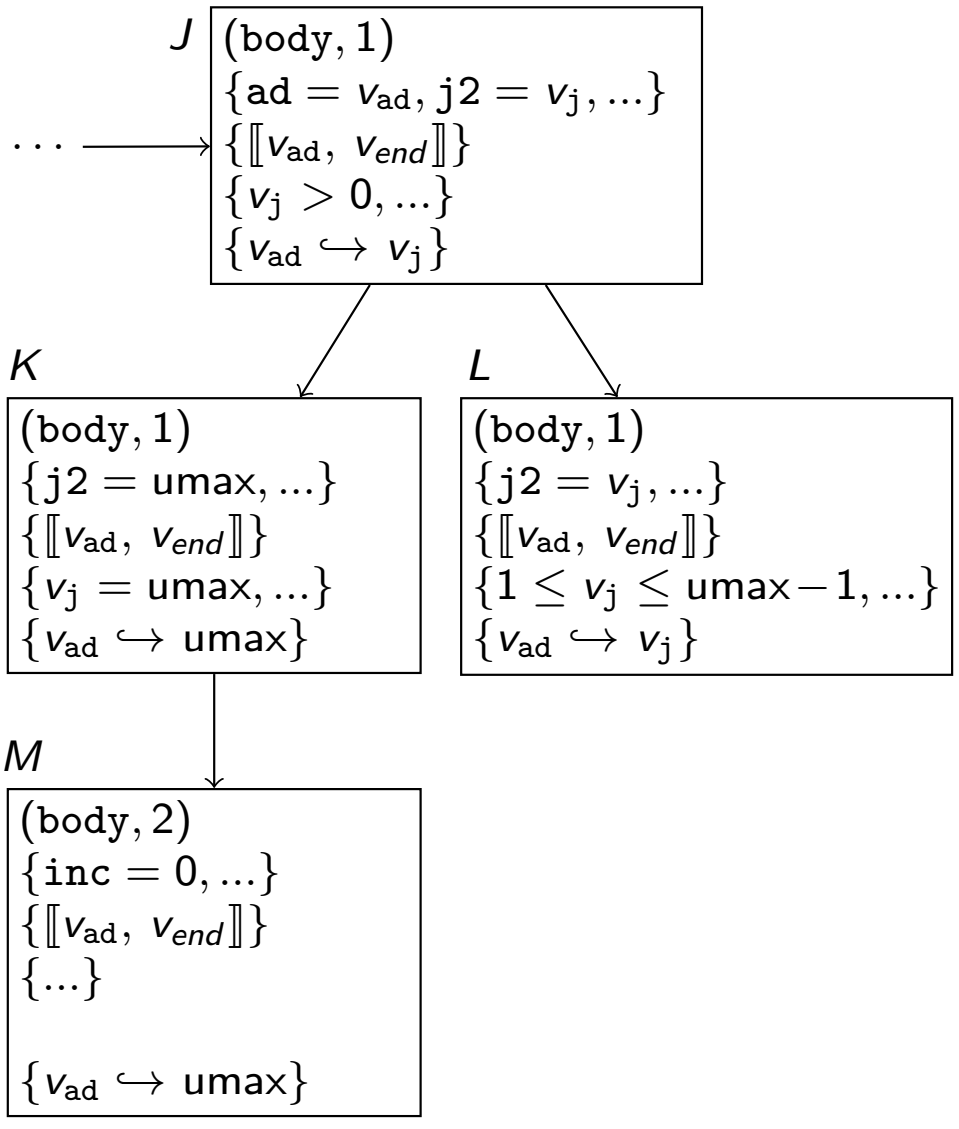

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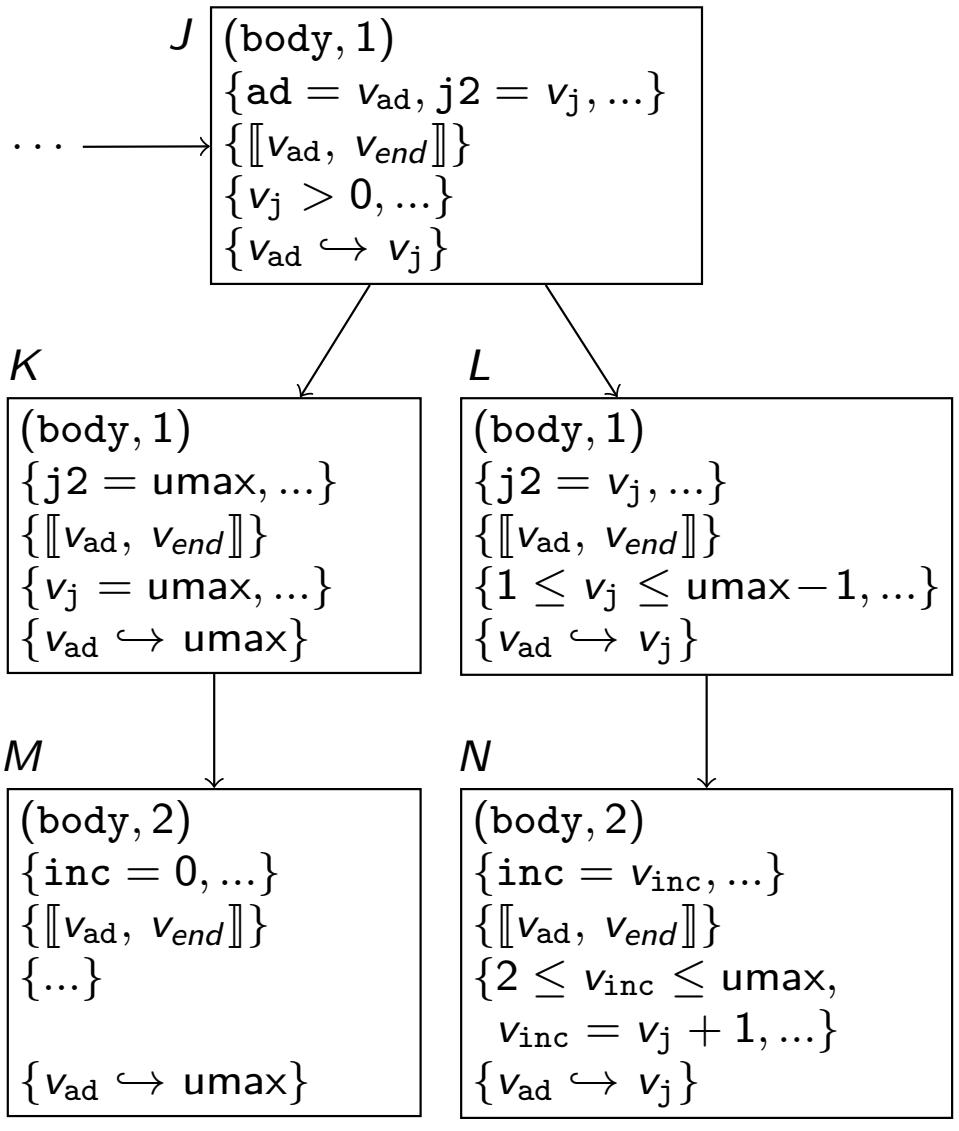

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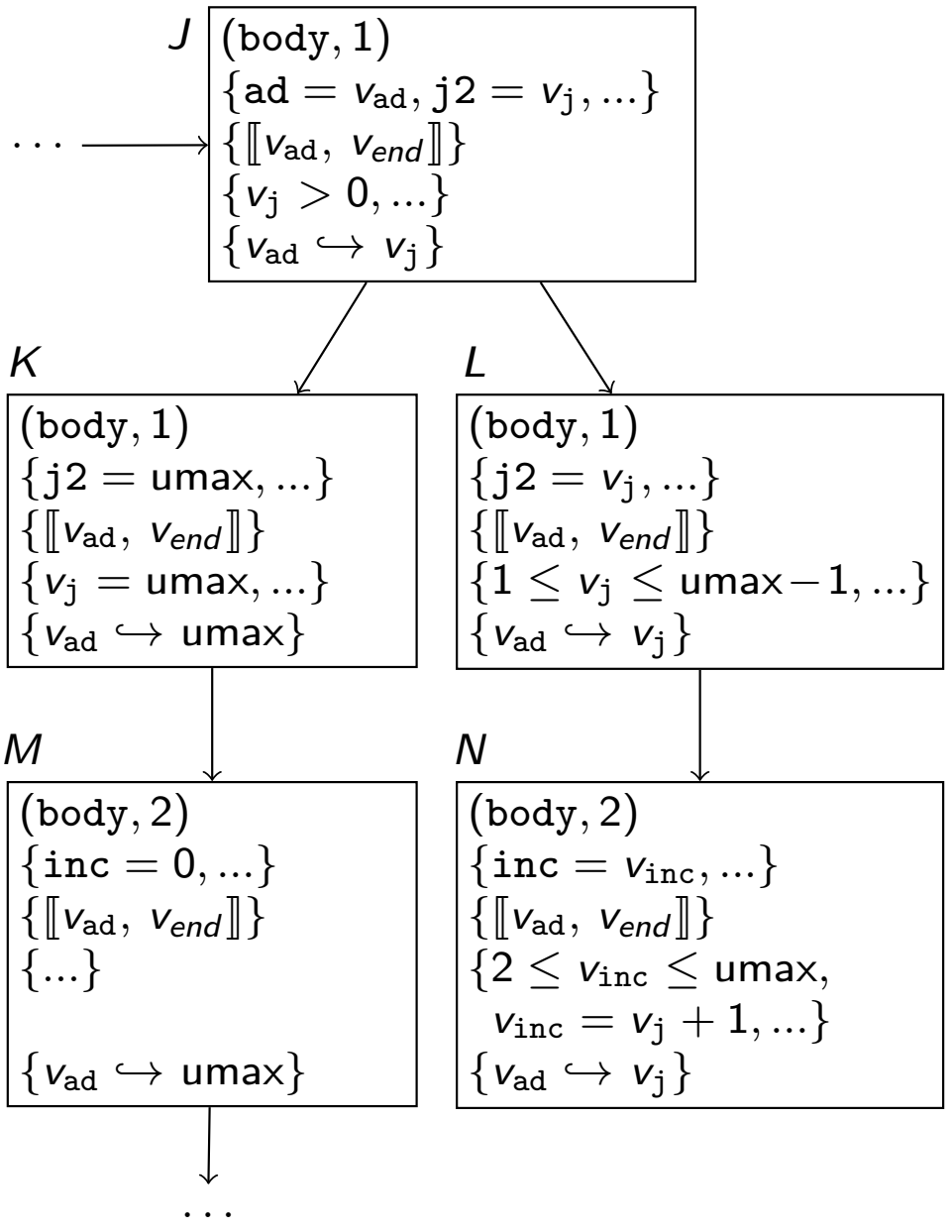

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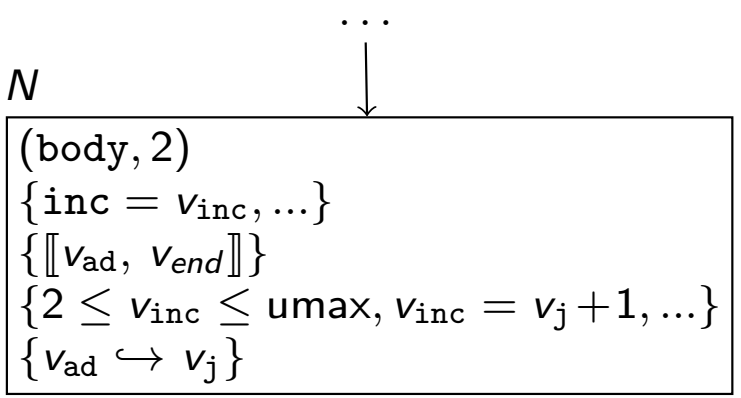

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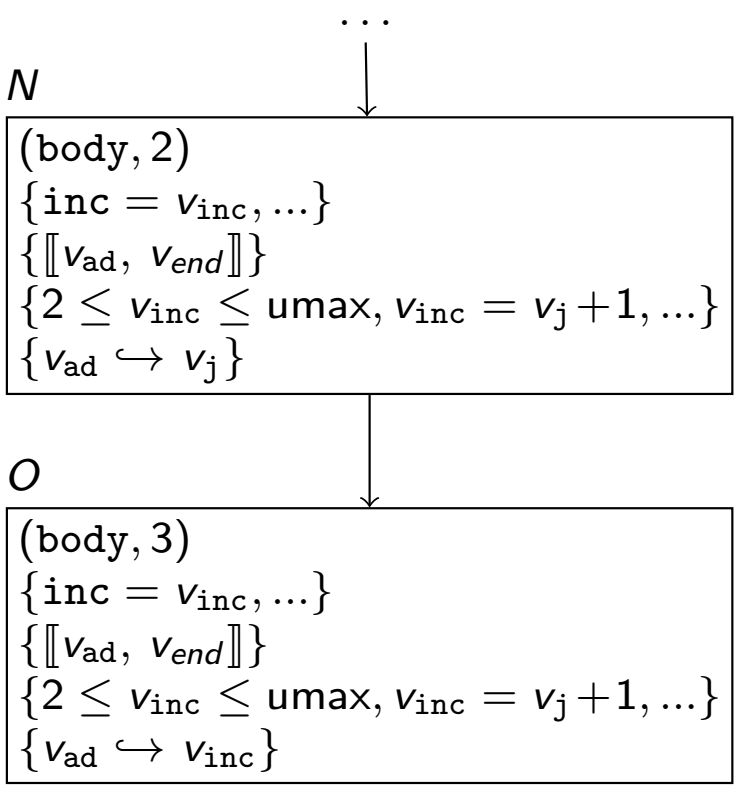

