

# A Case for Completion Modulo Equivalence

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*1st International Workshop on Confluence*

May 29, 2012, Nagoya, Japan

We present simple use cases from compiler writing for the use of completion modulo an equational theory, corresponding to the semantics of the target language.

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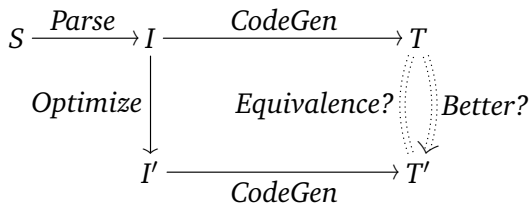
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CRSX is Higher Order Rewriting engine with special support for writing compilers.

- RTA 1996 free variable matching rules (w/Haskell interpreter)
- XML 2005 starts as internal optimizer in Java impl. of XPath
- HOR 2007 as executable HOR tool
- IBM 2008 Compiler specification language started
- HOR 2010 “Compromised Rewriting Systems” described as compiler source language
- RTA 2011 Environment extension and polymorphic sorts “tricks”
- HOR 2012 Issues for Real Programmers “+ Blessings of Completion”

See <http://crsx.sf.net>.



## 1 Optimization by Completion

- Peano
- Compilation

## 2 Conclusion

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$N ::= Z \mid S[N] ;$

$OP ::= + \mid \times ;$

$E ::= Op[OP, E, E] \mid Nat[N] ;$

$C ::= \mathbf{list}[INS] ;$

$INS ::= OP[OP] \mid PUSH[N] ;$

Note:  $\mathbf{list}[INS]$  *has members*  $(INS; \dots INS;)$

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$T[E, C] :: C ;$

$T[\text{Nat}[N], C] \rightarrow (\text{PUSH}[N]; C) ;$

$T[\text{Op}[\text{OP}, E_1, E_2], C] \rightarrow T[E_1, T[E_2, (\text{OP}[\text{OP}]; C)]] ;$

## Sample

$T[\text{Op}[\times, \text{Nat}[\text{S}[\text{Z}]], \text{Op}[\text{+}, \text{Nat}[\text{Z}], \text{Nat}[\text{S}[\text{S}[\text{Z}]]]], \text{O}]$

$\rightarrow T[\text{Nat}[\text{S}[\text{Z}]], T[\text{Op}[\text{+}, \text{Nat}[\text{Z}], \text{Nat}[\text{S}[\text{S}[\text{Z}]]], (\text{OP}[\times];)]]$

$\rightarrow (\text{PUSH}[\text{S}[\text{Z}]] ; T[\text{Nat}[\text{Z}], T[\text{Nat}[\text{S}[\text{S}[\text{Z}]]], (\text{OP}[\text{+}]; \text{OP}[\times];)]]])$

$\rightarrow \rightarrow (\text{PUSH}[\text{S}[\text{Z}]] ; \text{PUSH}[\text{Z}] ; \text{PUSH}[\text{S}[\text{S}[\text{Z}]]] ; \text{OP}[\text{+}]; \text{OP}[\times];)$

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$$T[\text{Op}[\times, \text{Nat}[\text{S}[\text{Z}]], \text{Op}[\text{+}, \text{Nat}[\text{Z}], \text{Nat}[\text{S}[\text{S}[\text{Z}]]]], ()]$$
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# Peano Optimization

$\text{Op}[+, \text{Nat}[Z], \mathbb{N}] \rightarrow \mathbb{N}$  ;

$\text{Op}[\times, \text{Nat}[S[Z]], \mathbb{N}] \rightarrow \mathbb{N}$  ;

– [Discard[N]]:

$\text{Op}[\times, \text{Nat}[Z], \mathbb{N}] \rightarrow \text{Nat}[Z]$  ;

## Sample

$T[\text{Op}[\times, \text{Nat}[S[Z]], \text{Op}[+, \text{Nat}[Z], \text{Nat}[S[S[Z]]]]], 0]$

