Satisfiability Encodings of DP Techniques for Maximal Completion

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41st TRS meeting @ Jozankei

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Bibliography

H. Sato and S. Winkler
A Satisfiability Encoding of Dependency Pair Techniques for Maximal Completion
WST 2014

Outline

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Knuth-Bendix Completion: Aim

Given input equalities $\mathcal{E}$, construct complete TRS $\mathcal{R}$ with $\leftrightarrow_{\mathcal{E}}^{*} = \leftrightarrow_{\mathcal{R}}^{*}$

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Group Theory

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$$\begin{array}{llll}
\mathcal{R}: & x \cdot 1 \to x & 1 \cdot x \to x & x \cdot (x^- \cdot y) \to y \\
& x \cdot x^- \to 1 & x^- \cdot x \to 1 & (x \cdot y) \cdot z \to x \cdot (y \cdot z) \\
& 1^- \to 1 & x^{--} \to x & x^- \cdot (x \cdot y) \to y \\
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\end{array}$$

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& (x \cdot y)^- \to y^- \cdot x^- & \mathrm{Y} \to \mathrm{Z} \quad &
\end{aligned}$$

PROPOSITION 1.31: (Cancellation laws) Let $(G, *)$ be a group, and let x, y, $z \in G$ such that either $x * y = x * z$ or $y * x = z * x$. Then $y = z$.

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\end{array}$$

WARNING 1.17: Let x and y be elements of a multiplicatively-written group G. Then $(xy)^2$ is not always the same as x^2y^2.

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for input ES $\mathcal{E}$ and ES $\mathcal{C}$,

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where $\mathfrak{R}(\mathcal{C})$ consists of terminating TRSs $\mathcal{R}$ such that $\mathcal{R} \subseteq \mathcal{C} \cup \mathcal{C}^{-1}$

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Theorem (KH11)

If $\varphi(\mathcal{E}) = \mathcal{R}$ then $\mathcal{R}$ is complete and $\leftrightarrow^*_{\mathcal{E}} = \leftrightarrow^*_{\mathcal{R}}$

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Maxcomp

finds $\mathfrak{R}(\mathcal{C})$ using MAXSAT call on

$$\bigvee_{s \approx t \in \mathcal{C}} [s > t] \vee [t > s]$$

encoding of reduction order $>$ (e.g. LPO or KBO)

Maxcomp: Drawback

Experiments

115 systems from Maxcomp distribution

method	#	avg. time
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Commuting group endomorphisms (CGE_2)

$$\mathcal{E}: \quad \begin{aligned} 1 \cdot x &\approx x & \mathsf{f}(x \cdot y) &\approx \mathsf{f}(x) \cdot \mathsf{f}(y) \\ x^{-} \cdot x &\approx 1 & \mathsf{g}(x \cdot y) &\approx \mathsf{g}(x) \cdot \mathsf{g}(y) \\ x \cdot (y \cdot z) &\approx (x \cdot y) \cdot z & \mathsf{f}(x) \cdot \mathsf{g}(y) &\approx \mathsf{g}(y) \cdot \mathsf{f}(x) \end{aligned}$$

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x^{-} \cdot x \to 1 & \mathsf{f}(x)^{-} \to \mathsf{f}(x^{-}) & (x \cdot y^{-}) \cdot y \to x \\
x \cdot x^{-} \to 1 & \mathsf{g}(x) \cdot \mathsf{g}(y) \to \mathsf{g}(x \cdot y) & \mathsf{f}(x) \cdot (\mathsf{f}(y) \cdot z) \to \mathsf{f}(x \cdot y) \cdot z \\
1^{-} \to 1 & (x \cdot y)^{-} \to y^{-} \cdot x^{-} & \mathsf{g}(x) \cdot (\mathsf{g}(y) \cdot z) \to \mathsf{g}(x \cdot y) \cdot z \\
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Aim: SAT encoding of termination techniques with enough power

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symmetric: $\mathcal{C} = \mathcal{C} \cup \mathcal{C}^{-1}$

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X^{def}_f for $f \in \mathcal{F}$

$\alpha(X^{\text{def}}_f) = \top$ if f is defined symbol

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rules trigger DPs

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$$\bigwedge_{\ell \to r \in \text{DPC}(\mathcal{C})} (S^{1}_{\ell \to r} \to [\ell \geqslant^{\text{pol}} r]) \wedge (\neg[\ell >^{\text{pol}} r] \to S^{2}_{\ell \to r})$$

reduction pair processors orient DPs weakly, and discard only strictly oriented ones

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$$\bigwedge_{i=2}^{3} \bigwedge_{\ell \to r \in \mathrm{DPC}(\mathcal{C})} (S^{i-1}_{\ell \to r} \to [\ell \geqslant^i r]) \wedge (\neg[\ell >^i r] \to S^i_{\ell \to r})$$

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$$\bigwedge_{\ell \to r \in \mathrm{DPC}(\mathcal{C})} \neg S^{3}_{\ell \to r}$$

finally no DP remains

Definitions

- dependency pair candidates
  $\mathsf{DPC}(\mathcal{C}) = \{\ell^{\#} \to u^{\#} \mid \ell \approx r \in \mathcal{C} \text{ is rewrite rule}, r \trianglerighteq u \text{ but } \ell \ntriangleright u\}$

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$(f(t_1, \ldots, t_n))^\# = f^\#(t_1, \ldots, t_n)$

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$$\mathcal{S} = \{S_{\ell\to r} \mid \ell \to r \in \mathcal{P}\} \qquad \mathcal{W} = \{W_{\ell\to r} \mid \ell \to r \in \mathcal{R}\}$$

  for pair of TRSs $(\mathcal{P}, \mathcal{R})$, and formula ϕ

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- for assignment α let TRS $\alpha(\mathcal{S}) = \{\ell \to r \mid S_{\ell\to r} \in \mathcal{S},\ \alpha(S_{\ell\to r}) = \top\}$

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  and DP problem $(\alpha(\mathcal{S}), \alpha(\mathcal{W}))$ is finite

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- DP processor encoding Proc maps D to set of DP problem
  encodings $\mathrm{Proc}(\mathsf{D}) = \{\mathsf{D}_1, \dots, \mathsf{D}_n\}$

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- for assignment α let TRS $\alpha(\mathcal{S}) = \{\ell \to r \mid S_{\ell\to r} \in \mathcal{S},\ \alpha(S_{\ell\to r}) = \top\}$
- assignment α is finite for $\mathsf{D} = (\mathcal{S}, \mathcal{W}, \phi)$ if $\alpha(\phi) = \top$ and DP problem $(\alpha(\mathcal{S}), \alpha(\mathcal{W}))$ is finite
- DP processor encoding Proc maps D to set of DP problem encodings $\mathrm{Proc}(\mathsf{D}) = \{\mathsf{D}_1, \ldots, \mathsf{D}_n\}$
- DP processor encoding Proc is sound if

  $$\left.\begin{array}{l}\mathrm{Proc}(\mathsf{D}) = \{\mathsf{D}_1, \ldots, \mathsf{D}_n\} \\ \alpha \text{ finite for all } D_i\end{array}\right\} \implies \alpha \text{ finite for } D$$

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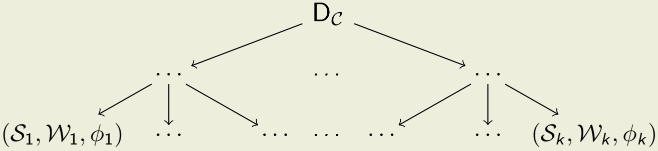
Lemma

Let $\mathcal{C}$ be an ES. Suppose there is a tree



where for every non-leaf node D *with children* $\mathsf{D}_1, \ldots, \mathsf{D}_n$ *there is sound processor encoding* Proc *such that* $\mathsf{Proc}(\mathsf{D}) = \{\mathsf{D}_1, \ldots, \mathsf{D}_n\}$.

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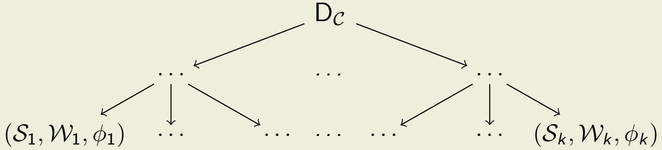

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If α satisfies

$$\phi = \bigwedge_{i=1}^{k} \phi_i \wedge \bigwedge_{S_{s \to t} \in \mathcal{S}_i} \neg S_{s \to t}$$

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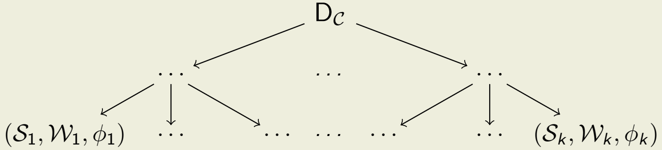

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Proof Idea

α is finite for all nodes, in particular the root. Termination of $\mathcal{R}$ follows from correctness of the DP framework [GTSK05].

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Let $(>, \geqslant)$ be reduction pair with SAT encodings $[\cdot > \cdot]$ and $[\cdot \geqslant \cdot]$.

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Definition (Reduction pair processor with rule removal)

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Then $\mathsf{Proc}_{(>,\geqslant)}(\mathcal{S}, \mathcal{W}, \phi) = \{(\mathcal{S}', \mathcal{W}', \phi \wedge T_S \wedge T_W)\}$

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$$T_S = \bigwedge_{S_{\ell\to r} \in \mathcal{S}} S_{\ell\to r} \to [\ell \geqslant r] \wedge (\neg[\ell > r] \to S'_{\ell\to r})$$

$$T_W = \bigwedge_{W_{\ell\to r} \in \mathcal{W}} W_{\ell\to r} \to [\ell \geqslant r] \wedge (\neg[\ell > r] \to W'_{\ell\to r})$$

Definition (Reduction pair processor with rule removal)

Let $(>, \geqslant)$ be reduction pair with SAT encodings $[\cdot > \cdot]$ and $[\cdot \geqslant \cdot]$.

Then $\text{Proc}_{(>,\geqslant)}(\mathcal{S}, \mathcal{W}, \phi) = \{(\mathcal{S}', \mathcal{W}', \phi \wedge T_S \wedge T_W)\}$

$$\mathcal{S}' = \{S'_{\ell\to r} \mid S_{\ell\to r} \in \mathcal{S}\} \qquad \mathcal{W}' = \{W'_{\ell\to r} \mid W_{\ell\to r} \in \mathcal{W}\}$$

$$T_S = \bigwedge_{S_{\ell\to r}\in\mathcal{S}} S_{\ell\to r} \to [\ell \geqslant r] \wedge (\neg[\ell > r] \to S'_{\ell\to r})$$

$$T_W = \bigwedge_{W_{\ell\to r}\in\mathcal{W}} W_{\ell\to r} \to [\ell \geqslant r] \wedge (\neg[\ell > r] \to W'_{\ell\to r})$$

Luckily ...

encodings $[\cdot \geqslant \cdot]$ for RPO, KBO, WPO (+ argument filterings, usable rules), polynomial, matrix interpretations, ... are well studied

Example (CGE_2 snippet)

$\mathcal{C}$: 1: $\mathsf{f}(x^-) \approx \mathsf{f}(x)^-$ DPC($\mathcal{C}$): 3: $\mathsf{f}^\#(x^-) \to \mathsf{f}(x)^{-\#}$ 4: $\mathsf{f}^\#(x^-) \to \mathsf{f}^\#(x)$

2: $\mathsf{f}(x)^- \approx \mathsf{f}(x^-)$ 5: $\mathsf{f}(x)^{-\#} \to \mathsf{f}^\#(x^-)$ 6: $\mathsf{f}(x)^{-\#} \to x^{-\#}$

$\mathrm{D}_{\mathcal{C}} = (\mathcal{S}, \mathcal{W}, \phi)$ for $\mathcal{S} = \{S_3, S_4, S_5, S_6\}$ and $\mathcal{W} = \{W_1, W_2\}$.