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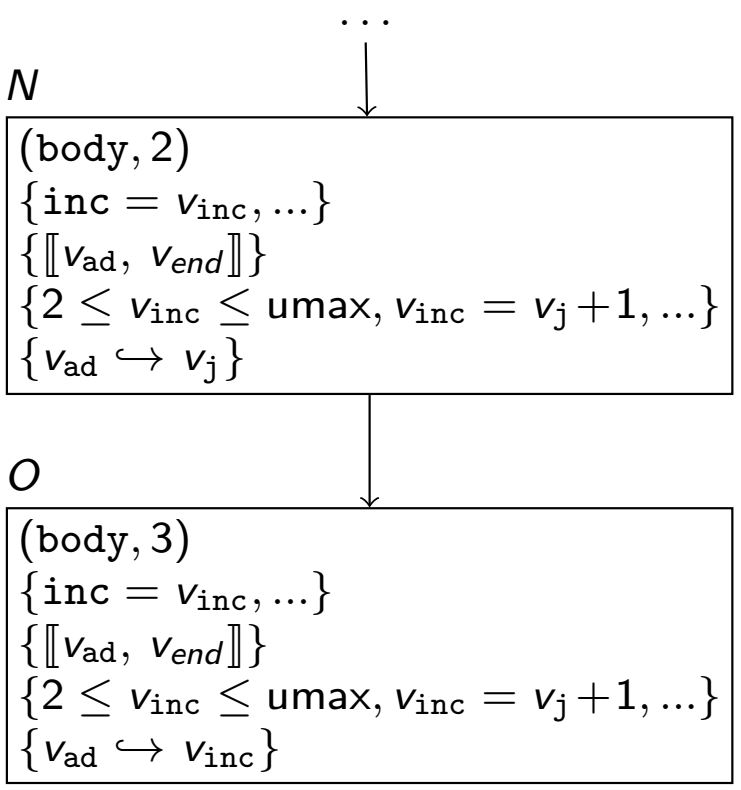

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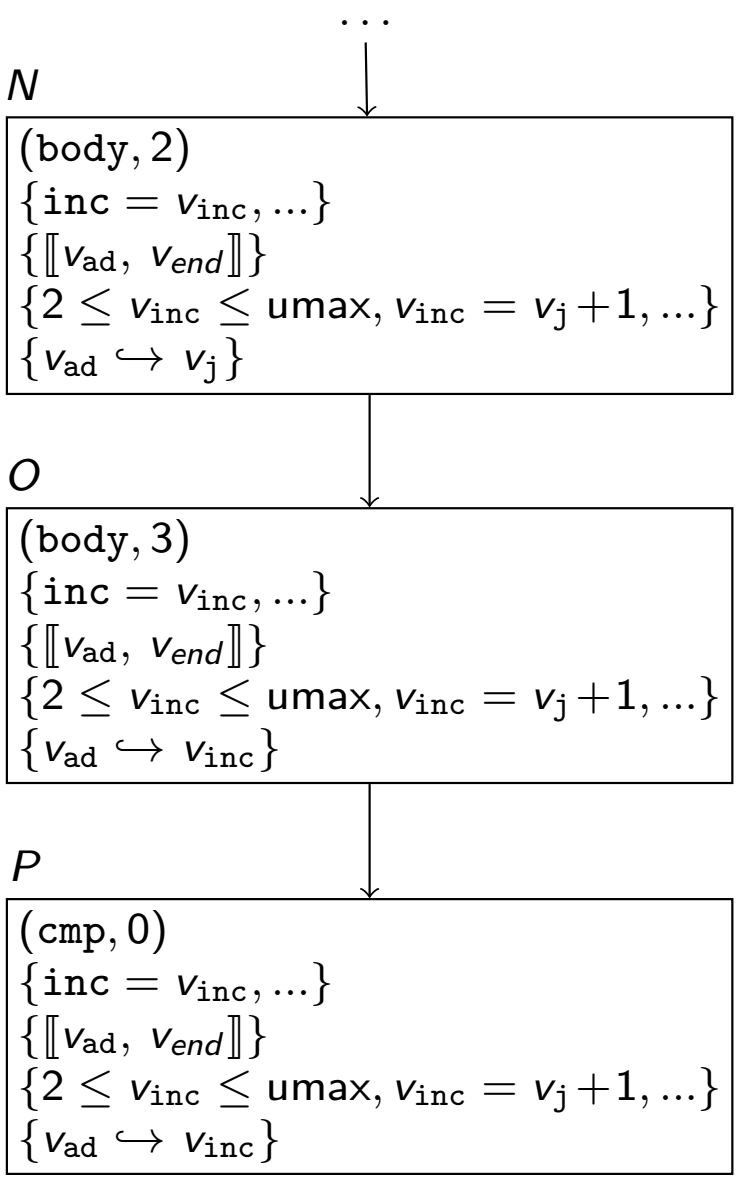

Generalization

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P

$(\texttt{cmp}, 0)$

$\{\texttt{inc} = v_{\texttt{inc}}, \ldots\}$

$\cdots \longrightarrow \{[\![v_{\texttt{ad}}, v_{end}]\!]\}$

$\{2 \leq v_{\texttt{inc}} \leq \mathsf{umax}, v_{\texttt{inc}} = v_{\texttt{j}} + 1, \ldots\}$

$\{v_{\texttt{ad}} \hookrightarrow v_{\texttt{inc}}\}$

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$$\cdots \longrightarrow P \begin{array}{|l} (\mathtt{cmp}, 0) \\ \{\mathtt{inc} = v_{\mathtt{inc}}, \ldots\} \\ \{[\![v_{\mathtt{ad}}, v_{end}]\!]\} \\ \{2 \leq v_{\mathtt{inc}} \leq \mathsf{umax}, v_{\mathtt{inc}} = v_{\mathtt{j}} + 1, \ldots\} \\ \{v_{\mathtt{ad}} \hookrightarrow v_{\mathtt{inc}}\} \end{array}$$

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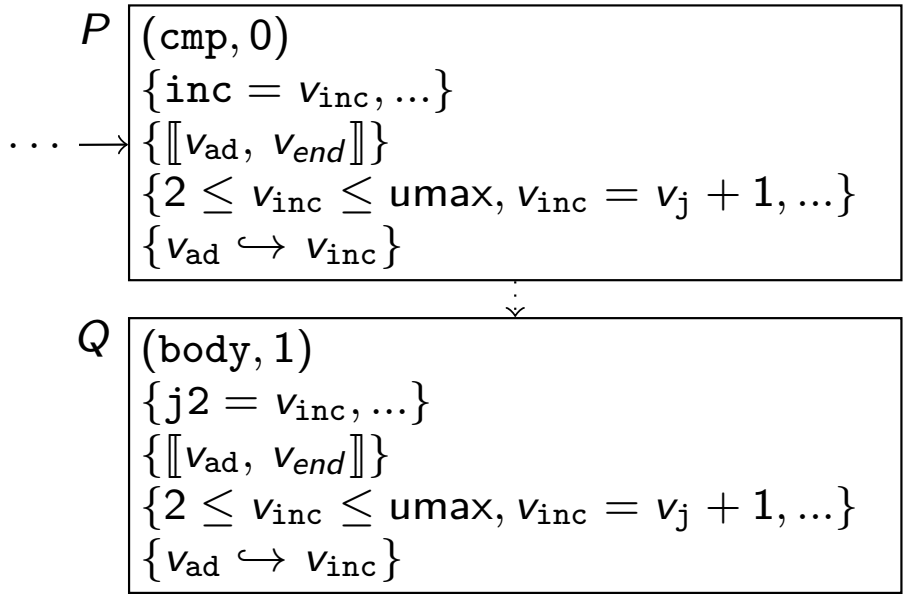

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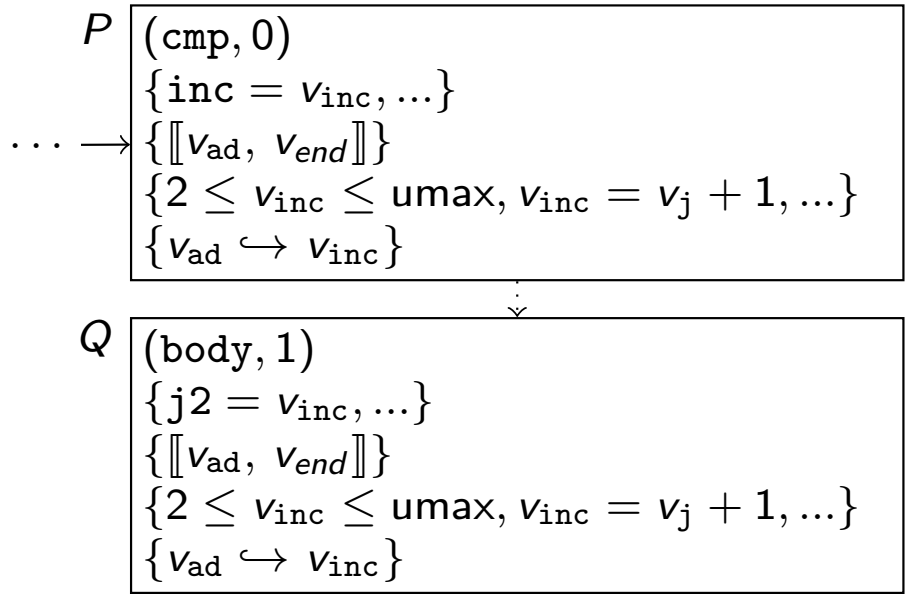
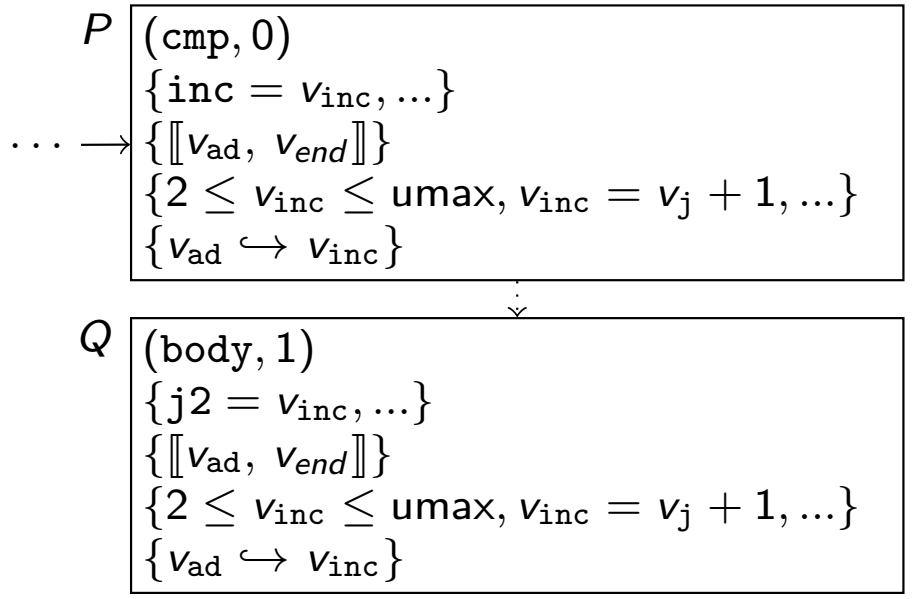
done:  0:ret void }
```



Generalization

```
define i32 @g(i32 j) {
entry: 0:ad = alloca i32
       1:store i32 j, i32* ad
       2:br label cmp
cmp:   0:j1 = load i32* ad
       1:j1p = icmp ugt i32 j1, 0
       2:br i1 j1p, label body,
                    label done
body:  0:j2 = load i32* ad
       1:inc = add i32 j2, 1
       2:store i32 inc, i32* ad
       3:br label cmp
done:  0:ret void }
```



Generalization

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       1:store i32 j, i32* ad
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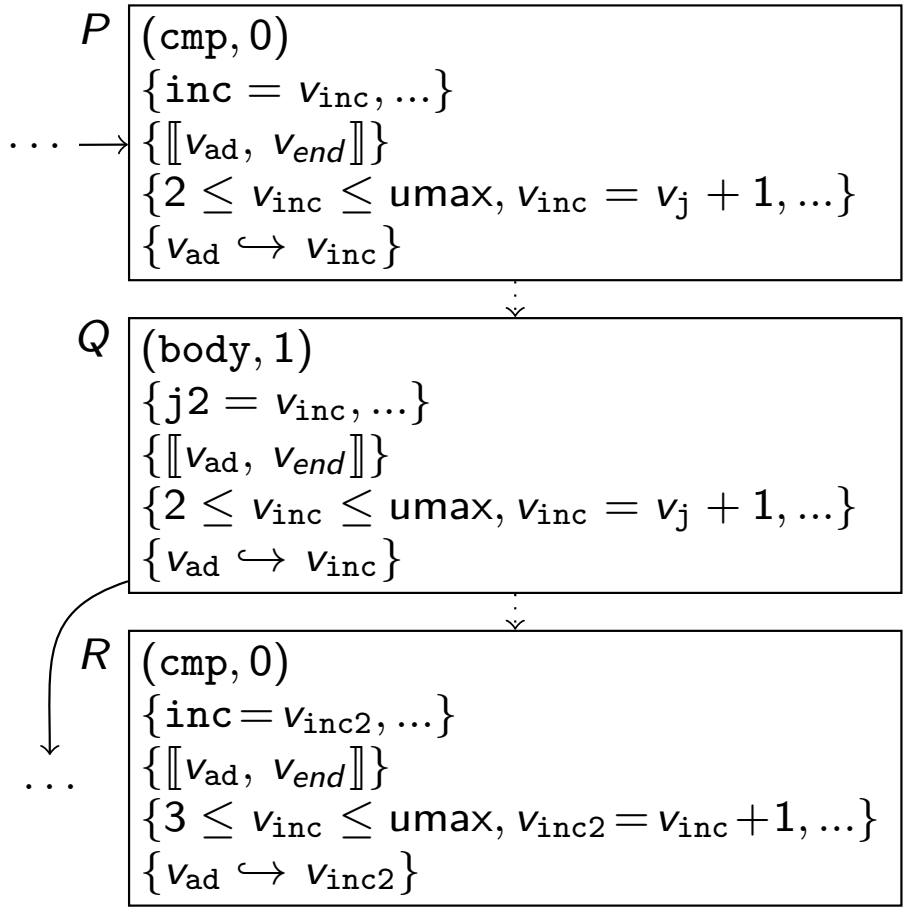

Generalization

