A Case for Completion Modulo Equivalence

Kristoffer H. Rose

IBM Thomas J. Watson Research Center

1st International Workshop on Confluence
May 29, 2012, Nagoya, Japan

Abstract

We present simple use cases from compiler writing for the use of completion modulo an equational theory, corresponding to the semantics of the target language.

CRSX is Higher Order Rewriting engine with special support for writing compilers.

- **RTA 1996** free variable matching rules (w/Haskell interpreter)
- **XML 2005** starts as internal optimizer in Java impl. of XPath
- **HOR 2007** as executable HOR tool
- **IBM 2008** Compiler specification language started
- **HOR 2010** "Compromised Rewriting Systems" described as compiler source language
- **RTA 2011** Environment extension and polymorphic sorts "tricks"
- **HOR 2012** Issues for Real Programmers "+ Blessings of Completion"

See `http://crsx.sf.net`.

Compiling by Rewriting

$$
\begin{tikzcd}
S \arrow[r, "Parse"] & I \arrow[rr, "CodeGen"] \arrow[d, "Optimize"'] & & T \arrow[d, dotted, dash, bend right, "Equivalence?"'] \arrow[d, dotted, bend left, "Better?"] \\
& I' \arrow[rr, "CodeGen"'] & & T'
\end{tikzcd}
$$

Plan

1. Optimization by Completion
   - Peano
   - Compilation
2. Conclusion

Plan

1. Optimization by Completion
   - Peano
   - Compilation
2. Conclusion

Peano Sorts

$\text{N} ::= \text{Z} \mid \text{S}[\text{N}]$;
$\text{OP} ::= + \mid \times$;

$\text{E} ::= \text{Op}[\text{OP}, \text{E}, \text{E}] \mid \text{Nat}[\text{N}]$;

$\text{C} ::= \textbf{list}[\text{INS}]$;
$\text{INS} ::= \text{OP}[\text{OP}] \mid \text{PUSH}[\text{N}]$;

Note: **list**[INS] *has members* (INS; . . . INS;)

Peano Sorts

$\textsc{n} ::= \mathrm{Z} \mid \mathrm{S}[\textsc{n}]$;
$\textsc{op} ::= + \mid \times$;

$\textsc{e} ::= \mathrm{Op}[\textsc{op}, \textsc{e}, \textsc{e}] \mid \mathrm{Nat}[\textsc{n}]$;

$\textsc{c} ::= \mathbf{list}[\textsc{ins}]$;
$\textsc{ins} ::= \mathrm{OP}[\textsc{op}] \mid \mathrm{PUSH}[\textsc{n}]$;

Note: $\mathbf{list}[\textsc{ins}]$ *has members* $(\textsc{ins}; \ldots\ \textsc{ins};)$

Peano Sorts

$$\text{N} ::= \text{Z} \mid \text{S}[\text{N}]\ ;$$
$$\text{OP} ::= + \mid \times\ ;$$

$$\text{E} ::= \text{Op}[\text{OP}, \text{E}, \text{E}] \mid \text{Nat}[\text{N}]\ ;$$

$$\text{C} ::= \textbf{list}[\text{INS}]\ ;$$
$$\text{INS} ::= \text{OP}[\text{OP}] \mid \text{PUSH}[\text{N}]\ ;$$

Note: **list**[INS] *has members* (INS; ... INS;)

Peano Compilation

$\mathrm{T}[\textsc{e}, \textsc{c}] :: \textsc{c}$;

$\mathrm{T}[\mathrm{Nat}[\textsc{n}],\ \textsc{c}] \to (\mathrm{PUSH}[\textsc{n}]; \textsc{c})$;

$\mathrm{T}[\mathrm{Op}[\textsc{op}, \textsc{e}_1, \textsc{e}_2],\ \textsc{c}] \to \mathrm{T}[\textsc{e}_1, \mathrm{T}[\textsc{e}_2, (\mathrm{OP}[\textsc{op}]; \textsc{c})]]$;

Sample

$\mathrm{T}[\mathrm{Op}[\times, \mathrm{Nat}[\mathrm{S}[\mathrm{Z}]], \mathrm{Op}[+, \mathrm{Nat}[\mathrm{Z}], \mathrm{Nat}[\mathrm{S}[\mathrm{S}[\mathrm{Z}]]]]], ()]$

$\to \mathrm{T}[\mathrm{Nat}[\mathrm{S}[\mathrm{Z}]],\ \mathrm{T}[\mathrm{Op}[+, \mathrm{Nat}[\mathrm{Z}], \mathrm{Nat}[\mathrm{S}[\mathrm{S}[\mathrm{Z}]]]],\ (\mathrm{OP}[\times];)]]$