Example (CGE_2 snippet)

$\mathcal{C}$: 1: $\mathsf{f}(x^-) \approx \mathsf{f}(x)^-$ 2: $\mathsf{f}(x)^- \approx \mathsf{f}(x^-)$

DPC($\mathcal{C}$): 3: $\mathsf{f}^\#(x^-) \to \mathsf{f}(x)^{-\#}$ 4: $\mathsf{f}^\#(x^-) \to \mathsf{f}^\#(x)$ 5: $\mathsf{f}(x)^{-\#} \to \mathsf{f}^\#(x^-)$ 6: $\mathsf{f}(x)^{-\#} \to x^{-\#}$

$\mathsf{D}_\mathcal{C} = (\mathcal{S}, \mathcal{W}, \phi)$ for $\mathcal{S} = \{S_3, S_4, S_5, S_6\}$ and $\mathcal{W} = \{W_1, W_2\}$.

Let $(>, \geqslant)$ be reduction pair associated to some LPO.
Then $\mathsf{Proc}_{(>,\geqslant)}$ maps $\mathsf{D}_\mathcal{C}$ to $(\mathcal{S}', \mathcal{W}', \phi \wedge T_S \wedge T_W)$, where

$$T_S = (S_3 \to [\mathsf{f}^\#(x^-) \geqslant \mathsf{f}(x)^{-\#}]) \wedge (\neg[\mathsf{f}^\#(x^-) > \mathsf{f}(x)^{-\#}] \to S_3')$$

Example (CGE$_2$ snippet)

$\mathcal{C}$: 1: $\mathsf{f}(x^-) \approx \mathsf{f}(x)^-$ DPC($\mathcal{C}$): 3: $\mathsf{f}^\#(x^-) \to \mathsf{f}(x)^{-\#}$ 4: $\mathsf{f}^\#(x^-) \to \mathsf{f}^\#(x)$

2: $\mathsf{f}(x)^- \approx \mathsf{f}(x^-)$ 5: $\mathsf{f}(x)^{-\#} \to \mathsf{f}^\#(x^-)$ 6: $\mathsf{f}(x)^{-\#} \to x^{-\#}$

$\mathsf{D}_\mathcal{C} = (\mathcal{S}, \mathcal{W}, \phi)$ for $\mathcal{S} = \{S_3, S_4, S_5, S_6\}$ and $\mathcal{W} = \{W_1, W_2\}$.

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$$(S_4 \to [\mathsf{f}^\#(x^-) \geqslant \mathsf{f}^\#(x)]) \wedge (\neg[\mathsf{f}^\#(x^-) > \mathsf{f}^\#(x)] \to S_4')$$

Example (CGE_2 snippet)

$\mathcal{C}$: 1: $\mathsf{f}(x^-) \approx \mathsf{f}(x)^-$ DPC($\mathcal{C}$): 3: $\mathsf{f}^\#(x^-) \to \mathsf{f}(x)^{-\#}$ 4: $\mathsf{f}^\#(x^-) \to \mathsf{f}^\#(x)$

2: $\mathsf{f}(x)^- \approx \mathsf{f}(x^-)$ 5: $\mathsf{f}(x)^{-\#} \to \mathsf{f}^\#(x^-)$ 6: $\mathsf{f}(x)^{-\#} \to x^{-\#}$

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$$(S_4 \to [\mathsf{f}^\#(x^-) \geqslant \mathsf{f}^\#(x)]) \wedge (\neg[\mathsf{f}^\#(x^-) > \mathsf{f}^\#(x)] \to S_4') \wedge \ldots$$

Example (CGE_2 snippet)

$$\mathcal{C}:\quad 1:\ \mathsf{f}(x^-) \approx \mathsf{f}(x)^- \qquad \mathrm{DPC}(\mathcal{C}):\quad 3:\ \mathsf{f}^\#(x^-) \to \mathsf{f}(x)^{-\#} \quad 4:\ \mathsf{f}^\#(x^-) \to \mathsf{f}^\#(x)$$

$$2:\ \mathsf{f}(x)^- \approx \mathsf{f}(x^-) \qquad 5:\ \mathsf{f}(x)^{-\#} \to \mathsf{f}^\#(x^-) \quad 6:\ \mathsf{f}(x)^{-\#} \to x^{-\#}$$

$\mathrm{D}_\mathcal{C} = (\mathcal{S}, \mathcal{W}, \phi)$ for $\mathcal{S} = \{S_3, S_4, S_5, S_6\}$ and $\mathcal{W} = \{W_1, W_2\}$.

Let $(>, \geqslant)$ be reduction pair associated to some LPO.
Then $\mathrm{Proc}_{(>,\geqslant)}$ maps $\mathrm{D}_\mathcal{C}$ to $(\mathcal{S}', \mathcal{W}', \phi \wedge T_S \wedge T_W)$, where

$$\begin{aligned} T_S = {} & (S_3 \to [\mathsf{f}^\#(x^-) \geqslant \mathsf{f}(x)^{-\#}]) \wedge (\neg[\mathsf{f}^\#(x^-) > \mathsf{f}(x)^{-\#}] \to S_3') \wedge \\ & (S_4 \to [\mathsf{f}^\#(x^-) \geqslant \mathsf{f}^\#(x)]) \wedge (\neg[\mathsf{f}^\#(x^-) > \mathsf{f}^\#(x)] \to S_4') \wedge \ldots \\ T_W = {} & (W_1 \to [\mathsf{f}(x^-) \geqslant \mathsf{f}(x)^-] \wedge W_1') \wedge \\ & (W_2 \to [\mathsf{f}(x)^- \geqslant \mathsf{f}(x^-)] \wedge W_2') \end{aligned}$$

DG with Cycle Analysis

Idea

assign number X_p^{w} to DP p such that cycle $p_1 \to p_2 \to \cdots \to p_n \to p_1$ issues unsatisfiable constraint $X_{p_1}^{\mathsf{w}} > X_{p_2}^{\mathsf{w}} > \cdots > X_{p_n}^{\mathsf{w}} > X_{p_1}^{\mathsf{w}}$

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Definition

$\mathrm{Proc}_{DG}(\mathcal{S}, \mathcal{W}, \phi) = \{(\mathcal{S}', \mathcal{W}', \phi \qquad\qquad)\}$

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Definition

$\mathrm{Proc}_{DG}(\mathcal{S}, \mathcal{W}, \phi) = \{(\mathcal{S}', \mathcal{W}', \phi \qquad\qquad)\}$

$$\mathcal{S}' = \{S'_{\ell\to r} \mid S_{\ell\to r} \in \mathcal{S}\}$$
$$\mathcal{W}' = \{W'_{\ell\to r} \mid S_{\ell\to r} \in \mathcal{S}\} \cup \{W'_{\ell\to r} \mid W_{\ell\to r} \in \mathcal{W}\}$$

DG with Cycle Analysis

Idea

assign number X^{w}_p to DP p such that cycle $p_1 \to p_2 \to \cdots \to p_n \to p_1$ issues unsatisfiable constraint $X^{\mathrm{w}}_{p_1} > X^{\mathrm{w}}_{p_2} > \cdots > X^{\mathrm{w}}_{p_n} > X^{\mathrm{w}}_{p_1}$

Definition

$$\mathrm{Proc}_{DG}(\mathcal{S}, \mathcal{W}, \phi) = \{(\mathcal{S}', \mathcal{W}', \phi \wedge T_S \qquad)\}$$

$$\mathcal{S}' = \{S'_{\ell\to r} \mid S_{\ell\to r} \in \mathcal{S}\}$$

$$\mathcal{W}' = \{W'_{\ell\to r} \mid S_{\ell\to r} \in \mathcal{S}\} \cup \{W'_{\ell\to r} \mid W_{\ell\to r} \in \mathcal{W}\}$$

$$T_S = \bigwedge_{S_{p_1}, S_{p_2} \in \mathcal{S}} S_{p_1} \wedge S_{p_2} \wedge X^{\mathrm{edge}}_{p_1,p_2} \wedge \neg S'_{p_1} \wedge \neg S'_{p_2} \to X^{\mathrm{w}}_{p_1} > X^{\mathrm{w}}_{p_2}$$

where $X^{\mathrm{edge}}_{s\to t, u\to v} = \mathrm{root}(t) = \mathrm{root}(u)\ ?\ \top : \bot$

DG with Cycle Analysis

Idea

assign number X^{w}_p to DP p such that cycle $p_1 \to p_2 \to \cdots \to p_n \to p_1$ issues unsatisfiable constraint $X^{\mathsf{w}}_{p_1} > X^{\mathsf{w}}_{p_2} > \cdots > X^{\mathsf{w}}_{p_n} > X^{\mathsf{w}}_{p_1}$

Definition

$\mathsf{Proc}_{DG}(\mathcal{S}, \mathcal{W}, \phi) = \{(\mathcal{S}', \mathcal{W}', \phi \wedge T_S \wedge T_W)\}$

$$
\begin{aligned}
\mathcal{S}' &= \{S'_{\ell\to r} \mid S_{\ell\to r} \in \mathcal{S}\} \\
\mathcal{W}' &= \{W'_{\ell\to r} \mid S_{\ell\to r} \in \mathcal{S}\} \cup \{W'_{\ell\to r} \mid W_{\ell\to r} \in \mathcal{W}\} \\
T_S &= \bigwedge_{S_{p_1}, S_{p_2} \in \mathcal{S}} S_{p_1} \wedge S_{p_2} \wedge X^{\mathsf{edge}}_{p_1,p_2} \wedge \neg S'_{p_1} \wedge \neg S'_{p_2} \to X^{\mathsf{w}}_{p_1} > X^{\mathsf{w}}_{p_2} \\
T_W &= \bigwedge_{S_{\ell\to r} \in \mathcal{S}} S_{\ell\to r} \to W'_{\ell\to r} \wedge \bigwedge_{W_{\ell\to r} \in \mathcal{W}} W_{\ell\to r} \to W'_{\ell\to r}
\end{aligned}
$$

where $X^{\mathsf{edge}}_{s\to t, u\to v} = \mathsf{root}(t) = \mathsf{root}(u)\ ?\ \top : \bot$

Example (CGE_2 snippet)

$\mathcal{C}$: $1: \mathsf{f}(x^-) \approx \mathsf{f}(x)^-$ $\quad$ $\mathrm{DPC}(\mathcal{C})$: $3: \mathsf{f}^\#(x^-) \to \mathsf{f}(x)^{-\#}$ $\quad$ $4: \mathsf{f}^\#(x^-) \to \mathsf{f}^\#(x)$