```
define i32 @g(i32 j) {
entry: 0:ad = alloca i32
       1:store i32 j, i32* ad
       2:br label cmp
cmp:   0:j1 = load i32* ad
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                    label done
body:  0:j2 = load i32* ad
       1:inc = add i32 j2, 1
       2:store i32 inc, i32* ad
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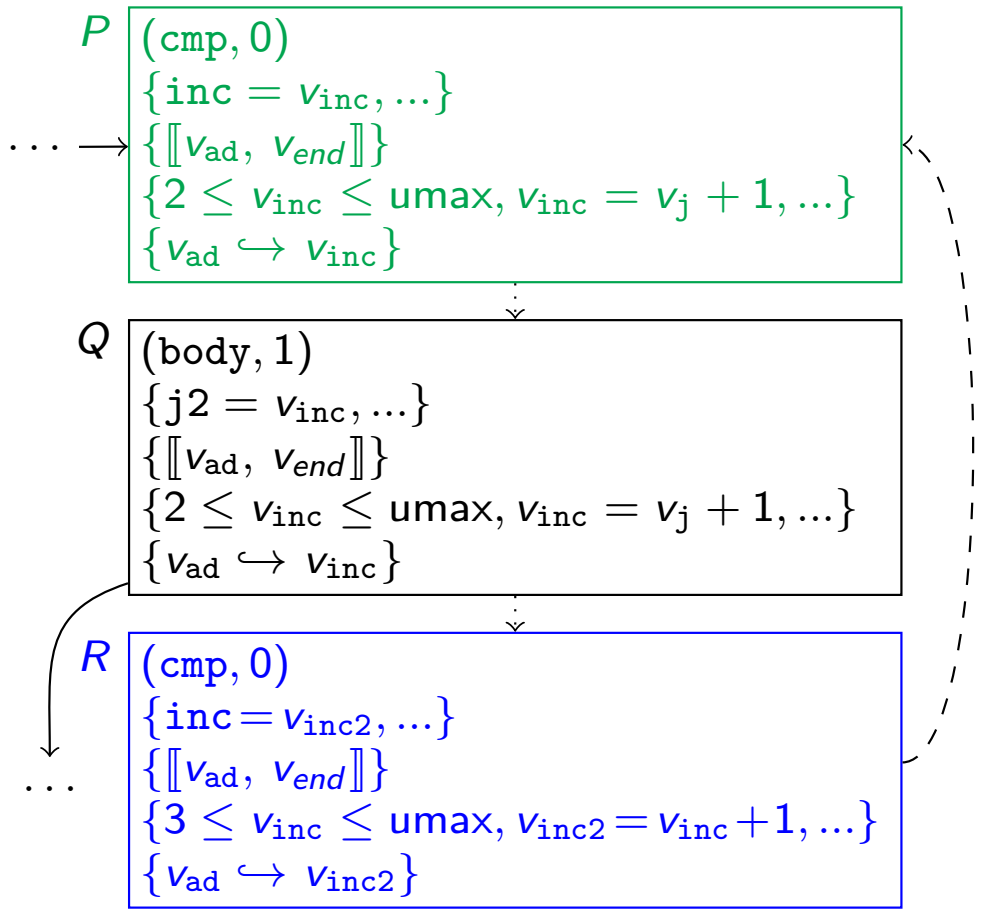
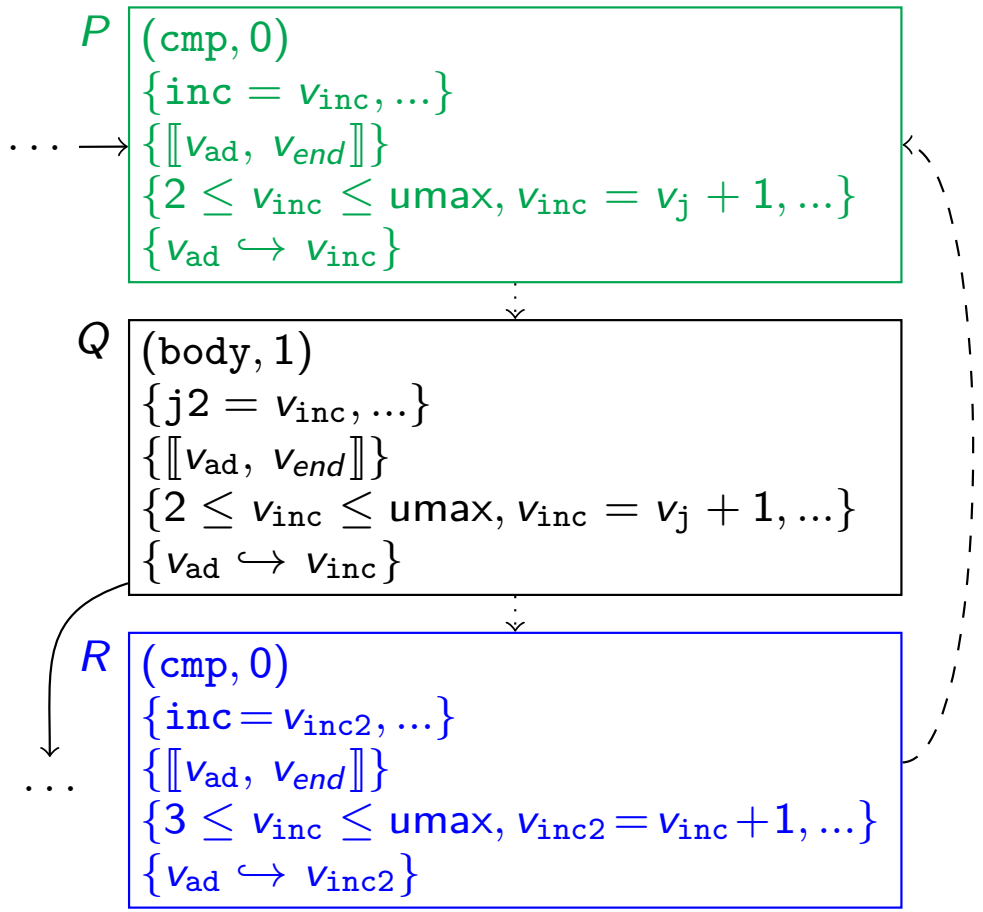
done:  0:ret void }
```



P is *generalization* of R with μ

Generalization

```
define i32 @g(i32 j) {
entry: 0:ad = alloca i32
       1:store i32 j, i32* ad
       2:br label cmp
cmp:   0:j1 = load i32* ad
       1:j1p = icmp ugt i32 j1, 0
       2:br i1 j1p, label body,
                    label done
body:  0:j2 = load i32* ad
       1:inc = add i32 j2, 1
       2:store i32 inc, i32* ad
       3:br label cmp
done:  0:ret void }
```



P is *generalization* of R with μ

- $\mu(PV_P(\mathtt{x})) = PV_R(\mathtt{x})$ for all program variables $\mathtt{x}$

Generalization

```
define i32 @g(i32 j) {
entry: 0:ad = alloca i32
       1:store i32 j, i32* ad
       2:br label cmp
cmp:   0:j1 = load i32* ad
       1:j1p = icmp ugt i32 j1, 0
       2:br i1 j1p, label body,
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       1:inc = add i32 j2, 1
       2:store i32 inc, i32* ad
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done:  0:ret void }
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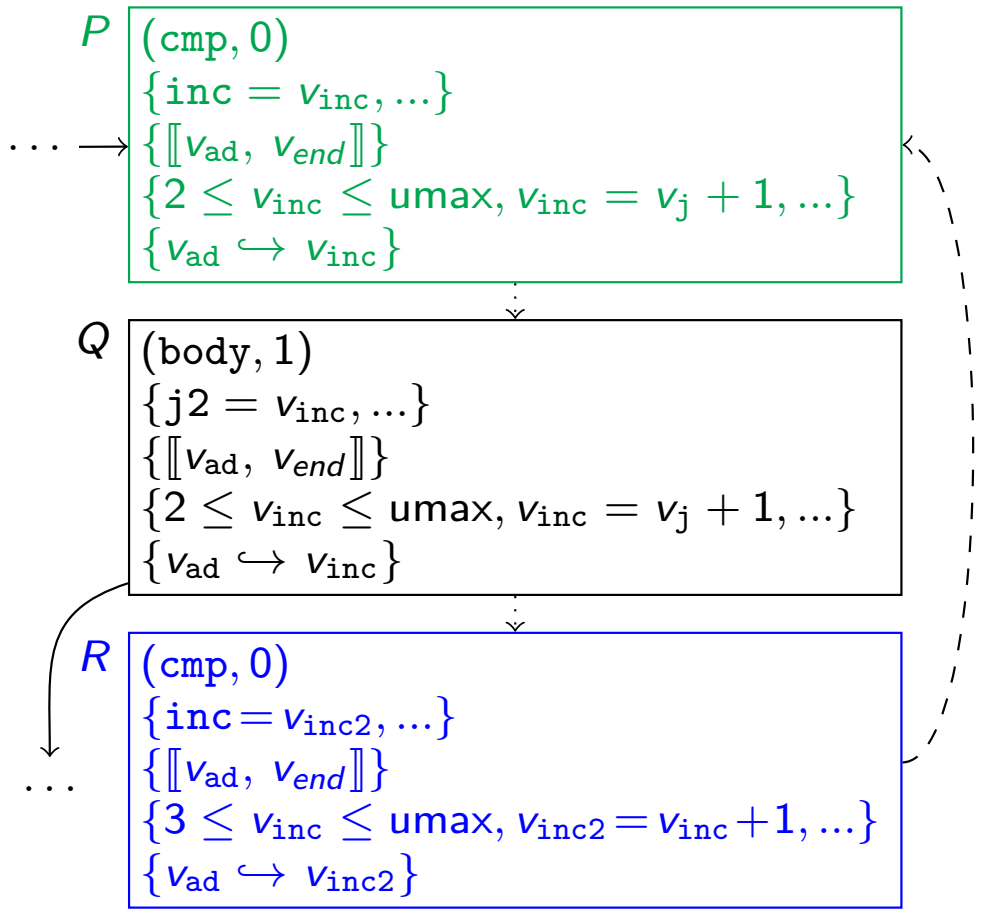

P is *generalization* of R with $\mu(v_j) = v_{inc}$, $\mu(v_{inc}) = v_{inc2}$

- $\mu(PV_P(x)) = PV_R(x)$ for all program variables x

Generalization

```
define i32 @g(i32 j) {
entry: 0:ad = alloca i32
       1:store i32 j, i32* ad
       2:br label cmp
cmp:   0:j1 = load i32* ad
       1:j1p = icmp ugt i32 j1, 0
       2:br i1 j1p, label body,
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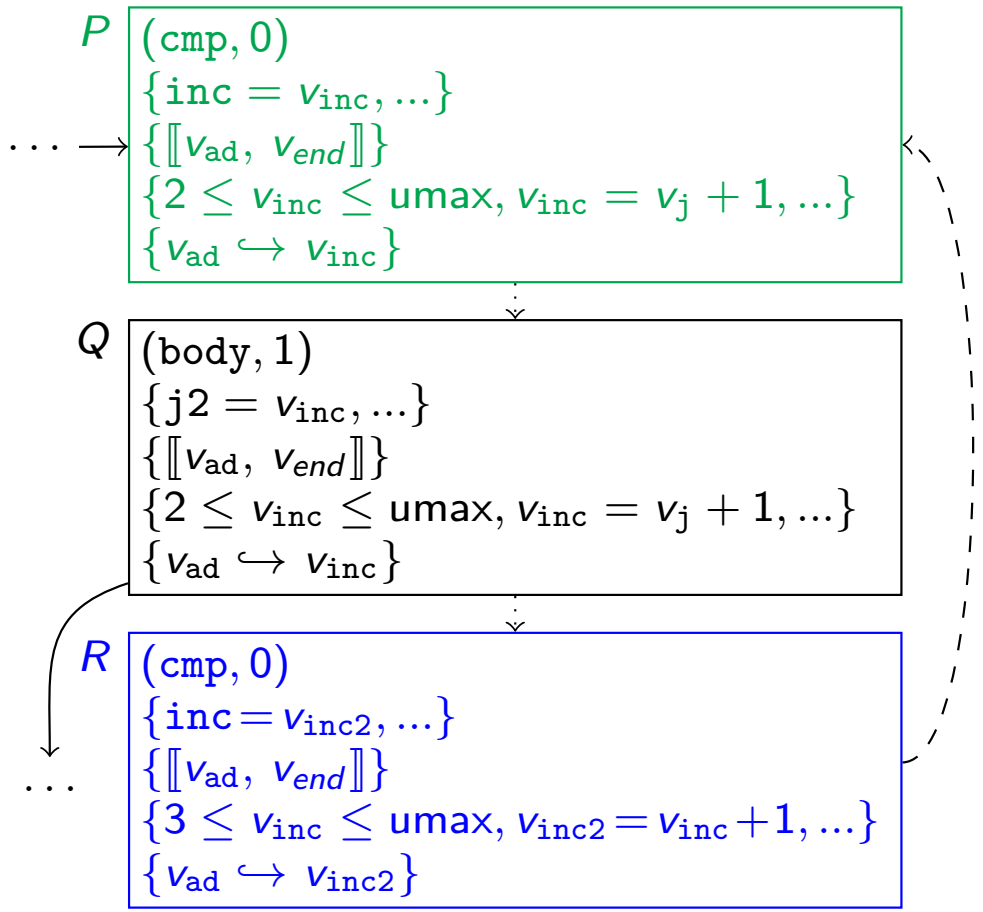

P is *generalization* of R with $\mu(v_j) = v_{inc}$, $\mu(v_{inc}) = v_{inc2}$

- $\mu(PV_P(x)) = PV_R(x)$ for all program variables x
- $[\![v_1, v_2]\!] \in AL_P$ implies $[\![\mu(v_1), \mu(v_2)]\!] \in AL_R$

Generalization