$\to (\mathrm{PUSH}[\mathrm{S}[\mathrm{Z}]];\ \mathrm{T}[\mathrm{Nat}[\mathrm{Z}],\ \mathrm{T}[\mathrm{Nat}[\mathrm{S}[\mathrm{S}[\mathrm{Z}]]],\ (\mathrm{OP}[+];\ \mathrm{OP}[\times];)]])$

$\to\to (\mathrm{PUSH}[\mathrm{S}[\mathrm{Z}]];\ \mathrm{PUSH}[\mathrm{Z}];\ \mathrm{PUSH}[\mathrm{S}[\mathrm{S}[\mathrm{Z}]]];\ \mathrm{OP}[+];\ \mathrm{OP}[\times];)$

Peano Compilation

$\mathrm{T}[\textsc{e}, \textsc{c}] :: \textsc{c}$;

$\mathrm{T}[\mathrm{Nat}[\textsc{n}],\ \textsc{c}] \to (\mathrm{PUSH}[\textsc{n}]; \textsc{c})$;

$\mathrm{T}[\mathrm{Op}[\textsc{op}, \textsc{e}_1, \textsc{e}_2],\ \textsc{c}] \to \mathrm{T}[\textsc{e}_1, \mathrm{T}[\textsc{e}_2, (\mathrm{OP}[\textsc{op}]; \textsc{c})]]$;

Sample

$\mathrm{T}[\mathrm{Op}[\times, \mathrm{Nat}[S[Z]], \mathrm{Op}[+, \mathrm{Nat}[Z], \mathrm{Nat}[S[S[Z]]]]], ()]$

$\to \mathrm{T}[\mathrm{Nat}[S[Z]],\ \mathrm{T}[\mathrm{Op}[+, \mathrm{Nat}[Z], \mathrm{Nat}[S[S[Z]]]],\ (\mathrm{OP}[\times];)]]$

$\to (\mathrm{PUSH}[S[Z]];\ \mathrm{T}[\mathrm{Nat}[Z],\ \mathrm{T}[\mathrm{Nat}[S[S[Z]]],\ (\mathrm{OP}[+];\ \mathrm{OP}[\times];)]])$

$\to\to (\mathrm{PUSH}[S[Z]];\ \mathrm{PUSH}[Z];\ \mathrm{PUSH}[S[S[Z]]];\ \mathrm{OP}[+];\ \mathrm{OP}[\times];)$

Peano Compilation

$\mathrm{T}[\textsc{e}, \textsc{c}] :: \textsc{c}\ ;$

$\mathrm{T}[\mathrm{Nat}[\textsc{n}],\ \textsc{c}] \to (\mathrm{PUSH}[\textsc{n}]; \textsc{c})\ ;$

$\mathrm{T}[\mathrm{Op}[\textsc{op}, \textsc{e}_1, \textsc{e}_2],\ \textsc{c}] \to \mathrm{T}[\textsc{e}_1, \mathrm{T}[\textsc{e}_2, (\mathrm{OP}[\textsc{op}]; \textsc{c})]]\ ;$

Sample

$\mathrm{T}[\mathrm{Op}[\times, \mathrm{Nat}[\mathrm{S}[\mathrm{Z}]], \mathrm{Op}[+, \mathrm{Nat}[\mathrm{Z}], \mathrm{Nat}[\mathrm{S}[\mathrm{S}[\mathrm{Z}]]]]], ()]$

$\to \mathrm{T}[\mathrm{Nat}[\mathrm{S}[\mathrm{Z}]],\ \mathrm{T}[\mathrm{Op}[+, \mathrm{Nat}[\mathrm{Z}], \mathrm{Nat}[\mathrm{S}[\mathrm{S}[\mathrm{Z}]]]],\ (\mathrm{OP}[\times];)]]$

$\to (\mathrm{PUSH}[\mathrm{S}[\mathrm{Z}]];\ \mathrm{T}[\mathrm{Nat}[\mathrm{Z}],\ \mathrm{T}[\mathrm{Nat}[\mathrm{S}[\mathrm{S}[\mathrm{Z}]]],\ (\mathrm{OP}[+];\ \mathrm{OP}[\times];)]])$

$\to\to (\mathrm{PUSH}[\mathrm{S}[\mathrm{Z}]];\ \mathrm{PUSH}[\mathrm{Z}];\ \mathrm{PUSH}[\mathrm{S}[\mathrm{S}[\mathrm{Z}]]];\ \mathrm{OP}[+];\ \mathrm{OP}[\times];)$

Peano Compilation

$T[\mathrm{E}, \mathrm{C}] :: \mathrm{C}$;

$T[Nat[\mathrm{N}],\ \mathrm{C}] \to (PUSH[\mathrm{N}]; \mathrm{C})$;

$T[Op[\mathrm{OP}, \mathrm{E}_1, \mathrm{E}_2],\ \mathrm{C}] \to T[\mathrm{E}_1, T[\mathrm{E}_2, (OP[\mathrm{OP}]; \mathrm{C})]]$;