$2: \mathsf{f}(x)^- \approx \mathsf{f}(x^-)$ $\quad$ $5: \mathsf{f}(x)^{-\#} \to \mathsf{f}^\#(x^-)$ $\quad$ $6: \mathsf{f}(x)^{-\#} \to x^{-\#}$

$\mathrm{D}_{\mathcal{C}} = (\mathcal{S}, \mathcal{W}, \phi)$ for $\mathcal{S} = \{S_3, S_4, S_5, S_6\}$, $\mathcal{W} = \{W_1, W_2\}$

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Proc_{DG} maps $\mathsf{D}_{\mathcal{C}}$ to $\mathsf{D} = (\mathcal{S}', \mathcal{W}', \phi \wedge T_S \wedge T_W)$, where
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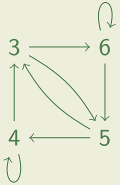

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$\mathcal{C}$: $1:\ \mathsf{f}(x^-) \approx \mathsf{f}(x)^-$ DPC($\mathcal{C}$): $3:\ \mathsf{f}^\#(x^-) \to \mathsf{f}(x)^{-\#}$ $4:\ \mathsf{f}^\#(x^-) \to \mathsf{f}^\#(x)$

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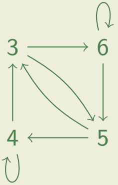

$\mathrm{D}_{\mathcal{C}} = (\mathcal{S}, \mathcal{W}, \phi)$ for $\mathcal{S} = \{S_3, S_4, S_5, S_6\}$, $\mathcal{W} = \{W_1, W_2\}$

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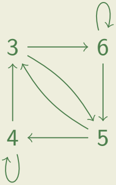
$$T_S = (S_4 \wedge S_3 \wedge \neg S_4' \wedge \neg S_3' \to X_4^{\mathsf{w}} > X_3^{\mathsf{w}})$$

Example (CGE_2 snippet)

$\mathcal{C}$: $1:\ \mathsf{f}(x^-) \approx \mathsf{f}(x)^-$ DPC($\mathcal{C}$): $3:\ \mathsf{f}^\#(x^-) \to \mathsf{f}(x)^{-\#}$ $4:\ \mathsf{f}^\#(x^-) \to \mathsf{f}^\#(x)$

$2:\ \mathsf{f}(x)^- \approx \mathsf{f}(x^-)$ $5:\ \mathsf{f}(x)^{-\#} \to \mathsf{f}^\#(x^-)$ $6:\ \mathsf{f}(x)^{-\#} \to x^{-\#}$



$\mathrm{D}_\mathcal{C} = (\mathcal{S}, \mathcal{W}, \phi)$ for $\mathcal{S} = \{S_3, S_4, S_5, S_6\}$, $\mathcal{W} = \{W_1, W_2\}$

Proc_{DG} maps $\mathrm{D}_\mathcal{C}$ to $\mathrm{D} = (\mathcal{S}', \mathcal{W}', \phi \wedge T_S \wedge T_W)$, where $\mathcal{S}' = \{S_3, \ldots, S_6\}$, $\mathcal{W}' = \{W_1, \ldots, W_6\}$, and

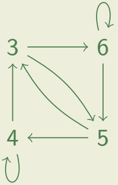
$$T_S = (S_4 \wedge S_3 \wedge \neg S_4' \wedge \neg S_3' \to X_4^{\mathrm{w}} > X_3^{\mathrm{w}}) \wedge (S_3 \wedge S_5 \wedge \neg S_3' \wedge \neg S_5' \to X_3^{\mathrm{w}} > X_5^{\mathrm{w}})$$

Example (CGE_2 snippet)

$\mathcal{C}$: $1: \mathsf{f}(x^-) \approx \mathsf{f}(x)^-$ $\quad 2: \mathsf{f}(x)^- \approx \mathsf{f}(x^-)$

DPC($\mathcal{C}$): $3: \mathsf{f}^\#(x^-) \to \mathsf{f}(x)^{-\#}$ $\quad 4: \mathsf{f}^\#(x^-) \to \mathsf{f}^\#(x)$ $\quad 5: \mathsf{f}(x)^{-\#} \to \mathsf{f}^\#(x^-)$ $\quad 6: \mathsf{f}(x)^{-\#} \to x^{-\#}$

$\mathsf{D}_\mathcal{C} = (\mathcal{S}, \mathcal{W}, \phi)$ for $\mathcal{S} = \{S_3, S_4, S_5, S_6\}$, $\mathcal{W} = \{W_1, W_2\}$



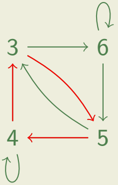
Proc_{DG} maps $\mathsf{D}_\mathcal{C}$ to $\mathsf{D} = (\mathcal{S}', \mathcal{W}', \phi \wedge T_S \wedge T_W)$, where $\mathcal{S}' = \{S_3, \ldots, S_6\}$, $\mathcal{W}' = \{W_1, \ldots, W_6\}$, and

$$\begin{aligned} T_S = {} & (S_4 \wedge S_3 \wedge \neg S_4' \wedge \neg S_3' \to X_4^{\mathsf{w}} > X_3^{\mathsf{w}}) \wedge \\ & (S_3 \wedge S_5 \wedge \neg S_3' \wedge \neg S_5' \to X_3^{\mathsf{w}} > X_5^{\mathsf{w}}) \wedge \\ & (S_5 \wedge S_4 \wedge \neg S_5' \wedge \neg S_4' \to X_5^{\mathsf{w}} > X_4^{\mathsf{w}}) \wedge \ldots \end{aligned}$$

Example (CGE_2 snippet)

$\mathcal{C}:\quad 1:\ \mathsf{f}(x^-) \approx \mathsf{f}(x)^- \qquad \mathsf{DPC}(\mathcal{C}):\quad 3:\ \mathsf{f}^\#(x^-) \to \mathsf{f}(x)^{-\#} \quad 4:\ \mathsf{f}^\#(x^-) \to \mathsf{f}^\#(x)$

$\qquad 2:\ \mathsf{f}(x)^- \approx \mathsf{f}(x^-) \qquad\qquad\qquad\qquad 5:\ \mathsf{f}(x)^{-\#} \to \mathsf{f}^\#(x^-) \quad 6:\ \mathsf{f}(x)^{-\#} \to x^{-\#}$



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Example (CGE_2 snippet)

$$\mathcal{C}: \quad 1: \mathsf{f}(x^-) \approx \mathsf{f}(x)^- \qquad \mathrm{DPC}(\mathcal{C}): \quad 3: \mathsf{f}^\#(x^-) \to \mathsf{f}(x)^{-\#} \quad 4: \mathsf{f}^\#(x^-) \to \mathsf{f}^\#(x)$$

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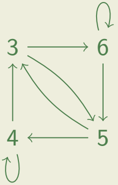
$D_\mathcal{C} = (\mathcal{S}, \mathcal{W}, \phi)$ for $\mathcal{S} = \{S_3, S_4, S_5, S_6\}$, $\mathcal{W} = \{W_1, W_2\}$



Proc_{DG} maps $D_\mathcal{C}$ to $D = (\mathcal{S}', \mathcal{W}', \phi \wedge T_S \wedge T_W)$, where $\mathcal{S}' = \{S_3, \ldots, S_6\}$, $\mathcal{W}' = \{W_1, \ldots, W_6\}$, and

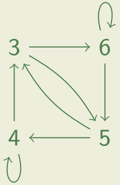
$$\begin{aligned} T_S = & (S_4 \wedge S_3 \wedge \neg S_4' \wedge \neg S_3' \to X_4^{\mathsf{w}} > X_3^{\mathsf{w}}) \wedge \\ & (S_3 \wedge S_5 \wedge \neg S_3' \wedge \neg S_5' \to X_3^{\mathsf{w}} > X_5^{\mathsf{w}}) \wedge \\ & (S_5 \wedge S_4 \wedge \neg S_5' \wedge \neg S_4' \to X_5^{\mathsf{w}} > X_4^{\mathsf{w}}) \wedge \ldots \end{aligned}$$

$$T_W = \bigwedge_{i=3}^{6} S_i \to W_i' \wedge \bigwedge_{i=1}^{2} W_i \to W_i'$$

Example (CGE_2 snippet)

$\mathcal{C}$: $1: \mathsf{f}(x^-) \approx \mathsf{f}(x)^-$ $\quad 2: \mathsf{f}(x)^- \approx \mathsf{f}(x^-)$

$\mathrm{DPC}(\mathcal{C})$: $3: \mathsf{f}^\#(x^-) \to \mathsf{f}(x)^{-\#}$ $\quad 4: \mathsf{f}^\#(x^-) \to \mathsf{f}^\#(x)$ $\quad 5: \mathsf{f}(x)^{-\#} \to \mathsf{f}^\#(x^-)$ $\quad 6: \mathsf{f}(x)^{-\#} \to x^{-\#}$



$\mathrm{D}_{\mathcal{C}} = (\mathcal{S}, \mathcal{W}, \phi)$ for $\mathcal{S} = \{S_3, S_4, S_5, S_6\}$, $\mathcal{W} = \{W_1, W_2\}$

Proc_{DG} maps $\mathrm{D}_{\mathcal{C}}$ to $\mathrm{D} = (\mathcal{S}', \mathcal{W}', \phi \wedge T_S \wedge T_W)$, where $\mathcal{S}' = \{S_3, \ldots, S_6\}$, $\mathcal{W}' = \{W_1, \ldots, W_6\}$, and

$$\begin{aligned} T_S = {} & (S_4 \wedge S_3 \wedge \neg S_4' \wedge \neg S_3' \to X_4^{\mathrm{w}} > X_3^{\mathrm{w}}) \wedge \\ & (S_3 \wedge S_5 \wedge \neg S_3' \wedge \neg S_5' \to X_3^{\mathrm{w}} > X_5^{\mathrm{w}}) \wedge \\ & (S_5 \wedge S_4 \wedge \neg S_5' \wedge \neg S_4' \to X_5^{\mathrm{w}} > X_4^{\mathrm{w}}) \wedge \ldots \end{aligned}$$

$$T_W = \bigwedge_{i=3}^{6} S_i \to W_i' \wedge \bigwedge_{i=1}^{2} W_i \to W_i'$$

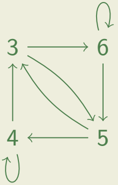
Proc_k: DG with k SCCs

Idea: limit #SCCs to at most k, and assign SCC number X_p^{scc} to DP p

Example (CGE$_2$ snippet)

$\mathcal{C}$: $1: \mathsf{f}(x^-) \approx \mathsf{f}(x)^-$ $\quad 2: \mathsf{f}(x)^- \approx \mathsf{f}(x^-)$

DPC($\mathcal{C}$): $3: \mathsf{f}^\#(x^-) \to \mathsf{f}(x)^{-\#}$ $\quad 4: \mathsf{f}^\#(x^-) \to \mathsf{f}^\#(x)$ $\quad 5: \mathsf{f}(x)^{-\#} \to \mathsf{f}^\#(x^-)$ $\quad 6: \mathsf{f}(x)^{-\#} \to x^{-\#}$



$\mathrm{D}_\mathcal{C} = (\mathcal{S}, \mathcal{W}, \phi)$ for $\mathcal{S} = \{S_3, S_4, S_5, S_6\}$, $\mathcal{W} = \{W_1, W_2\}$