```
define i32 @g(i32 j) {
entry: 0:ad = alloca i32
       1:store i32 j, i32* ad
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       3:br label cmp
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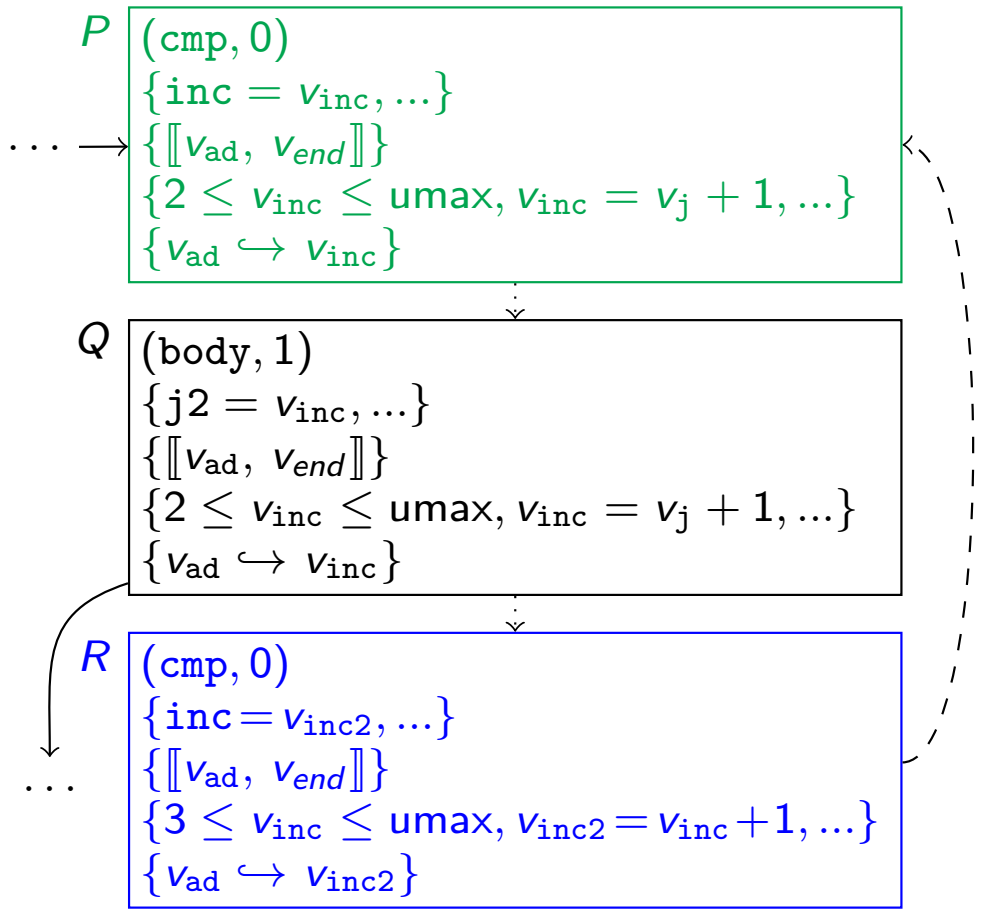
```



P is *generalization* of R with $\mu(v_j) = v_{inc}$, $\mu(v_{inc}) = v_{inc2}$

- $\mu(PV_P(x)) = PV_R(x)$ for all program variables x
- $[\![v_1, v_2]\!] \in AL_P$ implies $[\![\mu(v_1), \mu(v_2)]\!] \in AL_R$
- $\models \langle R \rangle \implies \mu(KB_P)$

Generalization

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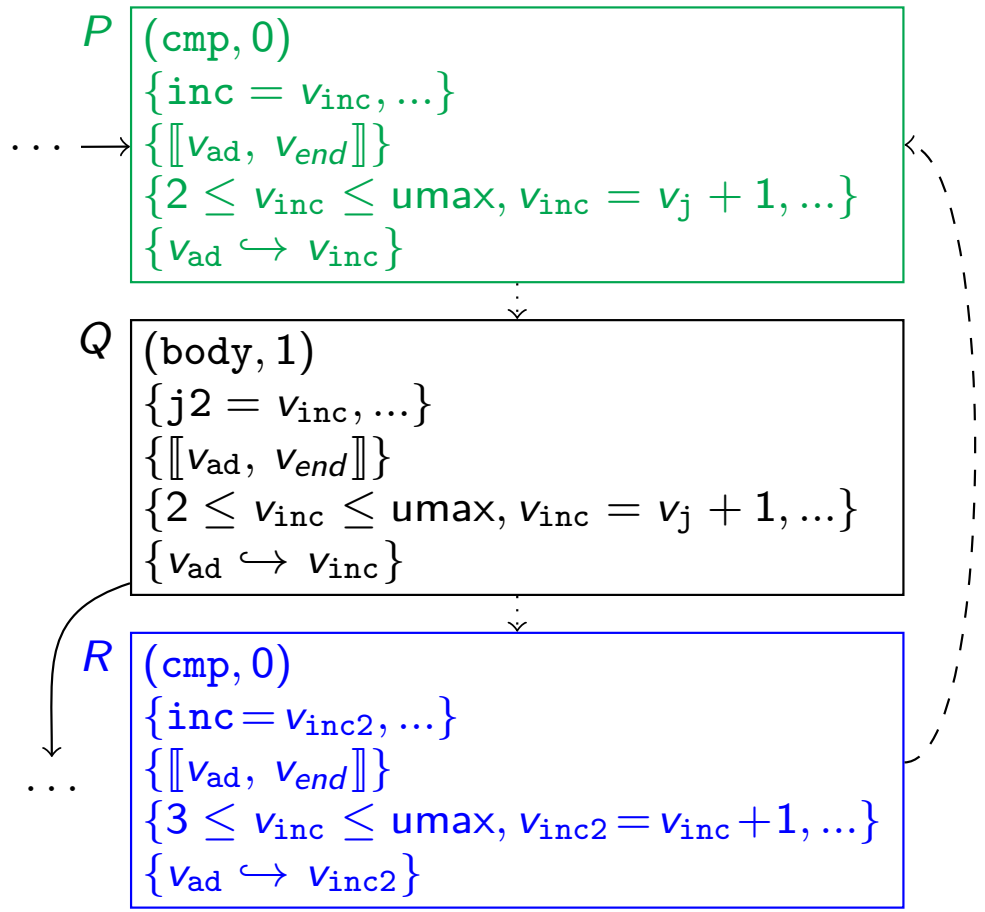
```



P is *generalization* of R with $\mu(v_j) = v_{inc}$, $\mu(v_{inc}) = v_{inc2}$

- $\mu(PV_P(x)) = PV_R(x)$ for all program variables x
- $[\![v_1, v_2]\!] \in AL_P$ implies $[\![\mu(v_1), \mu(v_2)]\!] \in AL_R$
- $\models \langle R \rangle \implies \mu(KB_P)$
- $v_1 \hookrightarrow v_2 \in PT_P$ implies $\mu(v_1) \hookrightarrow \mu(v_2) \in PT_R$

Safety

```
define i32 @g(i32 j) {
entry: 0:ad = alloca i32
       1:store i32 j, i32* ad
       2:br label cmp
cmp:   0:j1 = load i32* ad
       1:j1p = icmp ugt i32 j1, 0
       2:br i1 j1p, label body,
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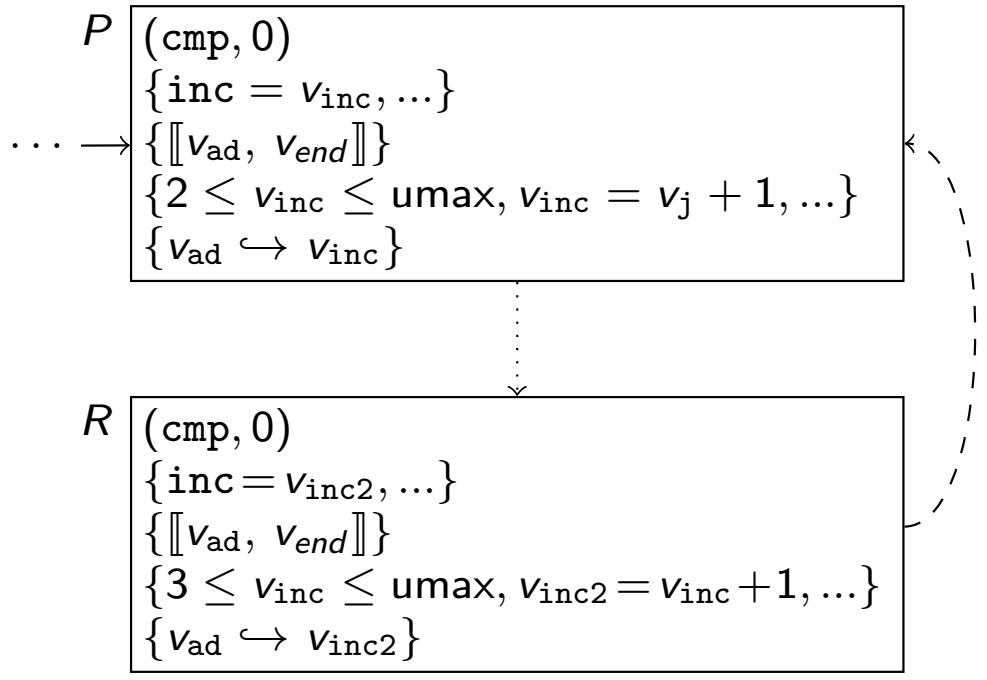
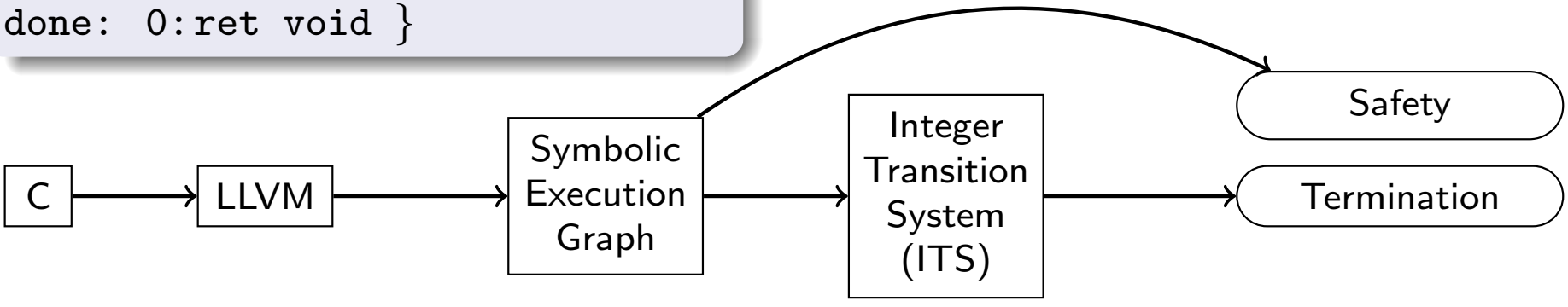
done:  0:ret void }
```





Safety

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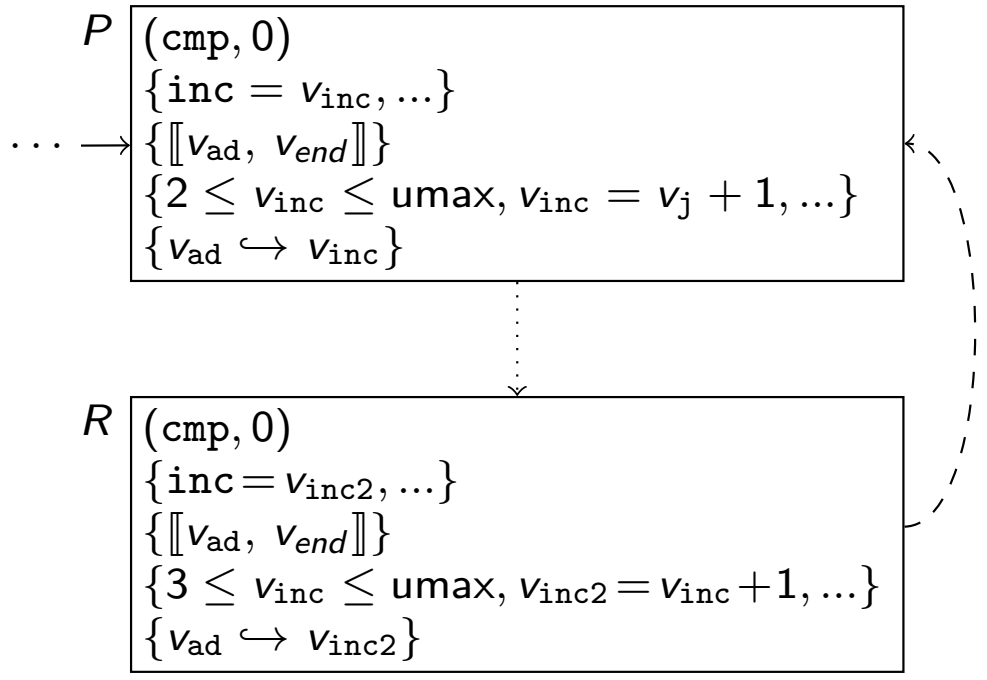