Sample

$T[Op[\times, Nat[S[Z]], Op[+, Nat[Z], Nat[S[S[Z]]]]], ()]$

$\to T[Nat[S[Z]],\ T[Op[+, Nat[Z], Nat[S[S[Z]]]],\ (OP[\times];)]]$

$\to (PUSH[S[Z]];\ T[Nat[Z],\ T[Nat[S[S[Z]]],\ (OP[+];\ OP[\times];)]])$

$\to\to (PUSH[S[Z]];\ PUSH[Z];\ PUSH[S[S[Z]]];\ OP[+];\ OP[\times];)$

Peano Compilation

$\mathrm{T}[\mathrm{E}, \mathrm{C}] :: \mathrm{C}$;

$\mathrm{T}[\mathrm{Nat}[\mathrm{N}],\ \mathrm{C}] \to (\mathrm{PUSH}[\mathrm{N}]; \mathrm{C})$;

$\mathrm{T}[\mathrm{Op}[\mathrm{OP}, \mathrm{E}_1, \mathrm{E}_2],\ \mathrm{C}] \to \mathrm{T}[\mathrm{E}_1, \mathrm{T}[\mathrm{E}_2, (\mathrm{OP}[\mathrm{OP}]; \mathrm{C})]]$;

Sample

$\mathrm{T}[\mathrm{Op}[\times, \mathrm{Nat}[\mathrm{S}[\mathrm{Z}]], \mathrm{Op}[+, \mathrm{Nat}[\mathrm{Z}], \mathrm{Nat}[\mathrm{S}[\mathrm{S}[\mathrm{Z}]]]]], ()]$

$\to \mathrm{T}[\mathrm{Nat}[\mathrm{S}[\mathrm{Z}]],\ \mathrm{T}[\mathrm{Op}[+, \mathrm{Nat}[\mathrm{Z}], \mathrm{Nat}[\mathrm{S}[\mathrm{S}[\mathrm{Z}]]]],\ (\mathrm{OP}[\times];)]]$

$\to (\mathrm{PUSH}[\mathrm{S}[\mathrm{Z}]];\ \mathrm{T}[\mathrm{Nat}[\mathrm{Z}],\ \mathrm{T}[\mathrm{Nat}[\mathrm{S}[\mathrm{S}[\mathrm{Z}]]],\ (\mathrm{OP}[+];\ \mathrm{OP}[\times];)]])$

$\to\to (\mathrm{PUSH}[\mathrm{S}[\mathrm{Z}]];\ \mathrm{PUSH}[\mathrm{Z}];\ \mathrm{PUSH}[\mathrm{S}[\mathrm{S}[\mathrm{Z}]]];\ \mathrm{OP}[+];\ \mathrm{OP}[\times];)$

Peano Compilation

$T[\textsc{e},\textsc{c}] :: \textsc{c}$;

$T[\mathrm{Nat}[\textsc{n}],\ \textsc{c}] \to (\mathrm{PUSH}[\textsc{n}];\textsc{c})$;

$T[\mathrm{Op}[\textsc{op},\textsc{e}_1,\textsc{e}_2],\ \textsc{c}] \to T[\textsc{e}_1, T[\textsc{e}_2, (\mathrm{OP}[\textsc{op}];\textsc{c})]]$;

Sample

$T[\mathrm{Op}[\times, \mathrm{Nat}[S[Z]], \mathrm{Op}[+, \mathrm{Nat}[Z], \mathrm{Nat}[S[S[Z]]]]], ()]$

$\to T[\mathrm{Nat}[S[Z]],\ T[\mathrm{Op}[+, \mathrm{Nat}[Z], \mathrm{Nat}[S[S[Z]]]],\ (\mathrm{OP}[\times];)]]$

$\to (\mathrm{PUSH}[S[Z]];\ T[\mathrm{Nat}[Z],\ T[\mathrm{Nat}[S[S[Z]]],\ (\mathrm{OP}[+];\ \mathrm{OP}[\times];)]])$

$\to\to (\mathrm{PUSH}[S[Z]];\ \mathrm{PUSH}[Z];\ \mathrm{PUSH}[S[S[Z]]];\ \mathrm{OP}[+];\ \mathrm{OP}[\times];)$

Peano Compilation

$T[\text{E}, \text{C}] :: \text{C}\ ;$

$T[\text{Nat}[\text{N}],\ \text{C}] \rightarrow (\text{PUSH}[\text{N}]; \text{C})\ ;$

$T[\text{Op}[\text{OP}, \text{E}_1, \text{E}_2],\ \text{C}] \rightarrow T[\text{E}_1, T[\text{E}_2, (\text{OP}[\text{OP}]; \text{C})]]\ ;$

Sample

$T[\text{Op}[\times, \text{Nat}[S[Z]], \text{Op}[+, \text{Nat}[Z], \text{Nat}[S[S[Z]]]]], ()]$