Proc$_{DG}$ maps $\mathrm{D}_\mathcal{C}$ to $\mathrm{D} = (\mathcal{S}', \mathcal{W}', \phi \wedge T_S \wedge T_W)$, where $\mathcal{S}' = \{S_3, \ldots, S_6\}$, $\mathcal{W}' = \{W_1, \ldots, W_6\}$, and

$$\begin{aligned} T_S = {} & (S_4 \wedge S_3 \wedge \neg S_4' \wedge \neg S_3' \to X_4^{\mathsf{w}} > X_3^{\mathsf{w}}) \wedge \\ & (S_3 \wedge S_5 \wedge \neg S_3' \wedge \neg S_5' \to X_3^{\mathsf{w}} > X_5^{\mathsf{w}}) \wedge \\ & (S_5 \wedge S_4 \wedge \neg S_5' \wedge \neg S_4' \to X_5^{\mathsf{w}} > X_4^{\mathsf{w}}) \wedge \ldots \end{aligned}$$

$$T_W = \bigwedge_{i=3}^{6} S_i \to W_i' \wedge \bigwedge_{i=1}^{2} W_i \to W_i'$$

Lemma

Proc$_{(>,\geqslant)}$, Proc$_{DG}$ *and* Proc$_k$ *are sound*

Example (CGE_2 in practice) iteration 1: 6 equalities

((x,y),z) = *(x,*(y,z)), *(-(x),x) = 1, *(1,x) = x, *(f(x),f(y)) = f(*(x,y)), *(g(x),g(y)) = g(*(x,y)), *(f(x),g(y)) = *(g(y),f(x))

Example (CGE_2 in practice) iteration 2: 19 equalities

*(f(1),f(x)) = f(x), *(f(-(x)),f(x)) = f(1), *(x,y) = *(*(x,1),y), *(*(-(*(x,y)),x),y) = 1, *(x,1) = *(*(x,-(y)),y), *(f(g(x)),f(g(y))) = f(g(*(x,y))), *(x,g(*(y,z))) = *(*(x,g(y)),g(z)), *(*(x,g(y)),f(z)) = *(*(x,f(z)),g(y)), *(f(f(x)),f(g(y))) = *(f(g(y)),f(f(x))), x = *(-(y),*(y,x)), *(f(x),*(f(y),z)) = *(f(*(x,y)),z), *(g(x),*(g(y),z)) = *(g(*(x,y)),z), *(*(x,y),z) = *(x,*(y,z)), *(-(x),x) = 1, *(1,x) = x, *(f(x),f(y)) = f(*(x,y)), *(g(x),g(y)) = g(*(x,y)), *(f(x),g(y)) = *(g(y),f(x)), *(*(f(x),g(y)),f(z)) = *(*(f(x),f(z)),g(y))

Example (CGE_2 in practice) iteration 3: 45 equalities

(-(g(x)),(g(*(x,y)),z)) = *(g(y),z), *(f(x),*(g(y),z)) = *(g(y),*(f(x),z)), *(-(f(x)),*(f(*(x,y)),z)) = *(f(y),z), *(f(x),*(g(y),f(z))) = *(g(y),f(*(x,z))), *(x,*(-(g(y)),g(*(y,z)))) = *(x,g(z)), *(-(*(x,g(y))),*(x,g(*(y,z)))) = g(z), x = *(-(1),x), x = *(-(-(x)),1), *(x,y) = *(-(-(x)),y), *(-(f(x)),f(*(x,y))) = f(y), *(-(g(x)),g(*(x,y))) = g(y), x = *(-(*(y,z)),*(y,*(z,x))), *(f(x),y) = *(f(1),*(f(x),y)), *(-(f(x)),*(g(y),f(x))) = g(y), *(f(1),x) = *(f(-(y)),*(f(y),x)), *(x,y) = *(*(x,-(1)),y), *(x,*(-(1),y)) = *(x,y), x = *(*(-(*(y,1)),y),x), x = *(-(*(y,1)),*(y,x)), *(x,y) = *(*(x,-(-(y))),1), *(x,*(-(-(y)),1)) = *(x,y), x = *(*(-(*(y,-(x))),y),1), x = *(-(*(y,-(x))),*(y,1)), *(*(-(g(x)),f(y)),g(x)) = f(y), *(*(*(-(1),-(*(x,y))),x),y) = 1, *(f(1),f(x)) = f(x), *(f(-(x)),f(x)) = f(1), *(x,y) = *(*(x,1),y), *(*(-(*(x,y)),x),y) = 1, *(x,1) = *(*(x,-(y)),y), *(f(g(x)),f(g(y))) = f(g(*(x,y))), *(x,g(*(y,z))) = *(*(x,g(y)),g(z)), *(*(x,g(y)),f(z)) = *(*(x,f(z)),g(y)), *(f(f(x)),f(g(y))) = *(f(g(y)),f(f(x))), x = *(-(y),*(y,x)), *(f(x),*(f(y),z)) = *(f(*(x,y)),z), *(g(x),*(g(y),z)) = *(g(*(x,y)),z), *(*(x,y),z) = *(x,*(y,z)), *(-(x),x) = 1, *(1,x) = x, *(f(x),f(y)) = f(*(x,y)), *(g(x),g(y)) = g(*(x,y)), *(f(x),g(y)) = *(g(y),f(x)), *(-(*(x,g(y))),*(x,*(g(y),f(z)))) = f(z), *(f(x),*(g(y),f(z))) = *(g(y),f(*(x,z)))