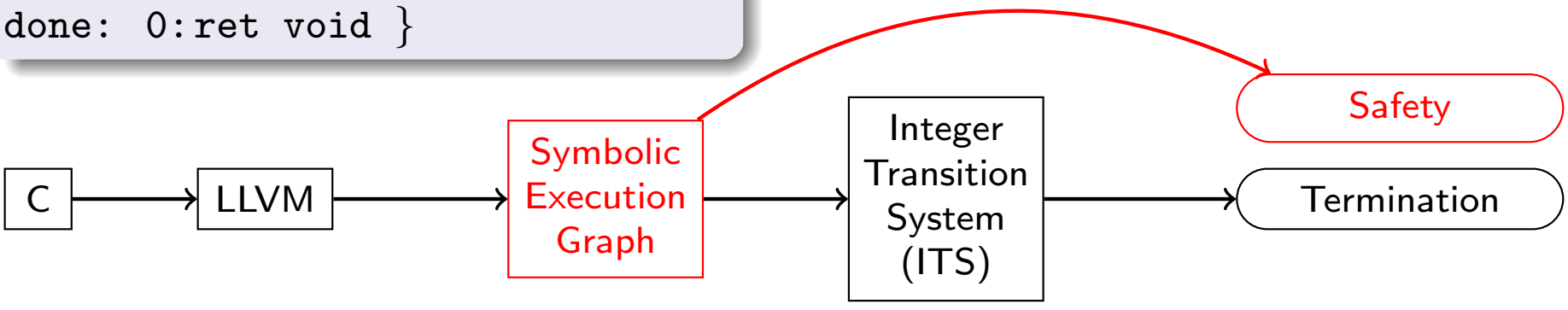

Safety

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define i32 @g(i32 j) {
entry: 0:ad = alloca i32
       1:store i32 j, i32* ad
       2:br label cmp
cmp:   0:j1 = load i32* ad
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       2:br i1 j1p, label body,
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       2:store i32 inc, i32* ad
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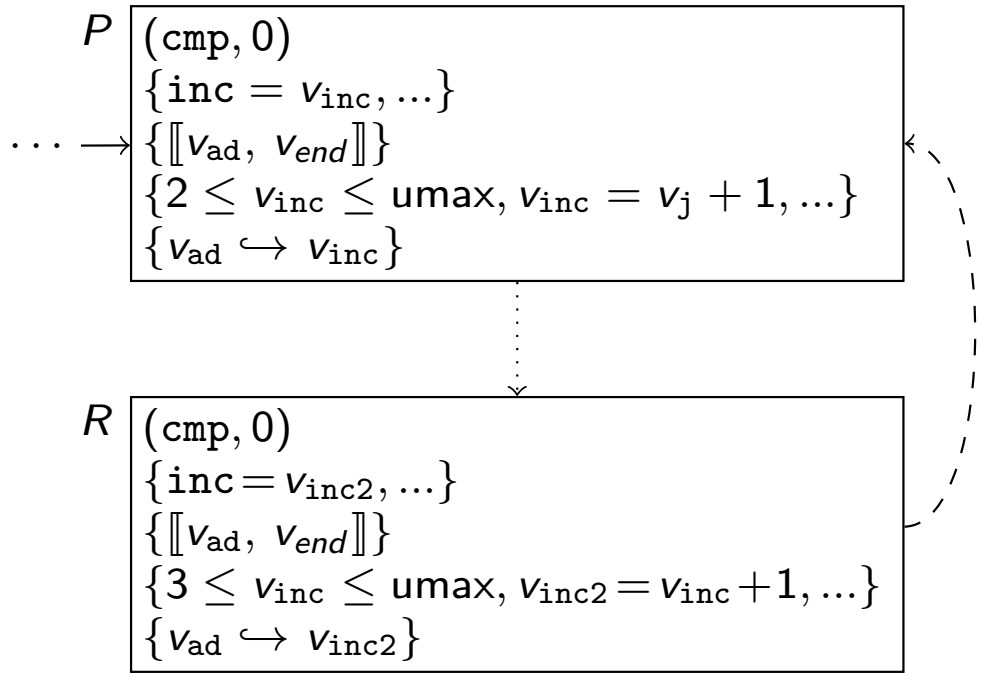
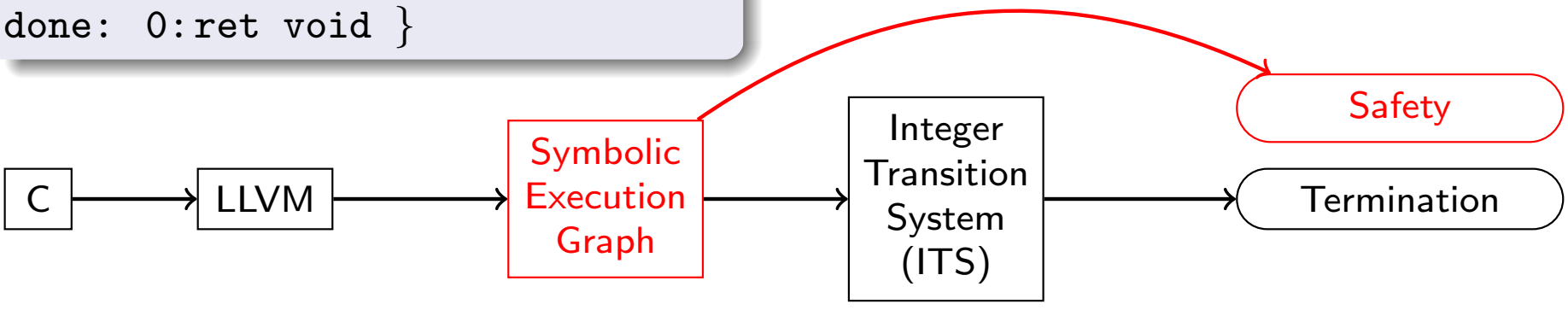
done:  0:ret void }
```





- Symbolic execution graph complete if leaves correspond to `return`

Safety

```
define i32 @g(i32 j) {
entry: 0:ad = alloca i32
       1:store i32 j, i32* ad
       2:br label cmp
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       2:br i1 j1p, label body,
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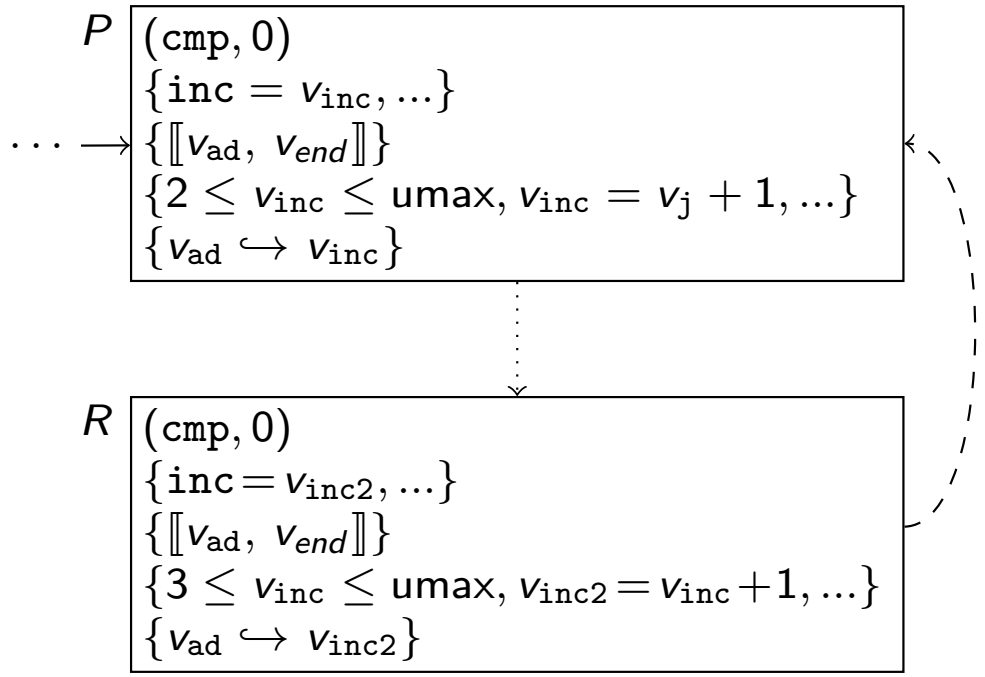

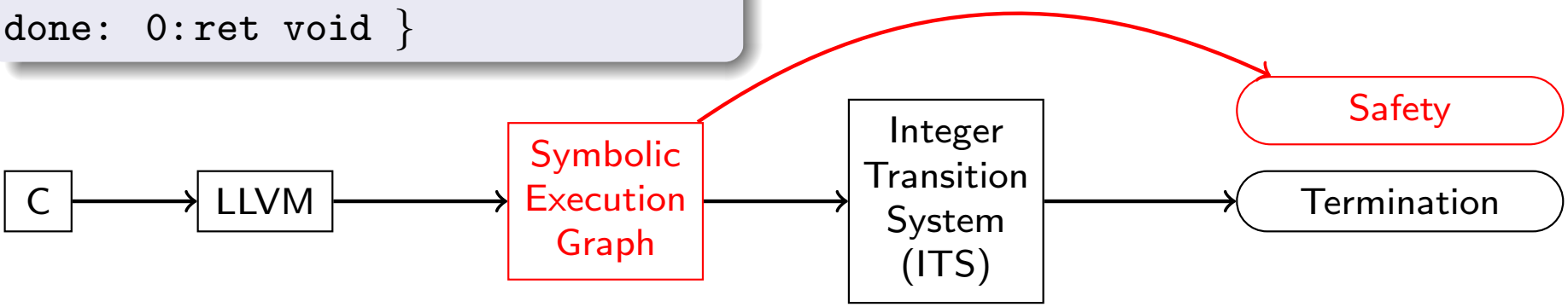

- Symbolic execution graph complete if leaves correspond to `return`
- Complete symbolic execution graph without *ERR* $\Longrightarrow$ Safety