$\rightarrow T[\text{Op}[+, \text{Nat}[S[S[Z]]], \text{Nat}[Z]], 0]$

$\rightarrow T[\text{Nat}[S[S[Z]]], 0]$

$\rightarrow (\text{PUSH}[S[S[Z]]];)$

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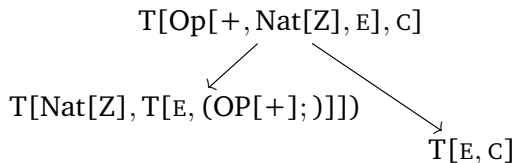
## Sample

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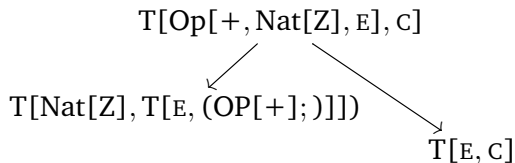
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## Observation

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$T[\text{Op}[+, \text{Nat}[Z], E_2], C] \rightarrow T[E_2, C] ;$

$T[\text{Op}[+, \text{Nat}[S[N_1]], E_2], C] \rightarrow T[\text{Nat}[S[N_1]], T[E_2, (\text{OP}[+]; C)]] ;$

– [Discard[ $E_2$ ]]:

$T[\text{Op}[\times, \text{Nat}[Z], E_2], C] \rightarrow (\text{PUSH}[Z]; C) ;$

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$E ::= \nu \mid \text{Int}[\text{integer}] \mid \text{Str}[\text{String}] \mid \text{Nil} \mid \text{Seq}[E, E]$   
 $\mid \text{IfNil}[E, E, E] \mid \text{Let}[E, x: E.E] ;$

$A ::= \text{list}[AI] ;$

$AI ::= L[AL] \mid J[AL] \mid \text{PU} \mid \text{POJN}[AL] \mid \text{POR}[AR]$   
 $\mid \text{SR}[AR] \mid \text{SI}[\text{integer}] \mid \text{SS}[\text{String}] ;$

$AL ::= \text{label} ;$

$AR ::= \text{register} ;$



# Compilation Rules

Idea:  $(\text{PU}; \{in:r_i; out:r_o\}T[E, (\text{POR}[r_o];)])$

$\{\rho\} = \{E: AR\} ;$

$\{\rho\} T[E, A] :: A ;$

$\{v:r\} T[v, A] \rightarrow (\text{SR}[r]; A) ;$

$\{\neg v\} T[v, A] \rightarrow (\text{SS}["\text{Error}"]; A) ;$

$\{\rho\} T[\text{Int}[I], A] \rightarrow (\text{SI}[I]; A) ;$

$\{\rho\} T[\text{Str}[s], A] \rightarrow (\text{SS}[s]; A) ;$

$\{\rho\} T[\text{Nil}, A] \rightarrow A ;$

$\{\rho\} T[\text{Seq}[E_1, E_2], A] \rightarrow \{\rho\}T[E_1, \{\rho\}T[E_2, A]] ;$

$-\text{[Fresh}[f, fi]] :$

$\{\rho\} T[\text{If}[E, E_1, E_2], A]$

$\rightarrow (\text{PU}; \{\rho\}T[E, (\text{POJN}[f]; \{\rho\}T[E_1, (\text{J}[fi]; \text{L}[f]; \{\rho\}T[E_2, (\text{L}[fi]; A)]))]) ;$

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$\{\rho\} T[\text{Let}[E_1, v_1.E_2[v_1]], A] \rightarrow (\text{PU}; \{\rho\}T[E_1, (\text{POR}[r]; \{\rho; x:r\}T[E_2[x], A])]) ;$

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If[Nil, E<sub>1</sub>, E<sub>2</sub>] → E<sub>2</sub> ;

Let[E, x.x] → E ;

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(PU; {in:r<sub>i</sub>; out:r<sub>o</sub>}T[Let[If[Nil, Nil, in], x.x], (POR[r<sub>o</sub>];)])

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$$\{S\}T[\text{If}[\text{Nil}, E_1, E_2], A] \rightarrow \{S\}T[E_2, A]$$

– [Fresh[f, fi]] :

$$\{S\}T[\text{If}[E \neq \text{Nil}, E_1, E_2], A]$$
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- **Compilers are complete modulo the target language semantic equations.**
- Optimizations are intermediate language (data) rewrites.
- Completion with priority to data rewrites integrates translation schemes and optimization rules.
- Peano integers never get boring!



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Thank You!