$\rightarrow T[\text{Nat}[S[Z]],\ T[\text{Op}[+, \text{Nat}[Z], \text{Nat}[S[S[Z]]]],\ (\text{OP}[\times];)]]$

$\rightarrow (\text{PUSH}[S[Z]];\ T[\text{Nat}[Z],\ T[\text{Nat}[S[S[Z]]],\ (\text{OP}[+];\ \text{OP}[\times];)]])$

$\rightarrow\rightarrow (\text{PUSH}[S[Z]];\ \text{PUSH}[Z];\ \text{PUSH}[S[S[Z]]];\ \text{OP}[+];\ \text{OP}[\times];)$

Peano Optimization

$\mathrm{Op}[+, \mathrm{Nat}[\mathrm{Z}], \mathrm{N}] \rightarrow \mathrm{N}\ ;$

$\mathrm{Op}[\times, \mathrm{Nat}[\mathrm{S}[\mathrm{Z}]], \mathrm{N}] \rightarrow \mathrm{N}\ ;$

$-[\mathrm{Discard}[\mathrm{N}]]:$
$\mathrm{Op}[\times, \mathrm{Nat}[\mathrm{Z}], \mathrm{N}] \rightarrow \mathrm{Nat}[\mathrm{Z}]\ ;$

Sample

$\mathrm{T}[\mathrm{Op}[\times, \mathrm{Nat}[\mathrm{S}[\mathrm{Z}]], \mathrm{Op}[+, \mathrm{Nat}[\mathrm{Z}], \mathrm{Nat}[\mathrm{S}[\mathrm{S}[\mathrm{Z}]]]]], ()]$

$\rightarrow \mathrm{T}[\mathrm{Op}[+, \mathrm{Nat}[\mathrm{S}[\mathrm{S}[\mathrm{Z}]]], \mathrm{Nat}[\mathrm{Z}]], ()]$

$\rightarrow \mathrm{T}[\mathrm{Nat}[\mathrm{S}[\mathrm{S}[\mathrm{Z}]]], ()]$

$\rightarrow (\mathrm{PUSH}[\mathrm{S}[\mathrm{S}[\mathrm{Z}]]];)$

Peano Optimization

$\mathrm{Op}[+, \mathrm{Nat}[\mathrm{Z}], \mathrm{N}] \to \mathrm{N}$;

$\mathrm{Op}[\times, \mathrm{Nat}[\mathrm{S}[\mathrm{Z}]], \mathrm{N}] \to \mathrm{N}$;

$-[\mathrm{Discard}[\mathrm{N}]]$:
$\mathrm{Op}[\times, \mathrm{Nat}[\mathrm{Z}], \mathrm{N}] \to \mathrm{Nat}[\mathrm{Z}]$;

Sample

$\mathrm{T}[\mathrm{Op}[\times, \mathrm{Nat}[\mathrm{S}[\mathrm{Z}]], \mathrm{Op}[+, \mathrm{Nat}[\mathrm{Z}], \mathrm{Nat}[\mathrm{S}[\mathrm{S}[\mathrm{Z}]]]]], ()]$

$\to \mathrm{T}[\mathrm{Op}[+, \mathrm{Nat}[\mathrm{S}[\mathrm{S}[\mathrm{Z}]]], \mathrm{Nat}[\mathrm{Z}]], ()]$

$\to \mathrm{T}[\mathrm{Nat}[\mathrm{S}[\mathrm{S}[\mathrm{Z}]]], ()]$

$\to (\mathrm{PUSH}[\mathrm{S}[\mathrm{S}[\mathrm{Z}]]];)$

Peano Optimization

$\mathrm{Op}[+, \mathrm{Nat}[Z], \mathrm{N}] \to \mathrm{N}$;

$\mathrm{Op}[\times, \mathrm{Nat}[S[Z]], \mathrm{N}] \to \mathrm{N}$;

$-[\mathrm{Discard}[\mathrm{N}]]$:
$\mathrm{Op}[\times, \mathrm{Nat}[Z], \mathrm{N}] \to \mathrm{Nat}[Z]$;

Sample

$\mathrm{T}[\mathrm{Op}[\times, \mathrm{Nat}[S[Z]], \mathrm{Op}[+, \mathrm{Nat}[Z], \mathrm{Nat}[S[S[Z]]]]], ()]$

$\to \mathrm{T}[\mathrm{Op}[+, \mathrm{Nat}[S[S[Z]]], \mathrm{Nat}[Z]], \ ()]$

$\to \mathrm{T}[\mathrm{Nat}[S[S[Z]]], \ ()]$

$\to (\mathrm{PUSH}[S[S[Z]]];)$

Peano Critical Pairs

$$\begin{tikzcd} & \mathrm{T}[\mathrm{Op}[+, \mathrm{Nat}[\mathrm{Z}], \textsc{e}], \textsc{c}] \arrow[dl] \arrow[dr] & \\ \mathrm{T}[\mathrm{Nat}[\mathrm{Z}], \mathrm{T}[\textsc{e}, (\mathrm{OP}[+];)]]]) & & \mathrm{T}[\textsc{e}, \textsc{c}] \end{tikzcd}$$

Observation

One branch is a “data reduction.”