Termination

```
define i32 @g(i32 j) {
entry: 0:ad = alloca i32
       1:store i32 j, i32* ad
       2:br label cmp
cmp:   0:j1 = load i32* ad
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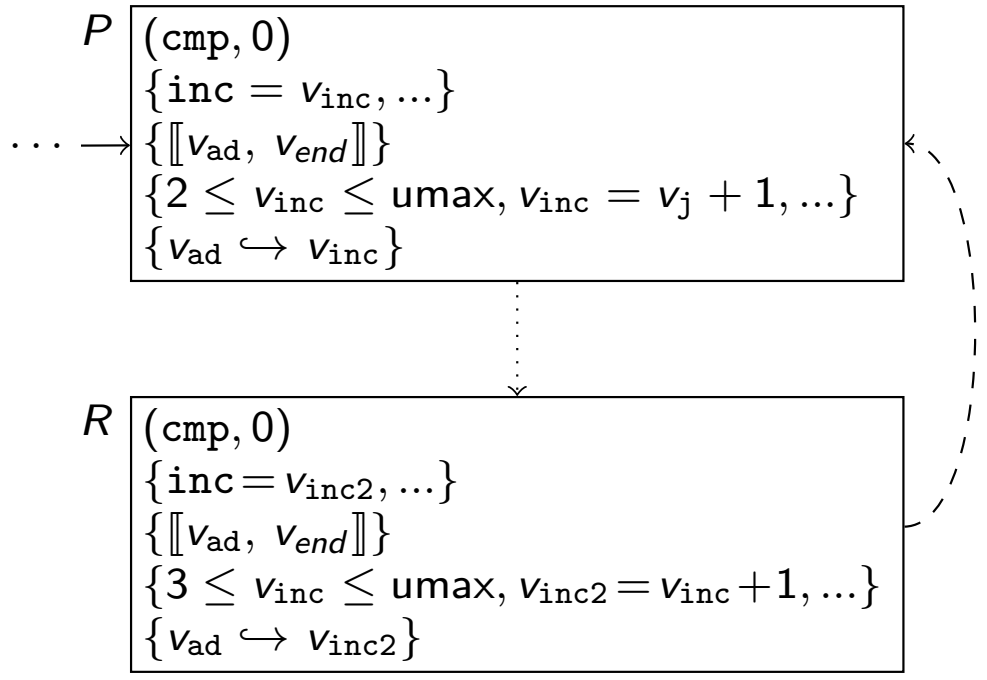
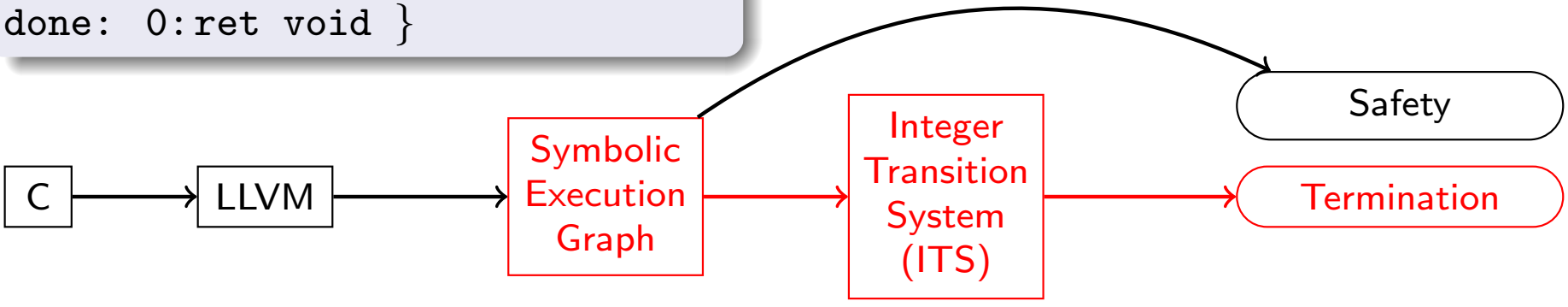
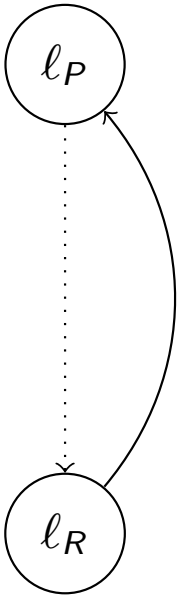
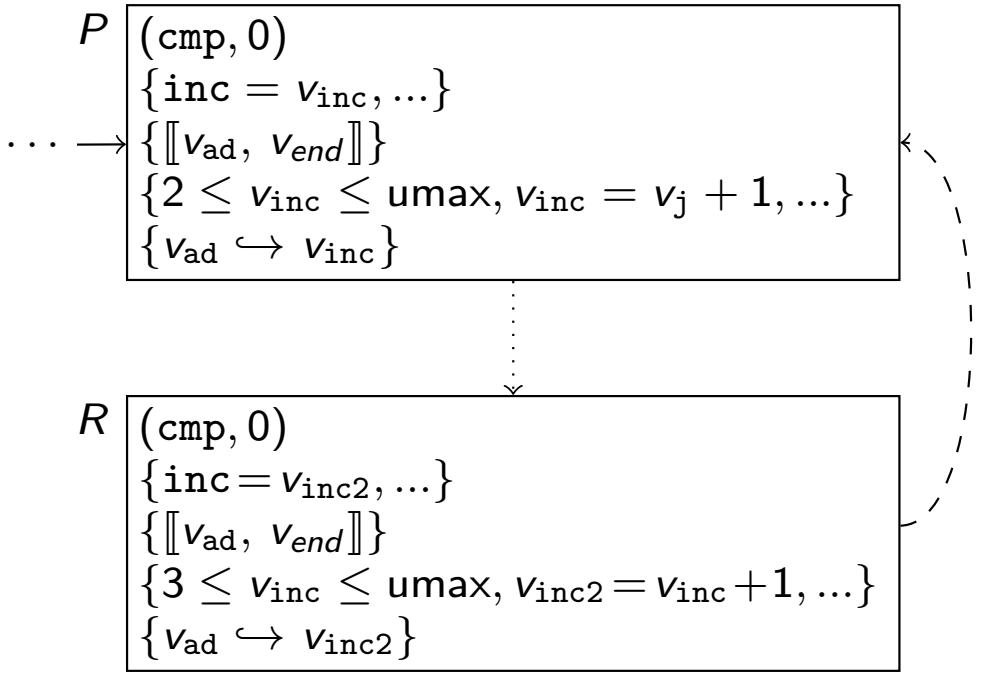
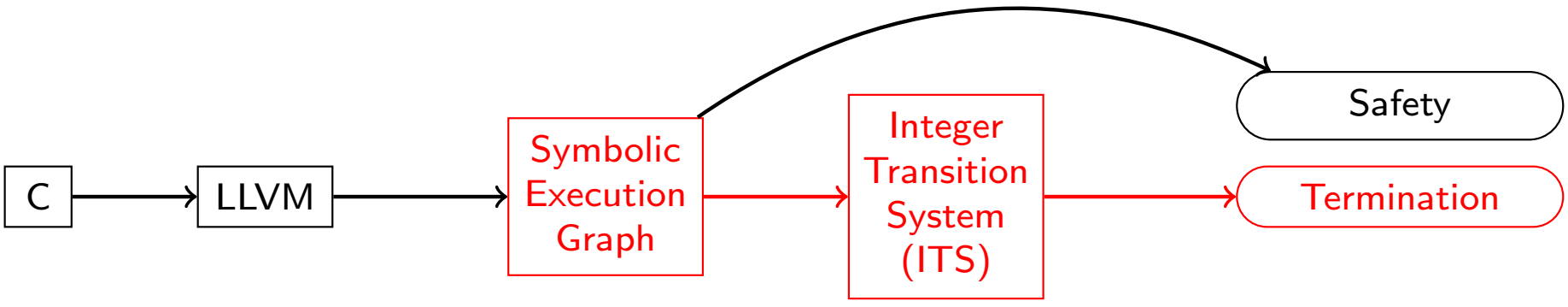




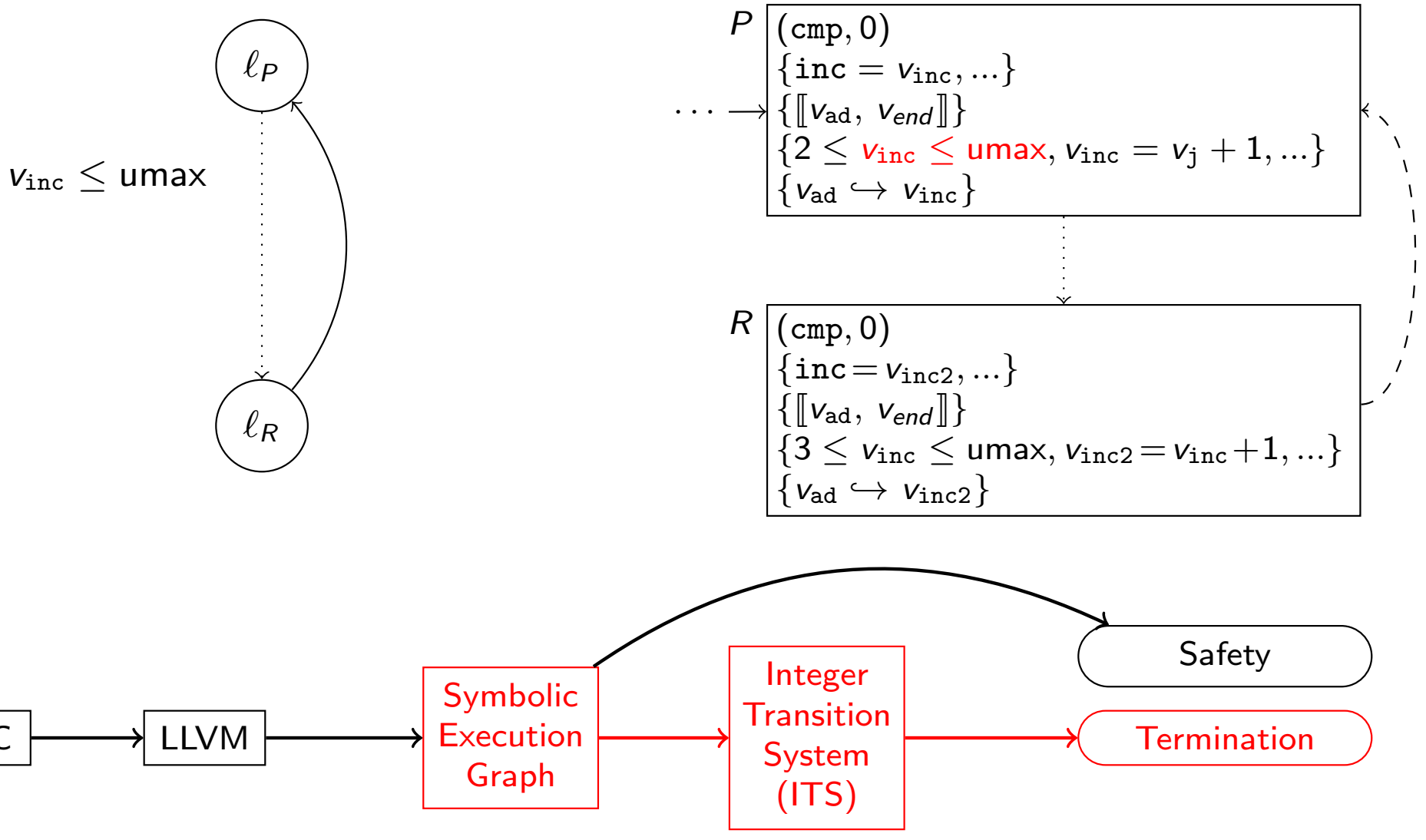
Termination



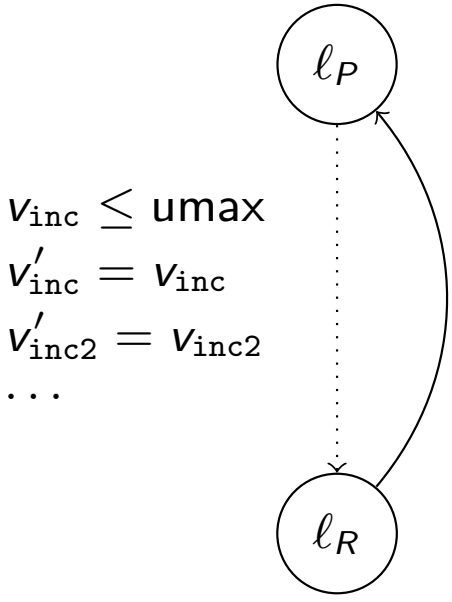


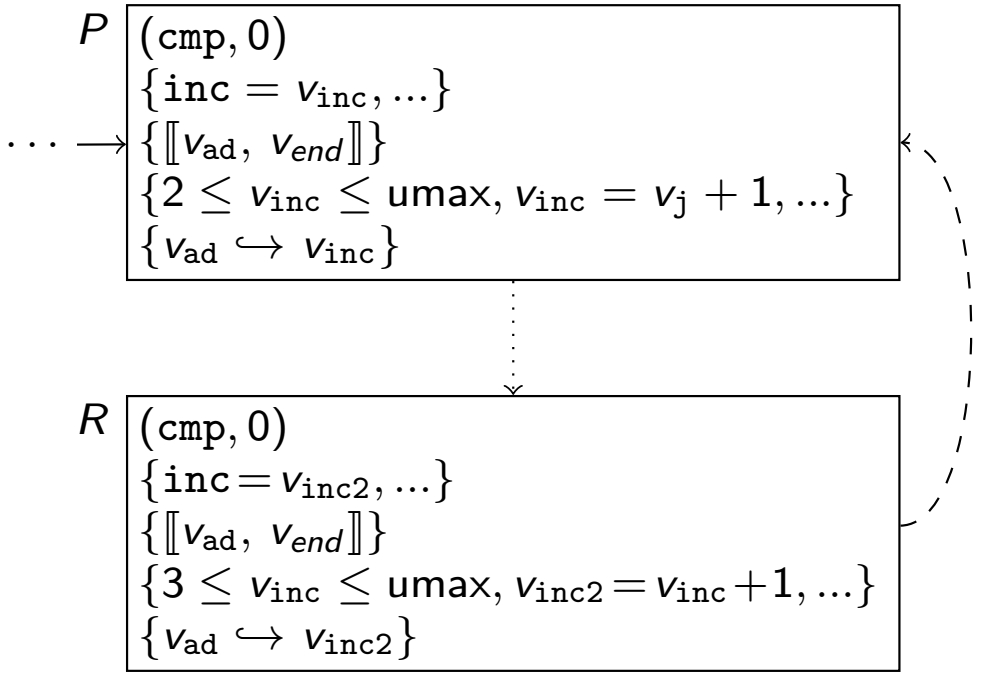


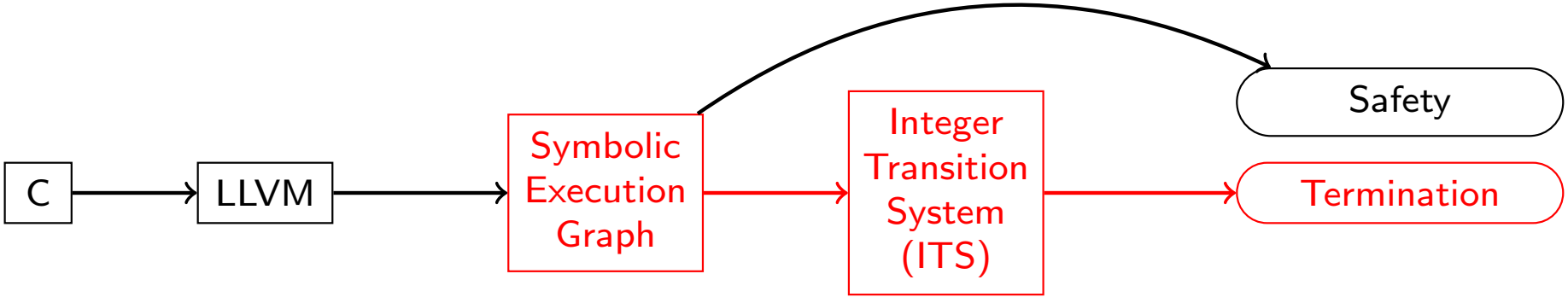
Termination



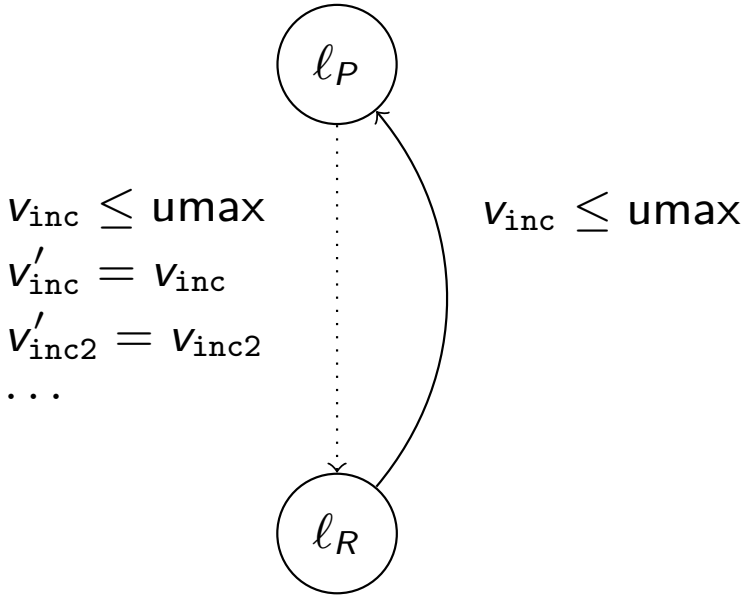
Termination

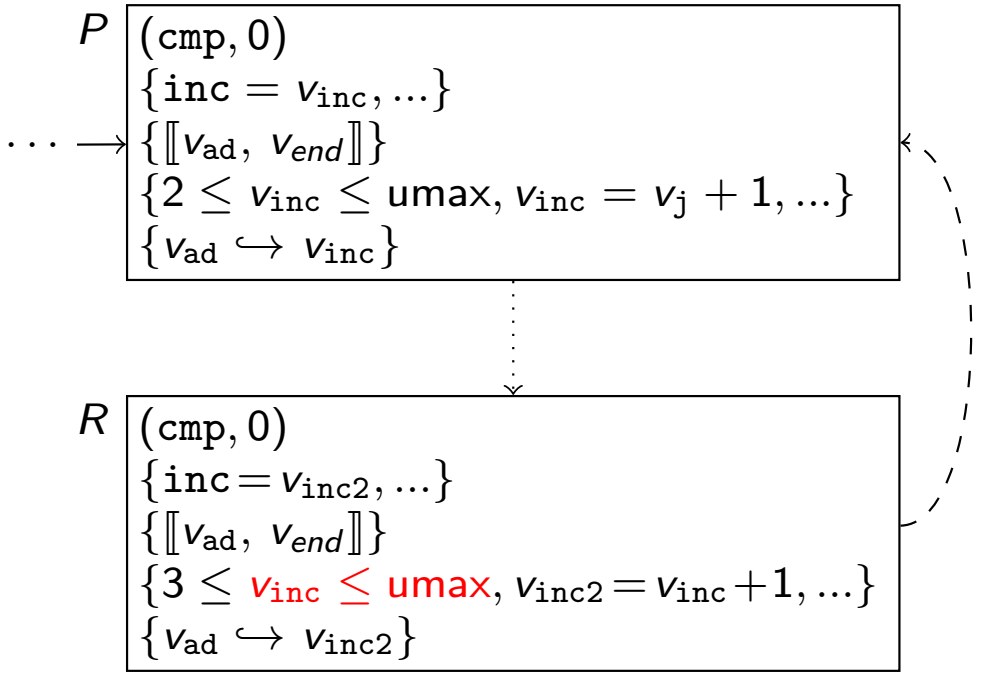