Example (CGE_2 in practice) — iteration 4: 238 equalities

x = *(-(*(y,-(1))),*(y,x)), *(f(x),y) = *(f(*(x,1)),y), *(-(f(-(-(x)))),f(x)) = f(1), *(-(f(1)),*(f(x),y)) = *(f(x),y), *(-(f(-(1))),*(f(x),y)) = *(f(x),y), *(-(f(-(x))),*(f(1),y)) = *(f(x),y), *(-(f(-(
(x)))),*(f(x),y)) = *(f(1),y), *(-(f(-(x))),*(f(y),z)) = *(f(*(x,y)),z), *(-(f(-(-(x)))),*(f(*(x,y)),z)) = *(f(y),z), *(-(*(f(x),1)),*(f(*(x,y)),z)) = *(f(y),z), *(-(f(x)),*(g(y),f(*(x,z)))) = *(g(y),f(z)), *(-
(g(-(1))),g(x)) = g(x), *(-(f(-(x))),f(1)) = f(x), *(-(g(-(x))),g(1)) = g(x), x = *(x,1), x = *(-(*(y,-(x))),y), *(-(*(-(*(x,-(y))),x)),y) = 1, *(x,y) = *(-(*(z,-(x))),*(z,y)), x = *(-(y),*(-(*(z,-
(y))),*(z,x))), *(x,1) = *(-(*(-(*(x,-(y))),1)),y), *(-(f(-(*(x,-(y))))),f(y)) = f(*(x,1)), *(-(g(-(*(x,-(y))))),g(y)) = g(*(x,1)), *(x,-(x)) = 1, *(-(*(x,1)),x) = 1, x = *(y,*(-(y),x)), x = *(-(*(
(x),1)),1), *(-(f(1)),f(x)) = f(x), *(-(g(1)),g(x)) = g(x), *(-(f(-(1))),f(x)) = f(x), *(-(g(x)),*(g(*(x,y)),z)) = *(g(y),z), *(f(x),*(g(y),z)) = *(g(y),*(f(x),z)), *(-(f(x)),*(f(*(x,y)),z)) = *(f(y),z),
(f(x),(g(y),f(z))) = *(g(y),f(*(x,z))), *(x,*(-(g(y)),g(*(y,z)))) = *(x,g(z)), *(-(*(x,g(y))),*(x,g(*(y,z)))) = g(z), x = *(-(1),x), x = *(-(-(x)),1), *(x,y) = *(-(-(x)),y), *(-(f(x)),f(*(x,y))) =
f(y), *(-(g(x)),g(*(x,y))) = g(y), x = *(-(*(y,z)),*(y,*(z,x))), *(f(x),y) = *(f(1),*(f(x),y)), *(-(f(x)),*(g(y),f(x))) = g(y), *(f(1),x) = *(f(-(y)),*(f(y),x)), *(x,y) = *(*(x,-(1)),y), *(x,*(-(1),y))
= *(x,y), x = *(*(-(*(y,1)),y),x), x = *(-(*(y,1)),*(y,x)), *(x,y) = *(*(x,-(-(y))),1), *(x,*(y,1)) = *(x,y), *(x,*(-(-(y)),1)) = *(x,y), *(x,y) = *(x,*(y,1)), x = *(*(-(*(y,-(x))),y),1), x =
(-((y,-(x))),*(y,1)), *(*(-(g(x)),f(y)),g(x)) = f(y), *(*(*(-(1),-(*(x,y))),x),y) = 1, *(f(1),f(x)) = f(x), *(f(-(x)),f(x)) = f(1), *(x,y) = *(*(x,1),y), *(*(-(*(x,y)),x),y) = 1, *(x,1) = *(*(x,-
(y)),y), *(f(g(x)),f(g(y))) = f(g(*(x,y))), *(x,g(*(y,z))) = *(*(x,g(y)),g(z)), *(*(x,g(y)),f(z)) = *(*(x,f(z)),g(y)), *(f(f(x)),f(g(y))) = *(f(g(y)),f(f(x))), x = *(-(y),*(y,x)), *(f(x),*(f(y),z)) =
(f((x,y)),z), *(g(x),*(g(y),z)) = *(g(*(x,y)),z), *(*(x,y),z) = *(x,*(y,z)), *(-(x),x) = 1, *(1,x) = x, *(f(x),f(y)) = f(*(x,y)), *(g(x),g(y)) = g(*(x,y)), *(f(x),g(y)) = *(g(y),f(x)), *(-(g(-
(1))),*(g(x),y)) = *(g(x),y), *(-(*(x,g(-(1)))),*(x,g(y))) = g(y), *(-(*(x,-(-(y)))),*(x,y)) = 1, *(-(g(-(-(x)))),*(g(x),y)) = *(g(1),y), *(-(*(x,g(-(-(y))))),*(x,g(y))) = g(1), *(-(g(-(-(x)))),g(x)) =
g(1), x = *(-(*(y,-(-(z)))),*(y,*(z,x))), *(-(g(-(-(x)))),*(g(*(x,y)),z)) = *(g(y),z), *(-(*(x,g(-(-(y))))),*(x,g(*(y,z)))) = g(z), *(-(f(-(-(x)))),f(*(x,y))) = f(y), *(-(g(-(-(x)))),g(*(x,y))) = g(y), *(x,y)
= *(-(*(-(*(z,x)),z)),y), *(x,*(y,z)) = *(-(*(x39,-(*(x,y)))),*(x39,z)), *(x,*(y,z)) = *(-(*(-(*(x,y)),1)),z), *(-(g(-(*(x,y)))),*(g(z),x39)) = *(g(*(x,*(y,z))),x39), *(-(*(x,g(-(*(y,z))))),*(x,g(x39))) =
g(*(y,*(z,x39))), *(-(f(-(*(x,y)))),f(z)) = f(*(x,*(y,z))), *(-(g(-(*(x,y)))),g(z)) = g(*(x,*(y,z))), *(-(*(x,f(1))),*(x,*(f(y),z))) = *(f(y),z), *(-(*(f(1),1)),*(f(x),y)) = *(f(x),y), *(-(f(1)),*(f(x),y)) =
*(f(x),y), *(-(g(f(1))),*(g(*(f(x),y)),z)) = *(g(*(f(x),y)),z), *(-(*(x,g(f(1)))),*(x,g(*(f(y),z)))) = g(*(f(y),z)), *(-(f(f(1))),f(*(f(x),y))) = f(*(f(x),y)), *(-(g(f(1))),g(*(f(x),y))) = g(*(f(x),y)), *(-(*(x,f(
(y)))),*(x,*(f(1),z))) = *(f(y),z), *(-(*(f(-(x)),1)),*(f(1),y)) = *(f(x),y), *(-(f(-(x))),*(f(1),y)) = *(f(x),y), *(-(g(f(-(x)))),*(g(*(f(1),y)),z)) = *(g(*(f(x),y)),z), *(-(*(x,g(f(-(y))))),*(x,g(*(f(1),z))))
= g(*(f(y),z)), *(-(f(f(-(x)))),f(*(f(1),y))) = f(*(f(x),y)), *(-(g(f(-(x)))),g(*(f(1),y))) = g(*(f(x),y)), x = *(-(*(-(*(y,1)),y)),x), *(x,y) = *(-(*(z,-(*(x,1)))),*(z,y)), *(x,y) = *(-(*(-(*(x,1)),1)),y),
(-(g(-((x,1)))),*(g(y),z)) = *(g(*(x,y)),z), *(-(*(x,g(-(*(y,1))))),*(x,g(z))) = g(*(y,z)), *(-(f(-(*(x,1)))),f(y)) = f(*(x,y)), *(-(g(-(*(x,1)))),g(y)) = g(*(x,y)), x = *(-(y),*(-(*(z,-(y))),*(z,x))),
(-((x,*(-(*(y,-(z))),y))),*(x,z)) = 1, *(-(*(-(*(x,-(y))),x)),y) = 1, *(-(g(*(-(*(x,-(y))),x))),*(g(y),z)) = *(g(1),z), *(-(*(x,g(*(-(*(y,-(z))),y))))),*(x,g(z))) = g(1), *(-(f(*(-(*(x,-(y))),x))),f(y))
= f(1), *(-(g(*(-(*(x,-(y))),x))),g(y)) = g(1), *(x,y) = *(-(*(z,-(x))),*(z,y)), *(x,y) = *(-(*(-(x),1)),y), *(-(g(-(x))),*(g(y),z)) = *(g(*(x,y)),z), *(-(*(x,g(-(y)))),*(x,g(z))) = g(*(y,z)), *(-(f(-
(x))),f(y)) = f(*(x,y)), *(-(g(-(x))),g(y)) = g(*(x,y)), *(-(*(g(x),f(*(y,z)))),*(g(*(x,x39)),f(*(y,z)))) = g(x39), *(-(*(x,-(g(y)))),*(x,*(g(z),x39))) = *(g(*(y,z)),x39), *(-(*(-(g(x)),1)),*(g(y),z)) =
(g((x,y)),z), *(-(g(-(g(x)))),*(g(*(g(y),z)),x39)) = *(g(*(g(*(x,y)),z)),x39), *(-(*(x,g(-(g(y))))),*(x,g(*(g(z),x39)))) = g(*(g(*(y,z)),x39)), *(-(f(-(g(x)))),f(*(g(y),z))) = f(*(g(*(x,y)),z)), *(-(g(-
(g(x)))),g(*(g(y),z))) = g(*(g(*(x,y)),z)), x = *(-(*(g(y),*(f(z),x39))),*(g(y),*(f(z),*(x39,x)))), *(-(*(x,f(y))),*(x,*(g(z),*(f(y),x39)))) = *(g(z),x39), *(-(*(f(x),1)),*(g(y),*(f(x),z))) = *(g(y),z),
(-(f(x)),(g(y),*(f(x),z))) = *(g(y),z), *(-(g(f(x))),*(g(*(g(y),*(f(x),z))),x39)) = *(g(*(g(y),z)),x39), *(-(*(x,g(f(y)))),*(x,g(*(g(z),*(f(y),x39))))) = g(*(g(z),x39)), *(-(f(f(x))),f(*(g(y),*(f(x),z))))
= f(*(g(y),z)), *(-(g(f(x))),g(*(g(y),*(f(x),z)))) = g(*(g(y),z)), x = *(-(g(y)),*(-(*(z,g(x39))),*(z,*(g(*(x39,y)),x)))), *(-(*(-(*(x,g(y))),x)),g(z)) = g(*(y,z)), *(x,g(*(y,z))) = *(-(*(x39,-
(*(x,g(y))))),*(x39,g(z))), *(x,g(*(y,z))) = *(-(*(-(*(x,g(y))),1)),g(z)), *(-(g(-(*(x,g(y))))),*(g(g(z)),x39)) = *(g(*(x,g(*(y,z)))),x39), *(-(*(x,g(-(*(y,g(z)))))),*(x,g(g(x39)))) = g(*(y,g(*(z,x39)))),
(-(f(-((x,g(y))))),f(g(z))) = f(*(x,g(*(y,z)))), *(-(g(-(*(x,g(y))))),g(g(z))) = g(*(x,g(*(y,z)))), *(-(*(x,g(y))),*(x,*(g(*(y,z)),x39))) = *(g(z),x39), *(-(*(x,-(f(y)))),*(x,f(z))) = f(*(y,z)), *(-
(*(-(f(x)),1)),f(y)) = f(*(x,y)), *(-(g(-(f(x)))),*(g(f(y)),z)) = *(g(f(*(x,y))),z), *(-(*(x,g(-(f(y))))),*(x,g(f(z)))) = g(f(*(y,z))), *(-(f(-(f(x)))),f(f(y))) = f(f(*(x,y))), *(-(g(-(f(x)))),g(f(y))) =
g(f(*(x,y))), *(-(*(x,-(g(y)))),*(x,g(z))) = g(*(y,z)), *(-(*(-(g(x)),1)),g(y)) = g(*(x,y)), *(-(g(-(g(x)))),*(g(g(y)),z)) = *(g(g(*(x,y))),z), *(-(*(x,g(-(g(y))))),*(x,g(g(z)))) = g(g(*(y,z))), *(-
(f(-(g(x)))),f(g(y))) = f(g(*(x,y))), *(-(g(-(g(x)))),g(g(y))) = g(g(*(x,y))), x = *(-(g(y)),*(-(f(z)),*(g(y),*(f(z),x)))), *(-(*(-(f(x)),g(y))),g(y)) = f(x), *(-(*(x,-(f(y)))),*(x,g(z))) = *(g(z),f(y)),
(-((-(f(x)),1)),g(y)) = *(g(y),f(x)), *(-(g(-(f(x)))),*(g(g(y)),z)) = *(g(*(g(y),f(x))),z), *(-(*(x,g(-(f(y))))),*(x,g(g(z)))) = g(*(g(z),f(y))), *(-(f(-(f(x)))),f(g(y))) = f(*(g(y),f(x))), *(-(g(-
(f(x)))),g(g(y))) = g(*(g(y),f(x))), *(-(*(g(x),1)),*(g(*(x,y)),z)) = *(g(y),z), *(-(g(g(x))),*(g(*(g(*(x,y)),z)),x39)) = *(g(*(g(y),z)),x39), *(-(*(x,g(g(y)))),*(x,g(*(g(*(y,z)),x39)))) = g(*(g(z),x39)),
(-(f(g(x))),f((g(*(x,y)),z))) = f(*(g(y),z)), *(-(g(g(x))),g(*(g(*(x,y)),z))) = g(*(g(y),z)), x = *(-(*(y,*(z,x39))),*(y,*(z,*(x39,x)))), x = *(-(*(y,*(z,1))),*(y,*(z,x))), x = *(-(*(y,*(z,-(x)))),*(y,*(z,1))),
(-(g((x,y))),*(g(*(x,*(y,z))),x39)) = *(g(z),x39), *(-(*(x,*(y,g(z)))),*(x,*(y,g(*(z,x39))))) = g(x39), *(-(*(x,g(*(y,z)))),*(x,g(*(y,*(z,x39))))) = g(x39), *(-(f(*(x,y))),f(*(x,*(y,z)))) = f(z), *(-
(g(*(x,y))),g(*(x,*(y,z)))) = g(z), x = *(-(*(-(*(y,x)),y)),1), *(-(g(-(x))),*(g(1),y)) = *(g(x),y), *(-(*(-(g(*(x,y))),g(x))),1) = g(y), *(-(*(x,g(-(y)))),*(x,g(1))) = g(y), *(-(g(1)),*(g(x),y)) = *(g(x),y),
(-((x,g(1))),*(x,g(y))) = g(y), *(-(*(x,f(y))),*(x,f(*(y,z)))) = f(z), *(-(*(f(x),1)),f(*(x,y))) = f(y), *(-(g(f(x))),*(g(f(*(x,y))),z)) = *(g(f(y)),z), *(-(*(x,g(f(y)))),*(x,g(f(*(y,z))))) = g(f(z)), *(-
(f(f(x))),f(f(*(x,y)))) = f(f(y)), *(-(g(f(x))),g(f(*(x,y)))) = g(f(y)), *(-(*(g(x),1)),g(*(x,y))) = g(y), *(-(g(g(x))),*(g(g(*(x,y))),z)) = *(g(g(y)),z), *(-(*(x,g(g(y)))),*(x,g(g(*(y,z))))) = g(g(z)), *(-
(f(g(x))),f(g(*(x,y)))) = f(g(y)), *(-(g(g(x))),g(g(*(x,y)))) = g(g(y)), *(-(*(x,f(y))),*(x,*(g(z),f(y)))) = g(z), *(-(*(f(x),1)),*(g(y),f(x))) = g(y), *(-(g(f(x))),*(g(*(g(y),f(x))),z)) = *(g(g(y)),z), *(-
(*(g(x),f(y))),*(g(*(x,z)),f(y))) = g(z), *(-(*(x,g(f(y)))),*(x,g(*(g(z),f(y))))) = g(g(z)), *(-(f(f(x))),f(*(g(y),f(x)))) = f(g(y)), *(-(g(f(x))),g(*(g(y),f(x)))) = g(g(y)), *(x,1) = *(-(*(y,-(*(x,-(z))))),*(y,z)),
(-(g(-((x,-(y))))),*(g(y),z)) = *(g(*(x,1)),z), *(-(*(x,g(-(*(y,-(z)))))),*(x,g(z))) = g(*(y,1)), *(-(f(-(*(x,y)))),*(f(z),x39)) = *(f(*(x,*(y,z))),x39), *(-(f(-(*(x,1)))),*(f(y),z)) = *(f(*(x,y)),z), *(-(f(-
(*(x,-(y))))),*(f(y),z)) = *(f(*(x,1)),z), *(-(f(-(g(x)))),*(f(*(g(y),z)),x39)) = *(f(*(g(*(x,y)),z)),x39), *(-(f(x)),*(g(y),*(f(*(x,z)),x39))) = *(g(y),*(f(z),x39)), *(-(f(f(x))),*(f(*(g(y),*(f(x),z))),x39))
= *(f(*(g(y),z)),x39), *(-(*(x,-(f(y)))),*(x,*(f(z),x39))) = *(f(*(y,z)),x39), *(-(*(-(f(x)),1)),*(f(y),z)) = *(f(*(x,y)),z), *(-(g(-(f(x)))),*(g(*(f(y),z)),x39)) = *(g(*(f(*(x,y)),z)),x39), *(-(f(-
(f(x)))),*(f(*(f(y),z)),x39)) = *(f(*(f(*(x,y)),z)),x39), *(-(*(x,g(-(f(y))))),*(x,g(*(f(z),x39)))) = g(*(f(*(y,z)),x39)), *(-(f(-(f(x)))),f(*(f(y),z))) = f(*(f(*(x,y)),z)), *(-(g(-(f(x)))),g(*(f(y),z))) =
g(*(f(*(x,y)),z)), *(-(f(-(*(x,g(y))))),*(f(g(z)),x39)) = *(f(*(x,g(*(y,z)))),x39), *(-(f(-(f(x)))),*(f(f(y)),z)) = *(f(f(*(x,y))),z), *(-(f(-(g(x)))),*(f(g(y)),z)) = *(f(g(*(x,y))),z), *(-(f(-(f(x)))),*(f(g(y)),z))
= *(f(*(g(y),f(x))),z), *(-(*(x,f(y))),*(x,*(f(*(y,z)),x39))) = *(f(z),x39), *(-(g(f(x))),*(g(*(f(*(x,y)),z)),x39)) = *(g(*(f(y),z)),x39), *(-(f(f(x))),*(f(*(f(*(x,y)),z)),x39)) = *(f(*(f(y),z)),x39),
(-((x,g(f(y)))),*(x,g(*(f(*(y,z)),x39)))) = g(*(f(z),x39)), *(-(f(f(x))),f(*(f(*(x,y)),z))) = f(*(f(y),z)), *(-(g(f(x))),g(*(f(*(x,y)),z))) = g(*(f(y),z)), *(-(f(g(x))),*(f(*(g(*(x,y)),z)),x39)) =
(f((g(y),z)),x39), *(-(f(*(x,y))),*(f(*(x,*(y,z))),x39)) = *(f(z),x39), *(-(f(f(x))),*(f(f(*(x,y))),z)) = *(f(f(y)),z), *(-(f(g(x))),*(f(g(*(x,y))),z)) = *(f(g(y)),z), *(-(f(f(x))),*(f(*(g(y),f(x))),z)) =
*(f(g(y)),z)