Peano Critical Pairs

$$
\begin{tikzcd}
 & \mathrm{T}[\mathrm{Op}[+, \mathrm{Nat}[\mathrm{Z}], \textsc{e}], \textsc{c}] \arrow[dl] \arrow[ddr] & \\
\mathrm{T}[\mathrm{Nat}[\mathrm{Z}], \mathrm{T}[\textsc{e}, (\mathrm{OP}[+];)]]]) & & \\
 & & \mathrm{T}[\textsc{e}, \textsc{c}]
\end{tikzcd}
$$

Observation

One branch is a “data reduction.”

Peano Completed

$\mathrm{T}[\mathrm{Op}[+, \mathrm{Nat}[\mathrm{Z}], \mathrm{E}_2],\ \mathrm{C}] \to \mathrm{T}[\mathrm{E}_2, \mathrm{C}]$;

$\mathrm{T}[\mathrm{Op}[+, \mathrm{Nat}[\mathrm{S}[\mathrm{N}_1]], \mathrm{E}_2],\ \mathrm{C}] \to \mathrm{T}[\mathrm{Nat}[\mathrm{S}[\mathrm{N}_1]], \mathrm{T}[\mathrm{E}_2, (\mathrm{OP}[+]; \mathrm{C})]]$;

$-[\mathrm{Discard}[\mathrm{E}_2]]$:
$\mathrm{T}[\mathrm{Op}[\times, \mathrm{Nat}[\mathrm{Z}], \mathrm{E}_2],\ \mathrm{C}] \to (\mathrm{PUSH}[\mathrm{Z}]; \mathrm{C})$;

$\mathrm{T}[\mathrm{Op}[\times, \mathrm{Nat}[\mathrm{S}[\mathrm{Z}]], \mathrm{E}_2],\ \mathrm{C}] \to \mathrm{T}[\mathrm{E}_2, \mathrm{C}]$;

$\mathrm{T}[\mathrm{Op}[\times, \mathrm{Nat}[\mathrm{S}[\mathrm{S}[\mathrm{N}_1]]], \mathrm{E}_2],\ \mathrm{C}] \to \mathrm{T}[\mathrm{Nat}[\mathrm{S}[\mathrm{S}[\mathrm{N}_1]]], \mathrm{T}[\mathrm{E}_2, (\mathrm{OP}[\times]; \mathrm{C})]]$;

Peano Completed

$\mathrm{T}[\mathrm{Op}[+, \mathrm{Nat}[\mathrm{Z}], \mathrm{E}_2], \ \mathrm{C}] \rightarrow \mathrm{T}[\mathrm{E}_2, \mathrm{C}] \ ;$

$\mathrm{T}[\mathrm{Op}[+, \mathrm{Nat}[\mathrm{S}[\mathrm{N}_1]], \mathrm{E}_2], \ \mathrm{C}] \rightarrow \mathrm{T}[\mathrm{Nat}[\mathrm{S}[\mathrm{N}_1]], \mathrm{T}[\mathrm{E}_2, (\mathrm{OP}[+]; \mathrm{C})]] \ ;$

$-[\mathrm{Discard}[\mathrm{E}_2]]$:
$\mathrm{T}[\mathrm{Op}[\times, \mathrm{Nat}[\mathrm{Z}], \mathrm{E}_2], \ \mathrm{C}] \rightarrow (\mathrm{PUSH}[\mathrm{Z}]; \mathrm{C}) \ ;$

$\mathrm{T}[\mathrm{Op}[\times, \mathrm{Nat}[\mathrm{S}[\mathrm{Z}]], \mathrm{E}_2], \ \mathrm{C}] \rightarrow \mathrm{T}[\mathrm{E}_2, \mathrm{C}] \ ;$

$\mathrm{T}[\mathrm{Op}[\times, \mathrm{Nat}[\mathrm{S}[\mathrm{S}[\mathrm{N}_1]]], \mathrm{E}_2], \ \mathrm{C}] \rightarrow \mathrm{T}[\mathrm{Nat}[\mathrm{S}[\mathrm{S}[\mathrm{N}_1]]], \mathrm{T}[\mathrm{E}_2, (\mathrm{OP}[\times]; \mathrm{C})]] \ ;$