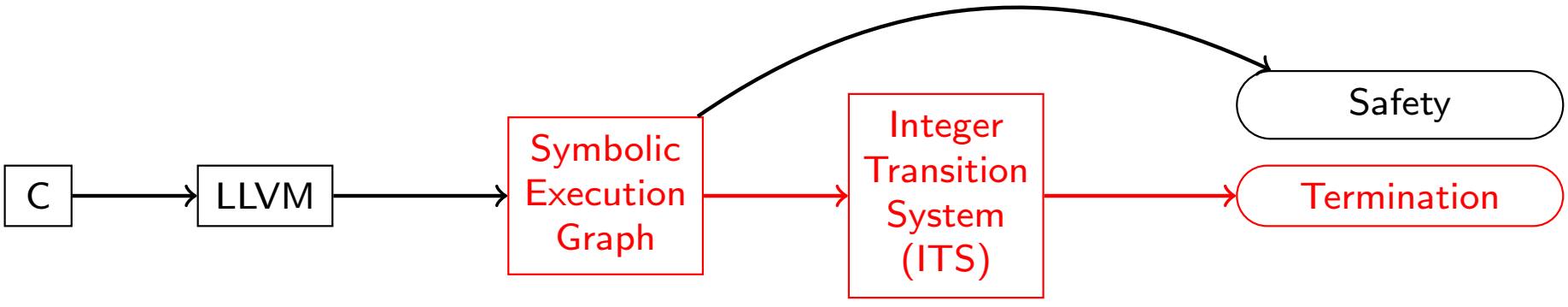




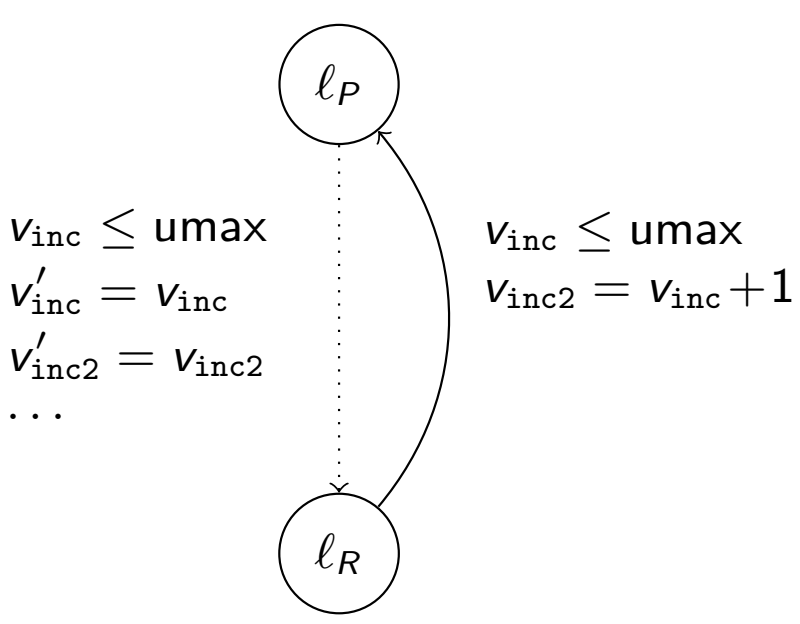
Termination

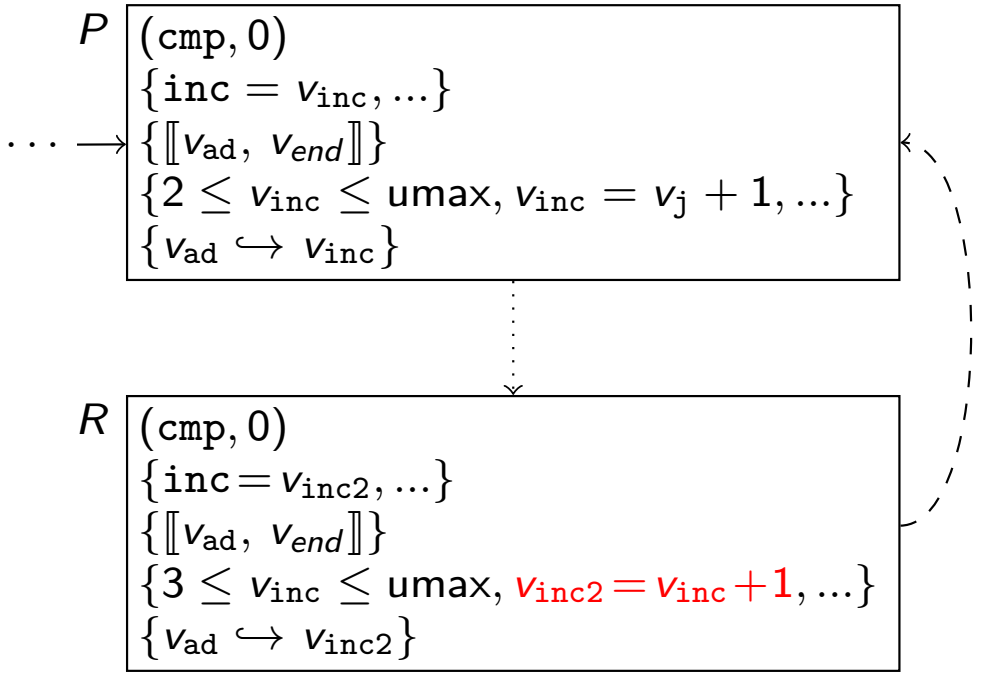


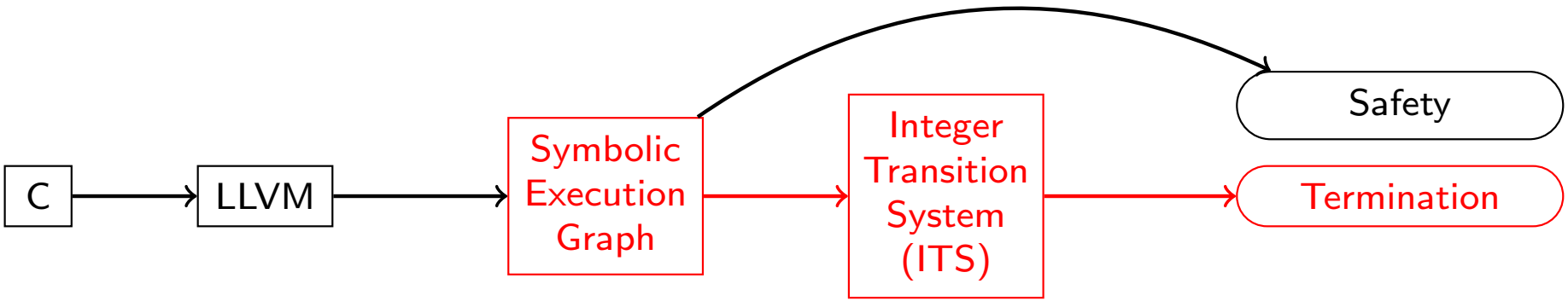




Termination







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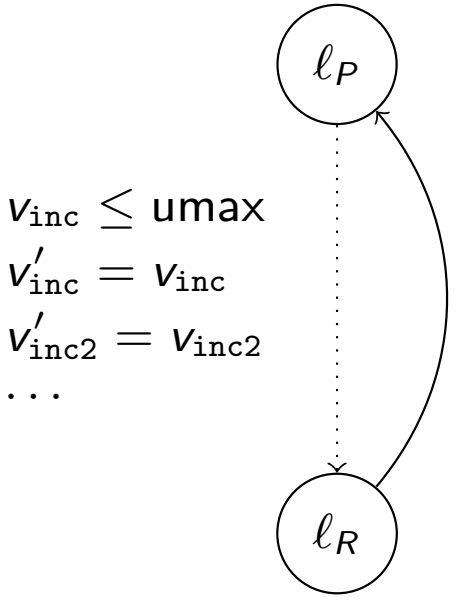
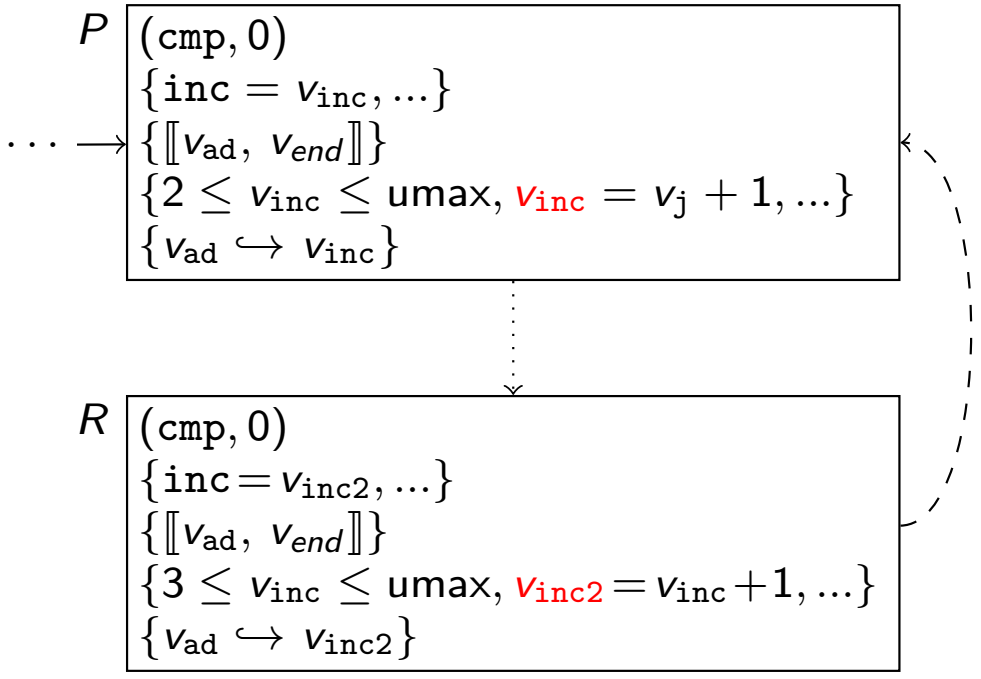
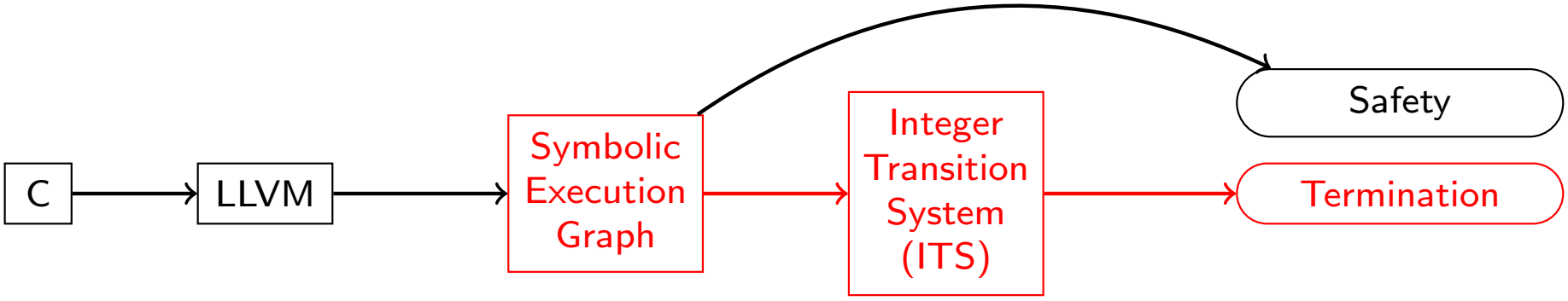
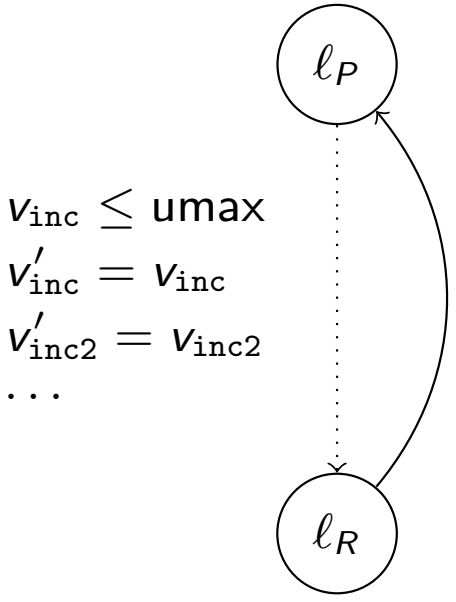
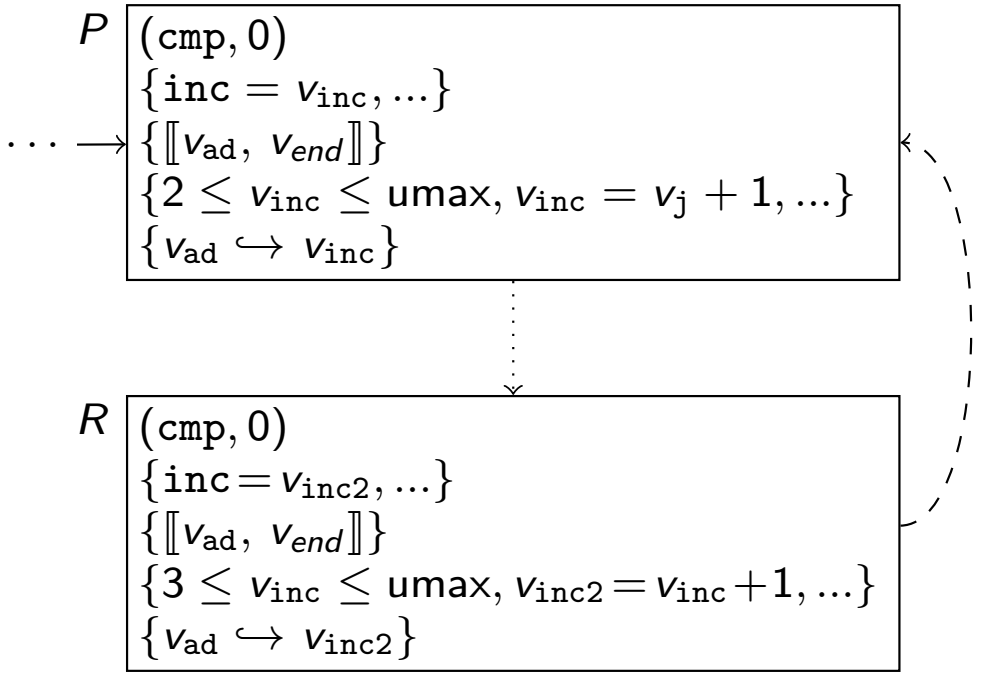


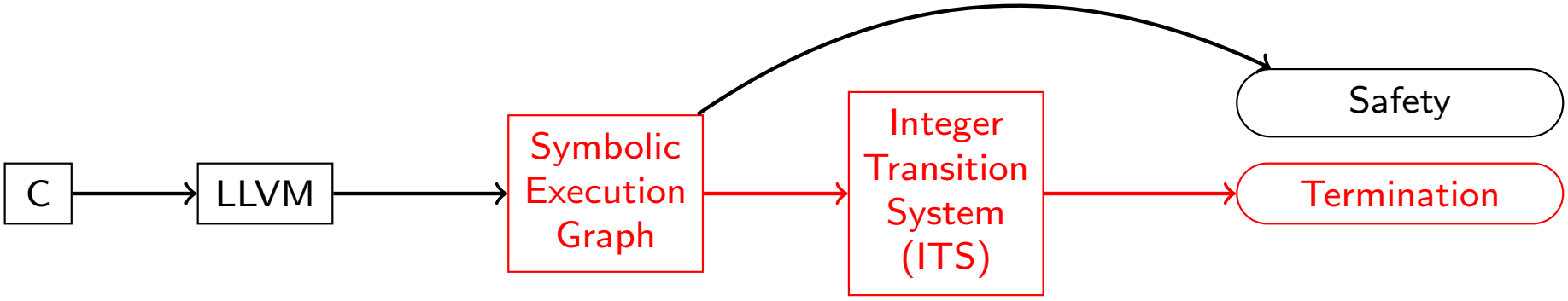




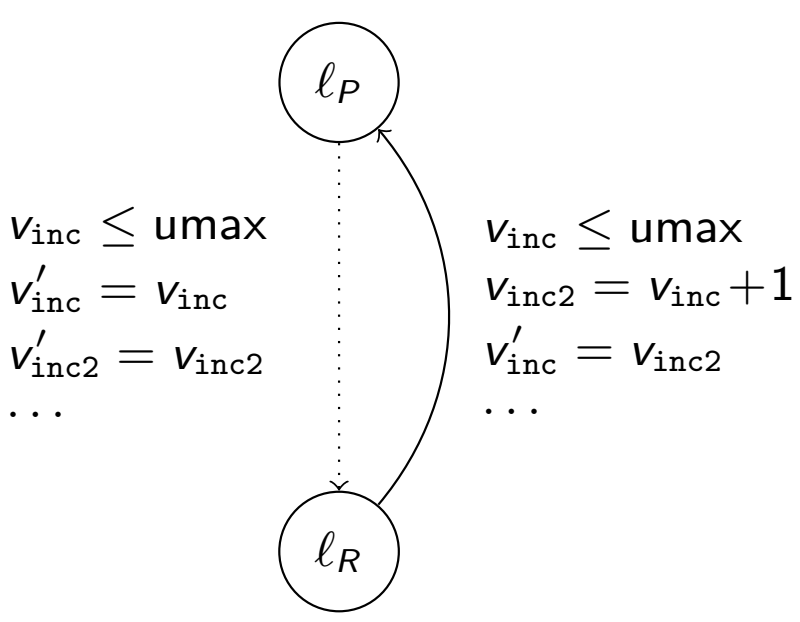
Termination

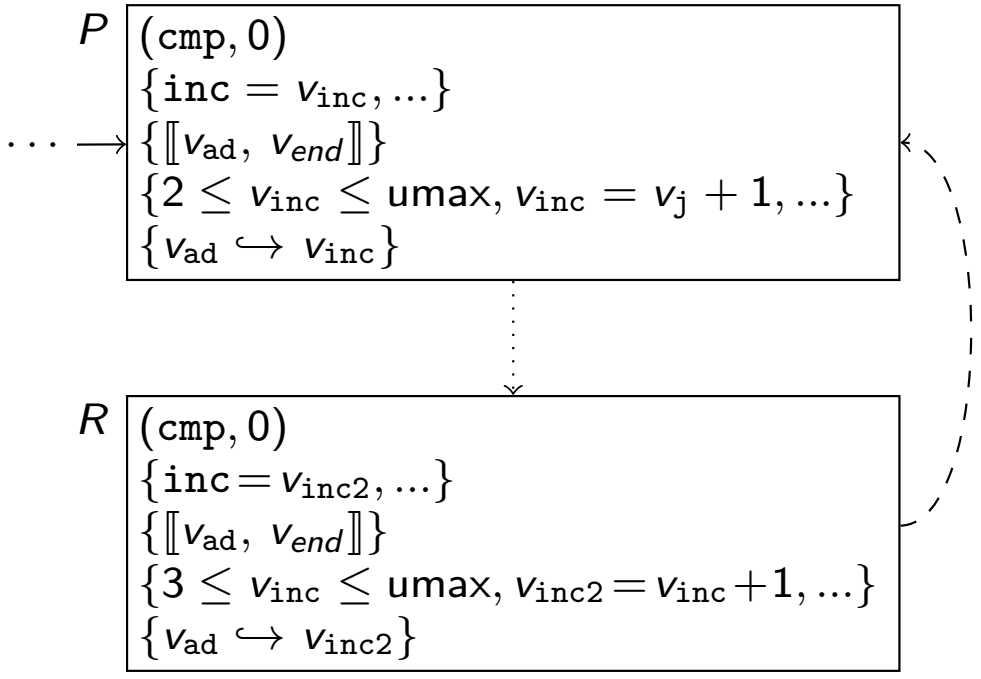


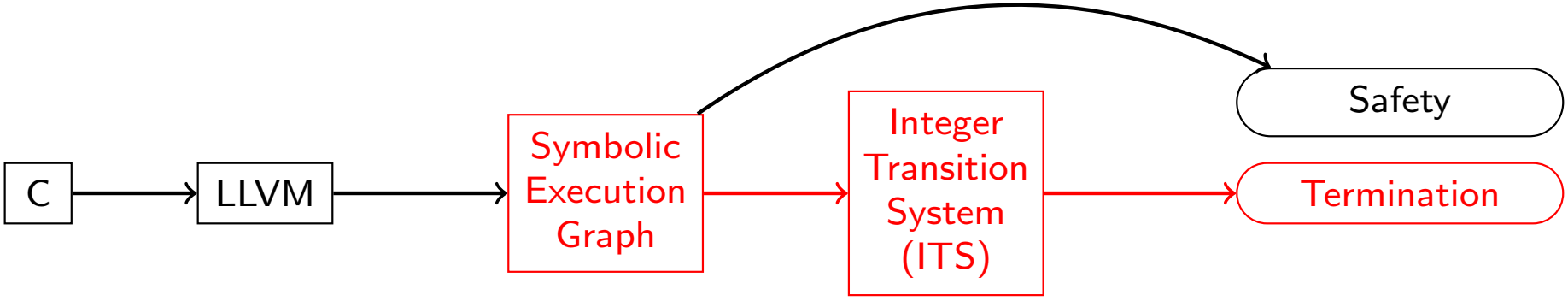




Termination







- ITS from cycles of symbolic execution graph