Example (CGE_2 in practice) iteration 5: 458 equalities

x = *(f(1),x), x = *(g(1),x), x = *(x,-(1)), -(f(-(x))) = f(x), x = *(f(y),*(f(-(y)),x)), *(-(f(-(x))),y) = *(f(x),y), *(-(f(-(x))),f(y)) = f(*(x,y)), [illegible]
1 = g(1), 1 = f(1), x = -(-(x)), *(f(*(x,y)),-(f(y))) = f(x), *(f(*(x,y)),-(f(y))) = -(f(-(x))), *(f(x),y) = *(f(*(x,z)),*(-(f(z)),y)), [illegible]
= g(-(1)), 1 = -(f(1)), 1 = f(-(1)), x = *(-(*(y,-(1))),*(y,x)), *(f(x),y) = *(f(*(x,1)),y), *(-(f(-(-(x)))),f(x)) = f(1), [illegible]
= *(f(*(x,y)),z), *(-(f(-(-(x)))),*(f(*(x,y)),z)) = *(f(y),z), *(-(*(f(x),1)),*(f(*(x,y)),z)) = *(f(y),z), *(-(f(x)),*(f(*(x,y)),z)) = *(f(y),z), [illegible]
(-((y,-(x))),y), *(-(*(-(*(x,-(y))),x)),y) = 1, *(x,y) = *(-(*(z,-(x))),*(z,y)), x = *(-(y),*(-(*(z,-(y))),*(z,x))), *(x,1) = *(-(*(-(*(x,-(y))),1)),y), [illegible]
x = *(-(*(-(x),1)),1), *(-(f(1)),f(x)) = f(x), *(-(g(1)),g(x)) = g(x), *(-(f(-(1))),f(x)) = f(x), *(-(g(x)),*(g(*(x,y)),z)) = *(g(y),z), [illegible]
= g(z), x = *(-(1),x), x = *(-(-(x)),1), *(x,y) = *(-(-(x)),y), *(-(f(x)),f(*(x,y))) = f(y), *(-(g(x)),g(*(x,y))) = g(y), [illegible]
(1),y)) = *(x,y), x = *(*(-(*(y,1)),y),x), x = *(-(*(y,1)),*(y,x)), *(x,y) = *(*(x,-(-(y))),1), *(x,-(-(y))) = *(x,y), *(x,*(y,1)) = *(x,y), [illegible]

[illegible]