Compilation Sorts

$$\begin{aligned}
\textsc{e} ::= {} & v \mid \mathrm{Int}[\mathbf{integer}] \mid \mathrm{Str}[\mathbf{String}] \mid \mathrm{Nil} \mid \mathrm{Seq}[\textsc{e}, \textsc{e}] \\
& \mid \mathrm{IfNil}[\textsc{e}, \textsc{e}, \textsc{e}] \mid \mathrm{Let}[\textsc{e},\ x{:}\ \textsc{e}.\textsc{e}] \ ;
\end{aligned}$$

$$\textsc{a} ::= \mathbf{list}[\textsc{ai}] \ ;$$

$$\begin{aligned}
\textsc{ai} ::= {} & \mathrm{L}[\textsc{al}] \mid \mathrm{J}[\textsc{al}] \mid \mathrm{PU} \mid \mathrm{POJN}[\textsc{al}] \mid \mathrm{POR}[\textsc{ar}] \\
& \mid \mathrm{SR}[\textsc{ar}] \mid \mathrm{SI}[\mathbf{integer}] \mid \mathrm{SS}[\mathbf{String}] \ ; \\
\textsc{al} ::= {} & \mathit{label} \ ; \\
\textsc{ar} ::= {} & \mathit{register} \ ;
\end{aligned}$$

Compilation Rules

Idea: $(\mathrm{PU};\ \{in{:}\,r_i;\ out{:}\,r_o\}\mathrm{T}[\mathrm{E},\ (\mathrm{POR}[r_o];)])$

$\{\rho\} = \{\mathrm{E}{:}\,\mathrm{AR}\}$;
$\{\rho\}\ \mathrm{T}[\mathrm{E},\mathrm{A}] :: \mathrm{A}$;

$\{v{:}\,r\}\ \mathrm{T}[v,\mathrm{A}] \to (\mathrm{SR}[r];\ \mathrm{A})$;
$\{\neg v\}\ \mathrm{T}[v,\mathrm{A}] \to (\mathrm{SS}[\texttt{"Error"}];\ \mathrm{A})$;

$\{\rho\}\ \mathrm{T}[\mathrm{Int}[\mathrm{I}],\mathrm{A}] \to (\mathrm{SI}[\mathrm{I}];\ \mathrm{A})$;
$\{\rho\}\ \mathrm{T}[\mathrm{Str}[\mathrm{S}],\mathrm{A}] \to (\mathrm{SS}[\mathrm{S}];\ \mathrm{A})$;
$\{\rho\}\ \mathrm{T}[\mathrm{Nil},\mathrm{A}] \to \mathrm{A}$;
$\{\rho\}\ \mathrm{T}[\mathrm{Seq}[\mathrm{E}_1,\mathrm{E}_2],\mathrm{A}] \to \{\rho\}\mathrm{T}[\mathrm{E}_1,\{\rho\}\mathrm{T}[\mathrm{E}_2,\mathrm{A}]]$;

$-[\mathrm{Fresh}[f,fi]]$:
$\{\rho\}\ \mathrm{T}[\mathrm{If}[\mathrm{E},\mathrm{E}_1,\mathrm{E}_2],\mathrm{A}]$
$\to (\mathrm{PU};\ \{\rho\}\mathrm{T}[\mathrm{E},\ (\mathrm{POJN}[f];\ \{\rho\}\mathrm{T}[\mathrm{E}_1,\ (\mathrm{J}[fi];\ \mathrm{L}[f];\ \{\rho\}\mathrm{T}[\mathrm{E}_2,\ (\mathrm{L}[fi];\ \mathrm{A})])])])$;

$-[\mathrm{Fresh}[x,r]]$:
$\{\rho\}\ \mathrm{T}[\mathrm{Let}[\mathrm{E}_1,v_1.\mathrm{E}_2[v_1]],\mathrm{A}] \to (\mathrm{PU};\ \{\rho\}\mathrm{T}[\mathrm{E}_1,\ (\mathrm{POR}[r];\ \{\rho; x{:}\,r\}\mathrm{T}[\mathrm{E}_2[x],\mathrm{A}])])$;

Compilation Rules

Idea: $(\mathrm{PU};\ \{in{:}\,r_{\mathrm{i}};\ out{:}\,r_{\mathrm{o}}\}\mathrm{T}[\mathrm{E},\ (\mathrm{POR}[r_{\mathrm{o}}];\,)])$

$\{\rho\} = \{\mathrm{E}{:}\ \mathrm{AR}\}$;
$\{\rho\}\ \mathrm{T}[\mathrm{E}, \mathrm{A}] :: \mathrm{A}$;

$\{v{:}\,r\}\ \mathrm{T}[v, \mathrm{A}] \to (\mathrm{SR}[r];\ \mathrm{A})$;
$\{\neg\, v\}\ \mathrm{T}[v, \mathrm{A}] \to (\mathrm{SS}[\texttt{"Error"}];\ \mathrm{A})$;