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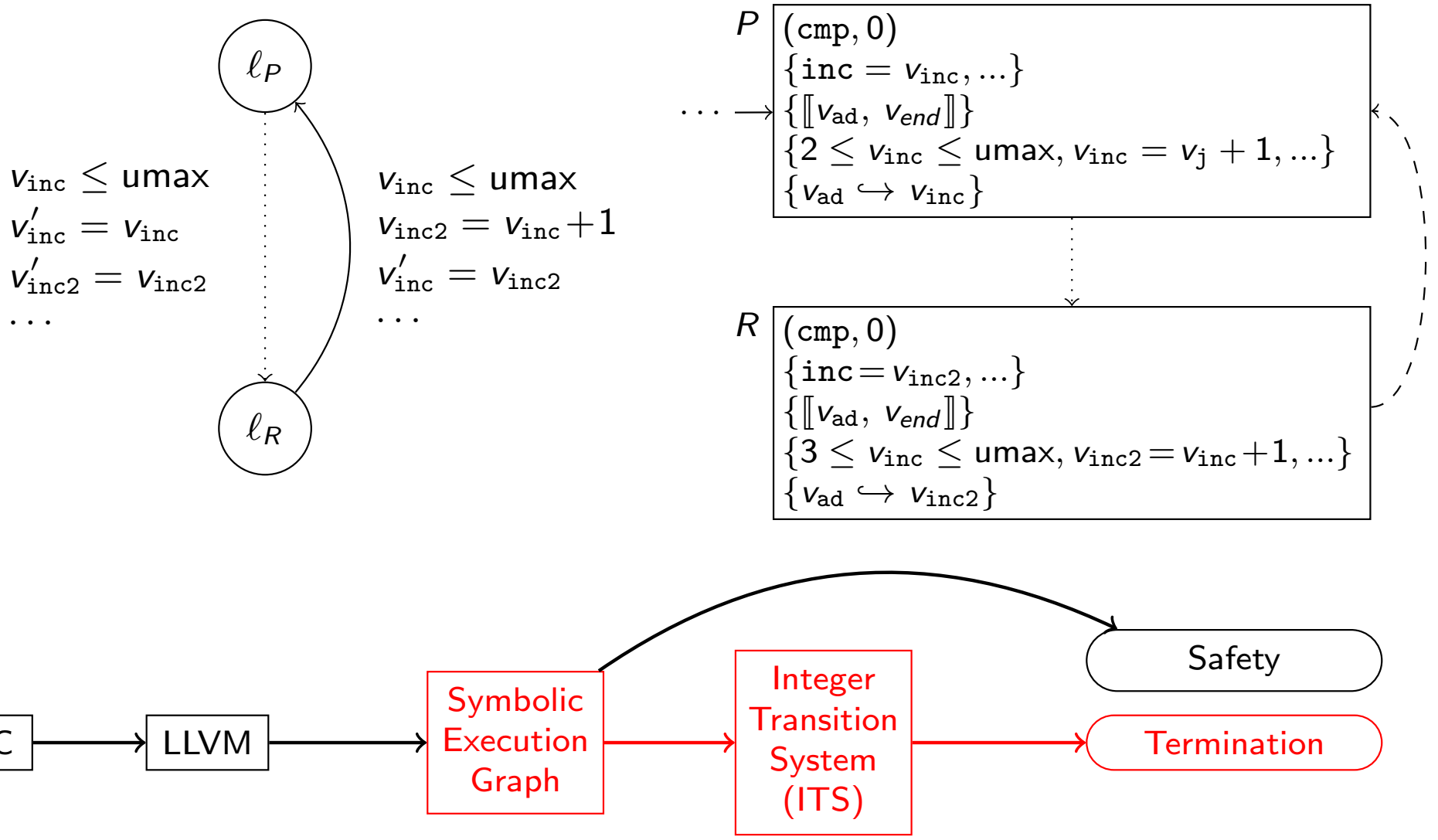

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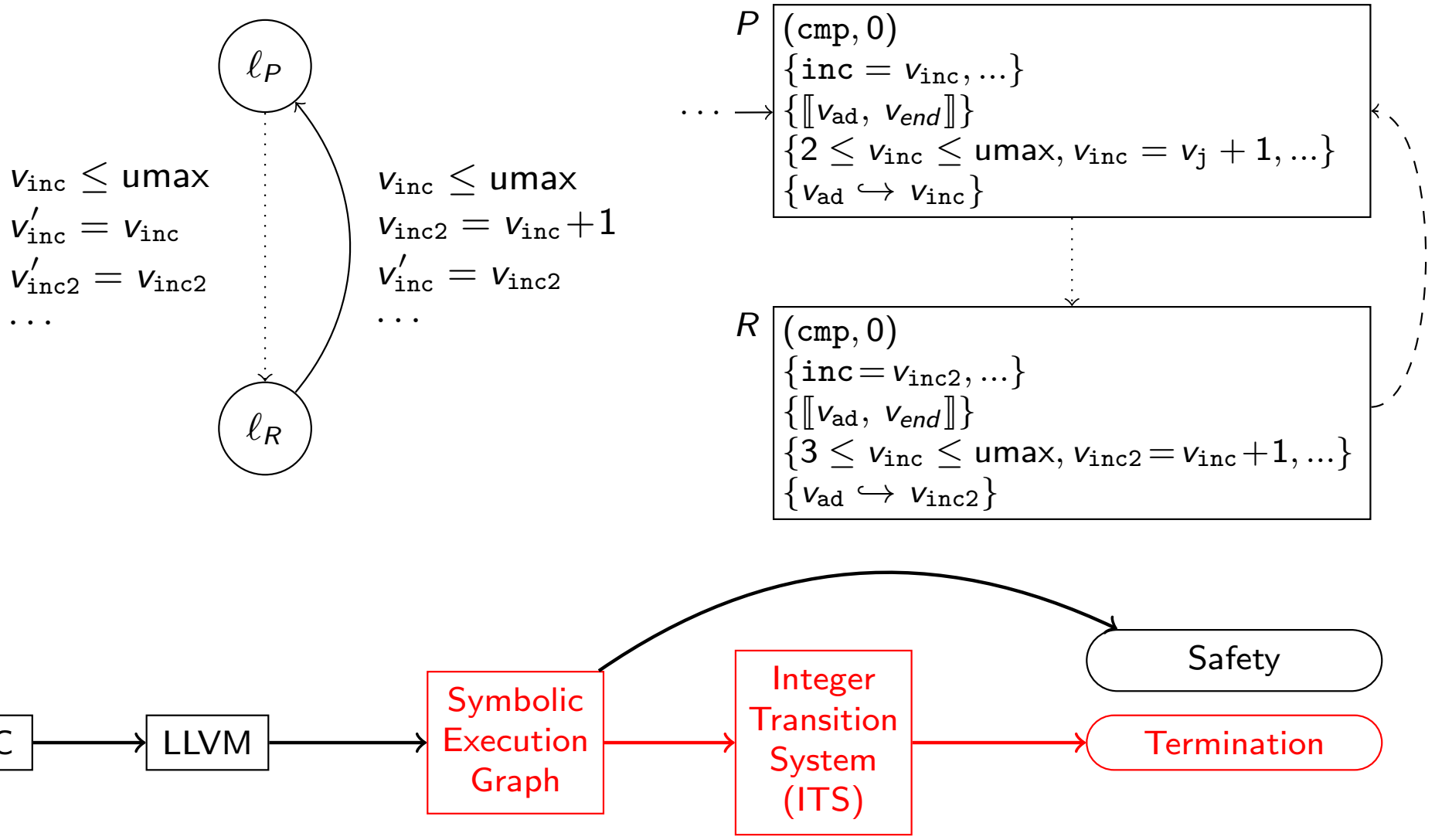

- ITS from cycles of symbolic execution graph
- ITS termination by existing tools $\Longrightarrow$ LLVM program terminates

Multiplication

- **Addition:** handle overflows by case analysis

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	T	**F**	**TO**	**RT**	**T**	**F**	**TO**	**RT**	**%**
AProVE	34	9	9	10.23	61	3	2	5.55	80.5
2LS	23	29	0	0.37	45	21	0	0.33	57.6
KITTeL	27	4	21	1.81	33	3	30	14.17	50.8
Juggernaut	10	19	23	34.12	22	26	18	6.22	27.1
Ultimate	–	–	–	–	11	54	1	12.77	16.7