Example (CGE$_2$ in practice) iteration 6: 356 equalities

(x,-(y)) = -((y,-(x))), *(x,*(y,-(*(x,y)))) = 1, *(x,y) = -(*(-(y),-(x))), -(g(-(x))) = g(x), -(g(-(f(x)))) = g(f(x)), *(-(*(x,y)),x) = -(y), -(f(x)) = f(-(x)), -(g(x)) = g(-(x)), x = -(*(-(*(y,x)),y)), -(f(f(x))) = f(f(-(x))), -(g(f(x))) = g(f(-(x))), f(-(g(
(x)))) = f(g(x)), *(-(*(x,f(y))),x) = f(-(y)), x = *(f(1),x), x = *(g(1),x), x = *(x,-(1)), -(f(-(x))) = f(x), x = *(f(y),*(f(-(y)),x)), *(-(f(-(x))),y) = *(f(x),y), *(-(f(-(x))),f(y)) = f(*(x,y)), *(f(x),*(f(1),y)) = *(f(x),y), *(f(x),y) = *(f(x),*(f(1),y)),
(-(f(-(x))),f((y,z))) = f(*(x,*(y,z))), *(f(x),*(f(y),-(-(z)))) = *(f(x),*(f(y),z)), 1 = g(1), 1 = f(1), x = -(-(x)), *(f(*(x,y)),-(f(y))) = f(x), *(f(*(x,y)),-(f(y))) = -(f(-(x))), *(f(x),y) = *(f(*(x,z)),*(-(f(z)),y)), *(f(*(x,y)),-(-(z))) = *(f(*(x,y)),z),
(f(x),(f(y),z)) = *(f(x),*(f(y),-(-(z)))), *(-(f(-(x))),y) = *(f(*(x,z)),*(-(f(z)),y)), 1 = -(1), 1 = -(-(1)), 1 = g(-(1)), 1 = -(f(1)), 1 = f(-(1)), x = *(-(*(y,-(1))),*(y,x)), *(f(x),y) = *(f(*(x,1)),y), *(-(f(-(-(x)))),f(x)) = f(1), *(-(f(1)),*(f(x),y))
= *(f(x),y), *(-(f(-(1))),*(f(x),y)) = *(f(x),y), *(-(f(-(x))),*(f(1),y)) = *(f(x),y), *(-(f(-(-(x)))),*(f(x),y)) = *(f(1),y), *(-(f(-(x))),*(f(y),z)) = *(f(*(x,y)),z), *(-(f(-(-(x)))),*(f(*(x,y)),z)) = *(f(y),z), *(-(*(f(x),1)),*(f(*(x,y)),z)) = *(f(y),z), *(-
(f(x)),*(f(*(x,y)),z)) = *(f(y),z), *(-(f(x)),*(g(y),f(*(x,z)))) = *(g(y),f(z)), *(-(g(-(1)),g(x)) = g(x), *(-(f(-(x)),f(1)) = f(x), *(-(g(-(x)),g(1)) = g(x), x = *(x,1), x = *(-(*(y,-(x))),y), *(-(*(-(*(x,-(y))),x)),y) = 1, *(x,y) = *(-(*(z,-(x))),*(z,y)),
x = *(-(y),*(-(*(z,-(y))),*(z,x))), *(x,1) = *(-(*(-(*(x,-(y))),1)),y), *(-(f(-(*(x,-(y))))),f(y)) = f(*(x,1)), *(-(g(-(*(x,-(y))))),g(y)) = g(*(x,1)), *(x,-(x)) = 1, *(-(*(x,1)),x) = 1, x = *(y,*(-(y),x)), x = *(-(*(-(x),1)),1), *(-(f(1)),f(x)) = f(x),
*(-(g(1)),g(x)) = g(x), *(-(f(-(1)),f(x)) = f(x), *(-(g(x)),*(g(*(x,y)),z)) = *(g(y),z), *(f(x),*(g(y),z)) = *(g(y),*(f(x),z)), *(f(x),*(g(y),f(z))) = *(g(y),f(*(x,z))), *(x,*(-(g(y)),g(*(y,z)))) = *(x,g(z)), *(-(*(x,g(y))),*(x,g(*(y,z)))) = g(z), x =
*(-(1),x), x = *(-(-(x)),1), *(x,y) = *(-(-(x)),y), *(-(f(x)),f(*(x,y))) = f(y), *(-(g(x)),g(*(x,y))) = g(y), x = *(-(*(y,z)),*(y,*(z,x))), *(f(x),y) = *(f(1),*(f(x),y)), *(-(f(x)),*(g(y),f(x))) = g(y), *(f(1),x) = *(f(-(y)),*(f(y),x)), *(x,y) = *(*(x,-(1)),y),
(x,(-(1),y)) = *(x,y), x = *(*(-(*(y,1)),y),x), x = *(-(*(y,1)),*(y,x)), *(x,y) = *(*(x,-(-(y))),1), *(x,-(-(y))) = *(x,y), *(x,*(y,1)) = *(x,y), *(x,*(-(-(y)),1)) = *(x,y), *(x,y) = *(x,-(-(y))), *(x,y) = *(x,*(y,1)), x = *(*(-(*(y,-(x))),y),1), x =
(-((y,-(x))),*(y,1)), *(*(-(g(x)),f(y)),g(x)) = f(y), *(*(*(-(1),-(*(x,y))),x),y) = 1, *(f(1),f(x)) = f(x), *(f(-(x)),f(x)) = f(1), *(x,y) = *(*(x,1),y), *(*(-(*(x,y)),x),y) = 1, *(x,1) = *(*(x,-(y)),y), *(f(g(x)),f(g(y))) = f(g(*(x,y))), *(x,g(*(y,z))) =
((x,g(y)),g(z)), *(*(x,g(y)),f(z)) = *(*(x,f(z)),g(y)), *(f(f(x)),f(g(y))) = *(f(g(y)),f(f(x))), x = *(-(y),*(y,x)), *(f(x),*(f(y),z)) = *(f(*(x,y)),z), *(g(x),*(g(y),z)) = *(g(*(x,y)),z), *(*(x,y),z) = *(x,*(y,z)), *(-(x),x) = 1, *(1,x) = x, *(f(x),f(y)) =
f(*(x,y)), *(g(x),g(y)) = g(*(x,y)), *(f(x),g(y)) = *(g(y),f(x)), *(x,y) = *(-(*(-(*(z,x)),z)),y), *(-(g(-(*(x,y)))),*(g(z),x39)) = *(g(*(x,*(y,z))),x39), *(-(*(x,g(-(*(y,z))))),*(x,g(x39))) = g(*(y,*(z,x39))), *(-(g(-(*(x,y)))),g(z)) = g(*(x,*(y,z))), *(-
(*(x,f(-(y)))),*(x,z)) = *(f(y),z), *(-(g(f(-(x)))),*(g(y),z)) = *(g(*(f(x),y)),z), *(-(*(x,g(f(-(y))))),*(x,g(z))) = g(*(f(y),z)), *(-(f(f(-(x)))),f(y)) = f(*(f(x),y)), *(-(g(f(-(x)))),g(y)) = g(*(f(x),y)), *(-(g(-(x))),*(g(y),z)) = *(g(*(x,y)),z), *(-(*(x,g(-
(y)))),*(x,g(z))) = g(*(y,z)), *(-(g(-(x))),g(y)) = g(*(x,y)), *(-(*(g(x),f(*(y,z)))),*(g(*(x,x39)),f(*(y,z)))) = g(x39), *(-(g(-(g(x)))),*(g(*(g(y),z)),x39)) = *(g(*(g(*(x,y)),z)),x39), *(-(*(x,g(-(g(y))))),*(x,g(*(g(z),x39)))) = g(*(g(*(y,z)),x39)),
(-(g(-(g(x)))),g((g(y),z))) = g(*(g(*(x,y)),z)), x = *(-(*(g(y),*(f(z),x39))),*(g(y),*(f(z),*(x39,x)))), *(-(*(x,f(y))),*(x,*(g(z),*(f(y),x39)))) = *(g(z),x39), *(-(f(x)),*(g(y),*(f(x),z))) = *(g(y),z), *(-(g(f(x))),*(g(*(g(y),*(f(x),z))),x39)) =
(g((g(y),z)),x39), *(-(*(x,g(f(y)))),*(x,g(*(g(z),*(f(y),x39))))) = g(*(g(z),x39)), *(-(f(f(x))),f(*(g(y),*(f(x),z)))) = f(*(g(y),z)), *(-(g(f(x))),g(*(g(y),*(f(x),z)))) = g(*(g(y),z)), x = *(-(g(y)),*(-(*(z,g(x39))),*(z,*(g(*(x39,y)),x)))), *(-(*(-
(*(x,g(y))),x)),g(z)) = g(*(y,z)), *(-(g(-(*(x,g(y))))),*(g(g(z)),x39)) = *(g(*(x,g(*(y,z)))),x39), *(-(*(x,g(-(*(y,g(z)))))),*(x,g(g(x39)))) = g(*(y,g(*(z,x39)))), *(-(g(-(*(x,g(y))))),g(g(z))) = g(*(x,g(*(y,z)))), *(-(*(x,g(y))),*(x,*(g(*(y,z)),x39)))
= *(g(z),x39), *(-(g(-(f(x)))),*(g(f(y)),z)) = *(g(f(*(x,y))),z), *(-(*(x,g(-(f(y))))),*(x,g(f(z)))) = g(f(*(y,z))), *(-(g(-(f(x)))),g(f(y))) = g(f(*(x,y))), *(-(g(-(g(x)))),*(g(g(y)),z)) = *(g(g(*(x,y))),z), *(-(*(x,g(-(g(y))))),*(x,g(g(z)))) =
g(g(*(y,z))), *(-(g(-(g(x)))),g(g(y))) = g(g(*(x,y))), x = *(-(g(y)),*(-(f(z)),*(g(y),*(f(z),x)))), *(-(*(-(f(x)),g(y))),g(y)) = f(x), *(-(g(-(f(x)))),*(g(g(y)),z)) = *(g(*(g(y),f(x))),z), *(-(*(x,g(-(f(y))))),*(x,g(g(z)))) = g(*(g(z),f(y))), *(-
(g(-(f(x)))),g(g(y))) = g(*(g(y),f(x))), *(-(g(g(x))),*(g(*(g(*(x,y)),z)),x39)) = *(g(*(g(y),z)),x39), *(-(*(x,g(g(y)))),*(x,g(*(g(*(y,z)),x39)))) = g(*(g(z),x39)), *(-(f(g(x))),f(*(g(*(x,y)),z))) = f(*(g(y),z)), *(-(g(g(x))),g(*(g(*(x,y)),z))) =
g(*(g(y),z)), x = *(-(*(y,*(z,x39))),*(y,*(z,*(x39,x)))), x = *(-(*(y,*(z,-(x)))),*(y,z)), *(-(g(*(x,y))),*(g(*(x,*(y,z))),x39)) = *(g(z),x39), *(-(*(x,*(y,g(z)))),*(x,*(y,g(*(z,x39))))) = g(x39), *(-(*(x,g(*(y,z)))),*(x,g(*(y,*(z,x39))))) = g(x39),
(-(f((x,y))),f(*(x,*(y,z)))) = f(z), *(-(g(*(x,y))),g(*(x,*(y,z)))) = g(z), *(-(g(-(x))),y) = *(g(x),y), -(*(-(g(*(x,y))),g(x))) = g(y), *(-(*(x,g(-(y)))),x) = g(y), *(-(*(x,f(y))),*(x,f(*(y,z)))) = f(z), *(-(g(f(x))),*(g(f(*(x,y))),z)) = *(g(f(y)),z),
(-((x,g(f(y)))),*(x,g(f(*(y,z))))) = g(f(z)), *(-(f(f(x))),f(f(*(x,y)))) = f(f(y)), *(-(g(f(x))),g(f(*(x,y)))) = g(f(y)), *(-(g(g(x))),*(g(g(*(x,y))),z)) = *(g(g(y)),z), *(-(*(x,g(g(y)))),*(x,g(g(*(y,z))))) = g(g(z)), *(-(f(g(x))),f(g(*(x,y)))) = f(g(y)),
(-(g(g(x))),g(g((x,y)))) = g(g(y)), *(-(*(x,f(y))),*(x,*(g(z),f(y)))) = g(z), *(-(g(f(x))),*(g(*(g(y),f(x))),z)) = *(g(g(y)),z), *(-(*(g(x),f(y))),*(g(*(x,z)),f(y))) = g(z), *(-(*(x,g(f(y)))),*(x,g(*(g(z),f(y))))) = g(g(z)), *(-(f(f(x))),f(*(g(y),f(x))))
= f(g(y)), *(-(g(f(x))),g(*(g(y),f(x)))) = g(g(y)), *(-(g(-(*(x,-(y))))),*(g(y),z)) = *(g(x),z), *(-(*(x,g(-(*(y,-(z)))))),*(x,g(z))) = g(y), *(-(f(x)),*(g(y),*(f(*(x,z)),x39))) = *(g(y),*(f(z),x39)), *(-(f(f(x))),*(f(*(g(y),*(f(x),z))),x39))
= *(f(*(g(y),z)),x39), *(-(g(-(f(x)))),*(g(*(f(y),z)),x39)) = *(g(*(f(*(x,y)),z)),x39), *(-(*(x,g(-(f(y))))),*(x,g(*(f(z),x39)))) = g(*(f(*(y,z)),x39)), *(-(g(-(f(x)))),g(*(f(y),z))) = g(*(f(*(x,y)),z)), *(-(f(g(x))),*(f(*(g(*(x,y)),z)),x39)) =
(f((g(y),z)),x39), *(-(f(*(x,y))),*(f(*(x,*(y,z))),x39)) = *(f(z),x39), *(-(f(f(x))),*(f(f(*(x,y))),z)) = *(f(f(y)),z), *(-(f(g(x))),*(f(g(*(x,y))),z)) = *(f(g(y)),z), *(-(f(f(x))),*(f(*(g(y),f(x))),z)) = *(f(g(y)),z), x = *(-(y),-(*(-(x),-(y)))), *(x,y) =
(-(z),(-(*(-(x),-(z))),y)), *(x,y) = *(-(*(-(*(y,-(z))),-(x))),z), x = *(-(*(-(*(y,*(x,-(z)))),y)),z), f(*(x,y)) = f(-(*(-(y),-(x)))), *(-(g(-(x))),g(y)) = g(-(*(-(y),-(x)))), *(-(x),*(-(*(g(y),-(x))),g(*(y,z)))) = g(z), *(-(*(-(*(g(*(x,y)),-(z))),g(x))),z) =