$\{\rho\}\ \mathrm{T}[\mathrm{Int}[\mathrm{I}], \mathrm{A}] \to (\mathrm{SI}[\mathrm{I}];\ \mathrm{A})$;
$\{\rho\}\ \mathrm{T}[\mathrm{Str}[\mathrm{S}], \mathrm{A}] \to (\mathrm{SS}[\mathrm{S}];\ \mathrm{A})$;
$\{\rho\}\ \mathrm{T}[\mathrm{Nil}, \mathrm{A}] \to \mathrm{A}$;
$\{\rho\}\ \mathrm{T}[\mathrm{Seq}[\mathrm{E}_1, \mathrm{E}_2], \mathrm{A}] \to \{\rho\}\mathrm{T}[\mathrm{E}_1, \{\rho\}\mathrm{T}[\mathrm{E}_2, \mathrm{A}]]$;

$-[\mathrm{Fresh}[f, \mathit{fi}]]$:
$\{\rho\}\ \mathrm{T}[\mathrm{If}[\mathrm{E}, \mathrm{E}_1, \mathrm{E}_2], \mathrm{A}]$
$\to (\mathrm{PU};\ \{\rho\}\mathrm{T}[\mathrm{E},\ (\mathrm{POJN}[f];\ \{\rho\}\mathrm{T}[\mathrm{E}_1,\ (\mathrm{J}[\mathit{fi}];\ \mathrm{L}[f];\ \{\rho\}\mathrm{T}[\mathrm{E}_2,\ (\mathrm{L}[\mathit{fi}];\ \mathrm{A})])])])$;

$-[\mathrm{Fresh}[x, r]]$:
$\{\rho\}\ \mathrm{T}[\mathrm{Let}[\mathrm{E}_1, v_1.\mathrm{E}_2[v_1]], \mathrm{A}] \to (\mathrm{PU};\ \{\rho\}\mathrm{T}[\mathrm{E}_1,\ (\mathrm{POR}[r];\ \{\rho; x{:}\,r\}\mathrm{T}[\mathrm{E}_2[x], \mathrm{A}])])$;

Compilation Optimization

$\mathrm{If}[\mathrm{Nil}, \mathrm{E}_1, \mathrm{E}_2] \to \mathrm{E}_2$;
$\mathrm{Let}[\mathrm{E}, x.x] \to \mathrm{E}$;
$\mathrm{Let}[\mathrm{E}_1, x_1.\mathrm{E}_2[]] \to \mathrm{E}_2$;

$(\mathrm{PU};\ \{in{:}\,r_\mathrm{i};\ out{:}\,r_\mathrm{o}\}\mathrm{T}[\mathrm{Let}[\mathrm{If}[\mathrm{Nil},\ \mathrm{Nil},\ in],\ x.x],\ (\mathrm{POR}[r_\mathrm{o}];)])$
$\to (\mathrm{PU};\ \{in{:}\,r_\mathrm{i};\ out{:}\,r_\mathrm{o}\}\mathrm{T}[\mathrm{If}[\mathrm{Nil},\ \mathrm{Nil},\ in],\ (\mathrm{POR}[r_\mathrm{o}];)])$
$\to (\mathrm{PU};\ \{in{:}\,r_\mathrm{i};\ out{:}\,r_\mathrm{o}\}\mathrm{T}[in,\ (\mathrm{POR}[r_\mathrm{o}];)])$
$\to (\mathrm{PU};\ \mathrm{SR}[r_\mathrm{in}];\ \mathrm{POR}[r_\mathrm{o}];)$

Compilation Optimization

$\text{If}[\text{Nil}, \mathrm{E}_1, \mathrm{E}_2] \to \mathrm{E}_2$;
$\text{Let}[\mathrm{E}, x.x] \to \mathrm{E}$;
$\text{Let}[\mathrm{E}_1, x_1.\mathrm{E}_2[]] \to \mathrm{E}_2$;

$(\text{PU};\ \{in{:}\,r_{\text{i}};\ out{:}\,r_{\text{o}}\}\text{T}[\text{Let}[\text{If}[\text{Nil},\ \text{Nil},\ in],\ x.x],\ (\text{POR}[r_{\text{o}}];)])$
$\to (\text{PU};\ \{in{:}\,r_{\text{i}};\ out{:}\,r_{\text{o}}\}\text{T}[\text{If}[\text{Nil},\ \text{Nil},\ in],\ (\text{POR}[r_{\text{o}}];)])$
$\to (\text{PU};\ \{in{:}\,r_{\text{i}};\ out{:}\,r_{\text{o}}\}\text{T}[in,\ (\text{POR}[r_{\text{o}}];)])$
$\to (\text{PU};\ \text{SR}[r_{\text{in}}];\ \text{POR}[r_{\text{o}}];)$

Compilation Optimization

$\{\mathrm{S}\}\mathrm{T}[\mathrm{If}[\mathrm{Nil}, \mathrm{E}_1, \mathrm{E}_2], \mathrm{A}] \rightarrow \{\mathrm{S}\}\mathrm{T}[\mathrm{E}_2, \mathrm{A}]$

$-[\mathrm{Fresh}[f, fi]]:$
$\{\mathrm{S}\}\mathrm{T}[\mathrm{If}[\mathrm{E} \not\equiv \mathrm{Nil}, \mathrm{E}_1, \mathrm{E}_2], \mathrm{A}]$
$\rightarrow (\mathrm{PU};\ \{\mathrm{S}\}\mathrm{T}[\mathrm{E},\ (\mathrm{POJN}[f];\ \{\mathrm{S}\}\mathrm{T}[\mathrm{E}_1,\ (\mathrm{J}[fi];\ \mathrm{L}[f];\ \{\mathrm{S}\}\mathrm{T}[\mathrm{E}_2,\ (\mathrm{L}[fi];\ \mathrm{A})])]$

$\{\mathrm{S}\}\mathrm{T}[\mathrm{Let}[\mathrm{E}, x.x], \mathrm{A}] \rightarrow \{\mathrm{S}\}\mathrm{T}[\mathrm{E}, \mathrm{A}]$;

$\{\mathrm{S}\}\mathrm{T}[\mathrm{Let}[\mathrm{E}_1, x_1.\mathrm{E}_2[]], \mathrm{A}] \rightarrow \mathrm{E}_2, \mathrm{A}]$;

$-[\mathrm{Fresh}[x, r]]:$
$\{\mathrm{S}\}\mathrm{T}[\mathrm{Let}[\mathrm{E}_1, v_1.\mathrm{E}_2[\,!v_1] \not\equiv v_1], \mathrm{A}] \rightarrow (\mathrm{PU};\ \{\mathrm{S}\}\mathrm{T}[\mathrm{E}_1,\ (\mathrm{POR}[r];\ \{\mathrm{S}\,!x{:}\,r\}\mathrm{T}[\mathrm{E}_2$

Compilation Optimization

$\{\mathrm{S}\}\mathrm{T}[\mathrm{If}[\mathrm{Nil}, \mathrm{E}_1, \mathrm{E}_2], \mathrm{A}] \rightarrow \{\mathrm{S}\}\mathrm{T}[\mathrm{E}_2, \mathrm{A}]$

$-[\mathrm{Fresh}[f, fi]]$:
$\{\mathrm{S}\}\mathrm{T}[\mathrm{If}[\mathrm{E} \not\equiv \mathrm{Nil}, \mathrm{E}_1, \mathrm{E}_2], \mathrm{A}]$
$\rightarrow (\mathrm{PU};\ \{\mathrm{S}\}\mathrm{T}[\mathrm{E},\ (\mathrm{POJN}[f];\ \{\mathrm{S}\}\mathrm{T}[\mathrm{E}_1,\ (\mathrm{J}[fi];\ \mathrm{L}[f];\ \{\mathrm{S}\}\mathrm{T}[\mathrm{E}_2,\ (\mathrm{L}[fi];\ \mathrm{A})])]$

$\{\mathrm{S}\}\mathrm{T}[\mathrm{Let}[\mathrm{E}, x.x], \mathrm{A}] \rightarrow \{\mathrm{S}\}\mathrm{T}[\mathrm{E}, \mathrm{A}]$;

$\{\mathrm{S}\}\mathrm{T}[\mathrm{Let}[\mathrm{E}_1, x_1.\mathrm{E}_2[]], \mathrm{A}] \rightarrow \mathrm{E}_2, \mathrm{A}]$;

$-[\mathrm{Fresh}[x, r]]$:
$\{\mathrm{S}\}\mathrm{T}[\mathrm{Let}[\mathrm{E}_1, v_1.\mathrm{E}_2[\,!v_1] \not\equiv v_1], \mathrm{A}] \rightarrow (\mathrm{PU};\ \{\mathrm{S}\}\mathrm{T}[\mathrm{E}_1,\ (\mathrm{POR}[r];\ \{\mathrm{S}\,!x{:}\,r\}\mathrm{T}[\mathrm{E}_2$

Plan

1. Optimization by Completion
   - Peano
   - Compilation

2. Conclusion

Conclusion?

- Compilers are complete modulo the target language semantic equations.
- Optimizations are intermediate language (data) rewrites.
- Completion with priority to data rewrites integrates translation schemes and optimization rules.
- Peano integers never get boring!

Conclusion?

- Compilers are complete modulo the target language semantic equations.
- Optimizations are intermediate language (data) rewrites.
- Completion with priority to data rewrites integrates translation schemes and optimization rules.
- Peano integers never get boring!

Conclusion?

- Compilers are complete modulo the target language semantic equations.
- Optimizations are intermediate language (data) rewrites.
- Completion with priority to data rewrites integrates translation schemes and optimization rules.
- Peano integers never get boring!

Conclusion?

- Compilers are complete modulo the target language semantic equations.
- Optimizations are intermediate language (data) rewrites.
- Completion with priority to data rewrites integrates translation schemes and optimization rules.
- Peano integers never get boring!

Thank You!