g(y), *(x,*(y,z)) = *(-(*(-(*(y,-(x39))),-(x))),*(x39,z)), *(-(g(-(*(x,-(y))))),*(g(*(y,z)),x39)) = *(g(*(x,z)),x39), *(-(*(x,g(-(*(y,-(z)))))),*(x,g(*(z,x39)))) = g(*(y,x39)), *(-(g(-(*(x,-(y))))),g(*(y,z))) = g(*(x,z)), *(x,*(-(y),z)) = *(-(*(y,-(x))),z),
*(-(x),y) = *(-(*(z,x)),*(z,y)), *(-(f(x)),y) = *(f(-(x)),y), *(-(f(x)),y) = *(f(z),*(-(f(*(x,z))),y)), *(-(f(x)),*(f(y),z)) = *(f(*(-(x),y)),z), *(-(f(x)),*(g(y),f(z))) = *(g(y),f(*(-(x),z))), *(-(g(x)),y) = *(g(z),*(-(g(*(x,z))),y)), *(-(g(x)),*(g(y),z)) =
(g((-(x),y)),z), *(f(x),y) = *(g(z),*(f(x),*(-(g(z)),y))), *(-(*(x,g(y))),*(x,g(z))) = g(*(-(y),z)), *(-(f(x)),f(y)) = f(*(-(x),y)), *(-(g(x)),g(y)) = g(*(-(x),y)), *(g(x),y) = *(g(*(x,z)),*(-(g(z)),y)), x = *(y,*(z,*(-(*(y,z)),x))), *(x,*(y,*(z,x39)))
= *(-(*(-(*(y,z)),-(x))),x39), *(x,*(f(y),z)) = *(-(*(f(-(y)),-(x))),z), *(-(f(f(-(x)))),*(f(y),z)) = *(f(*(f(x),y)),z), *(-(f(f(-(x)))),*(g(y),f(z))) = *(g(y),f(*(f(x),z))), *(x,*(y,z)) = *(-(*(-(y),-(x))),z), *(x,f(y)) = -(*(-(f(y)),-(x))), *(-(g(-(f(x)))),y)
= *(g(f(x)),y), *(-(*(x,g(-(f(y))))),x) = g(f(y)), *(x,*(f(y),z)) = *(-(*(-(f(y)),-(x))),z), *(-(g(-(f(x)))),*(g(y),z)) = *(g(*(f(x),y)),z), *(-(*(x,g(-(f(y))))),*(x,g(z))) = g(*(f(y),z)), *(-(g(-(f(x)))),g(y)) = g(*(f(x),y)), *(x,*(f(*(y,z)),x39))
= *(-(*(-(f(y)),-(x))),*(f(z),x39)), *(x,*(g(y),f(*(z,x39)))) = *(-(*(-(f(z)),-(x))),*(g(y),f(x39))), x = *(-(*(g(y),f(z))),*(-(f(x39)),*(g(y),*(f(*(x39,z)),x)))), *(-(*(-(f(x)),g(y))),*(g(y),f(z))) = f(*(x,z)), *(-(g(-(f(x)))),*(g(*(g(y),f(z))),x39))
= *(g(*(g(y),f(*(x,z)))),x39), *(-(*(x,g(-(f(y))))),*(x,g(*(g(z),f(x39))))) = g(*(g(z),f(*(y,x39)))), *(-(g(-(f(x)))),g(*(g(y),f(z)))) = g(*(g(y),f(*(x,z)))), x = *(-(*(-(g(-(y))),-(x))),g(y)), x = *(-(g(y)),*(-(g(-(y))),x)), x = *(-(g(-(g(-
(y))))),*(g(g(y)),x)), *(-(g(x)),*(-(g(-(x))),g(y))) = g(y), *(-(*(x,g(-(g(-(y)))))),*(x,g(g(y)))) = 1, *(-(g(-(g(-(x))))),g(g(x))) = 1, *(x,f(y)) = *(-(*(f(*(z,-(y))),-(x))),f(z)), *(-(f(f(*(x,-(y))))),*(g(z),f(f(x)))) = *(g(z),f(f(y))), *(-(g(f(*(x,-
(y))))),*(g(f(x)),z)) = *(g(f(y)),z), *(-(*(x,g(f(*(y,-(z)))))),*(x,g(f(y)))) = g(f(z)), *(-(f(f(*(x,-(y))))),f(f(x))) = f(f(y)), *(-(g(f(*(x,-(y))))),g(f(x))) = g(f(y)), *(x,g(y)) = *(-(*(-(g(-(*(z,-(y))))),-(x))),g(z)), x = *(-(g(y)),*(-(g(-(*(y,-
(z))))),*(g(z),x))), *(-(g(-(g(-(*(x,-(y))))))),*(g(g(x)),z)) = *(g(g(y)),z), *(-(g(x)),*(-(g(-(*(x,-(y))))),g(*(y,z)))) = g(z), *(-(*(-(g(-(*(x,-(*(y,z)))))),g(y))),g(x)) = g(z), *(-(*(x,g(-(g(-(*(y,-(z)))))))),*(x,g(g(y)))) = g(g(z)), *(-(g(-(g(-(*(x,-
(y))))))),g(g(x))) = g(g(y)), *(f(x),-(f(*(y,x)))) = -(f(y)), *(-(f(x)),g(y)) = *(g(y),f(-(x))), *(g(x),-(g(*(y,x)))) = -(g(y)), *(-(g(x)),y) = *(g(-(x)),y), *(g(x),*(f(y),-(g(x)))) = f(y), *(-(*(x,g(y))),x) = g(-(y)), *(g(*(x,y)),-(g(y))) = g(x),
(x,(g(*(y,z)),x39)) = *(-(*(-(g(y)),-(x))),*(g(z),x39)), *(x,*(g(y),z)) = *(-(*(f(x39),-(x))),*(g(y),*(f(x39),z))), *(f(*(-(f(x)),*(g(y),*(f(x),z)))),x39) = *(f(*(g(y),z)),x39), *(-(f(f(x))),*(g(y),f(*(g(z),*(f(x),x39))))) = *(g(y),f(*(g(z),x39))),
(x,(y,g(*(z,x39)))) = *(-(*(-(*(y,g(z))),-(x))),g(x39)), *(x,f(*(y,z))) = *(-(*(-(f(y)),-(x))),f(z)), *(x,g(*(y,z))) = *(-(*(-(g(y)),-(x))),g(z)), *(x,*(g(y),f(z))) = *(-(*(-(f(z)),-(x))),g(y)), *(x,*(g(y),z)) = *(-(*(g(x39),-(x))),*(g(*(x39,y)),z)), *(-
(f(g(x))),*(g(y),f(*(g(*(x,z)),x39)))) = *(g(y),f(*(g(z),x39))), *(x,y) = *(-(*(z,*(x39,-(x)))),*(z,*(x39,y))), *(-(f(*(x,y))),*(g(z),f(*(x,*(y,x39))))) = *(g(z),f(x39)), *(-(g(-(*(x,*(y,-(z)))))),g(z)) = g(*(x,y)), *(x,f(y)) = *(-(*(f(z),-(x))),f(*(z,y))),
(-(f(f(x))),(g(y),f(f(*(x,z))))) = *(g(y),f(f(z))), *(x,g(y)) = *(-(*(g(z),-(x))),g(*(z,y))), *(-(f(g(x))),*(g(y),f(g(*(x,z))))) = *(g(y),f(g(z))), *(x,g(y)) = *(-(*(f(z),-(x))),*(g(y),f(z))), *(-(f(f(x))),*(g(y),f(*(g(z),f(x))))) = *(g(y),f(g(z))),
(x,(f(y),z)) = *(-(*(f(x39),-(x))),*(f(*(x39,y)),z)), *(-(f(f(x))),*(g(y),f(*(f(*(x,z)),x39)))) = *(g(y),f(*(f(z),x39))), *(-(g(f(x))),*(g(*(f(*(x,y)),z)),x39)) = *(g(*(f(y),z)),x39), *(-(*(x,g(f(y)))),*(x,g(*(f(*(y,z)),x39)))) = g(*(f(z),x39)), *(-
(f(f(x))),f(*(f(*(x,y)),z))) = f(*(f(y),z)), *(-(g(f(x))),g(*(f(*(x,y)),z))) = g(*(f(y),z)), *(-(g(-(*(x,y)))),g(-(y))) = g(x), *(f(*(f(x),y)),*(-(f(*(-(f(z)),y))),x39)) = *(f(f(*(x,z))),x39), *(x,*(-(f(y)),z)) = *(-(*(f(*(x39,y)),-(x))),*(f(x39),z)),
(-((x,f(*(y,z)))),*(x,*(f(y),x39))) = *(-(f(z)),x39), *(-(f(x)),y) = *(-(f(*(z,x))),*(f(z),y)), *(f(*(f(x),y)),-(f(*(-(f(z)),y)))) = f(f(*(x,z))), *(-(f(f(*(x,y)))),*(f(*(f(x),z)),x39)) = *(f(*(-(f(y)),z)),x39), *(-(f(f(*(x,y)))),*(g(z),f(*(f(x),x39))))
= *(g(z),f(*(-(f(y)),x39))), *(-(g(f(*(x,y)))),*(g(*(f(x),z)),x39)) = *(g(*(-(f(y)),z)),x39), *(-(*(x,g(f(*(y,z))))),*(x,g(*(f(y),x39)))) = g(*(-(f(z)),x39)), *(-(f(f(*(x,y)))),f(*(f(x),z))) = f(*(-(f(y)),z)), *(-(g(f(*(x,y)))),g(*(f(x),z))) = g(*(-
(f(y)),z)), *(f(*(x,*(y,z))),*(-(f(z)),x39)) = *(f(*(x,y)),x39), *(f(x),*(-(f(y)),z)) = *(f(-(*(y,-(x)))),z), *(f(x),-(f(y))) = f(-(*(y,-(x)))), *(f(*(x,y)),*(-(f(*(z,y))),x39)) = *(f(-(*(z,-(x)))),x39), *(f(*(x,y)),-(f(*(z,y)))) = f(-(*(z,-(x)))),
*(f(x),y) = *(f(z),*(-(f(*(-(x),z))),y)), *(f(x),-(f(*(-(y),x)))) = f(y), *(f(x),*(-(f(*(y,*(z,x)))),x39)) = *(f(-(*(y,z))),x39), *(f(x),-(f(*(y,*(z,x))))) = f(-(*(y,z))), *(f(x),*(-(f(*(y,x))),z)) = *(f(-(y)),z), *(f(x),-(f(*(y,x)))) = f(-(y)), *(-
(*(x,f(y))),*(x,z)) = *(f(-(y)),z), *(-(g(-(*(x,f(y))))),g(f(-(y)))) = g(x), *(x,-(f(y))) = *(-(*(f(*(z,y)),-(x))),f(z)), *(-(*(x,f(*(y,z)))),*(x,f(y))) = -(f(z)), *(-(f(*(x,y))),f(x)) = -(f(y)), *(-(f(f(*(x,y)))),*(f(f(x)),z)) = *(f(-(f(y))),z), *(-
(f(f(*(x,y)))),*(g(z),f(f(x)))) = *(g(z),f(-(f(y)))), *(-(g(f(*(x,y)))),*(g(f(x)),z)) = *(g(-(f(y))),z), *(-(*(x,g(f(*(y,z))))),*(x,g(f(y)))) = g(-(f(z))), *(-(f(f(*(x,y)))),f(f(x))) = f(-(f(y))), *(-(g(f(*(x,y)))),g(f(x))) = g(-(f(y))), *(f(*(x,*(y,z))),-
(f(z))) = f(*(x,y)), *(f(*(f(x),y)),*(-(f(*(f(*(z,x)),y))),x39)) = *(f(-(f(z))),x39), *(f(*(f(x),y)),-(f(*(f(*(z,x)),y)))) = f(-(f(z))), *(f(*(g(x),f(y))),*(-(f(*(g(x),f(*(z,y))))),x39)) = *(f(-(f(z))),x39), *(f(*(g(x),f(y))),-(f(*(g(x),f(*(z,y)))))) = f(-
(f(z))), *(f(-(g(-(x)))),y) = *(f(g(x)),y), *(f(-(g(-(*(x,-(y)))))),z) = *(f(g(x)),*(-(f(g(y))),z)), *(f(g(x)),-(f(g(y)))) = f(-(g(-(*(x,-(y)))))), *(f(*(g(x),y)),*(-(f(*(g(*(z,x)),y))),x39)) = *(f(-(g(z))),x39), *(f(*(g(x),y)),-(f(*(g(*(z,x)),y)))) = f(-
(g(z))), *(f(*(g(x),*(f(y),z))),*(-(f(*(g(x),z))),x39)) = *(f(f(y)),x39), *(f(*(g(x),*(f(y),z))),-(f(*(g(x),z)))) = f(f(y)), *(f(-(*(x,g(y)))),z) = *(f(g(x39)),*(-(f(*(x,g(*(y,x39))))),z)), *(f(g(x)),-(f(*(y,g(*(z,x)))))) = f(-(*(y,g(z)))), *(f(-(f(x))),y) =
(f(f(z)),(-(f(f(*(x,z)))),y)), *(f(f(x)),-(f(f(*(y,x))))) = f(-(f(y))), *(f(-(g(x))),y) = *(f(g(z)),*(-(f(g(*(x,z)))),y)), *(f(g(x)),-(f(g(*(y,x))))) = f(-(g(y))), *(f(-(f(x))),y) = *(f(g(z)),*(-(f(*(g(z),f(x)))),y)), *(f(g(x)),-(f(*(g(x),f(y))))) = f(-(f(y))),
(f((f(*(x,y)),z)),*(-(f(*(f(y),z))),x39)) = *(f(f(x)),x39), *(f(*(f(*(x,y)),z)),-(f(*(f(y),z)))) = f(f(x)), *(f(*(g(*(x,y)),z)),*(-(f(*(g(y),z))),x39)) = *(f(g(x)),x39), *(f(*(g(*(x,y)),z)),-(f(*(g(y),z)))) = f(g(x)), *(f(*(x,y)),z) = *(f(*(x,*(y,x39))),*(-
(f(x39)),z)), *(f(f(x)),y) = *(f(f(*(x,z))),*(-(f(f(z))),y)), *(f(f(*(x,y))),-(f(f(y)))) = f(f(x)), *(f(g(x)),y) = *(f(g(*(x,z))),*(-(f(g(z))),y)), *(f(g(*(x,y))),-(f(g(y)))) = f(g(x)), *(f(*(g(x),f(y))),*(-(f(g(x))),z)) = *(f(f(y)),z), *(f(*(g(x),f(y))),-
(f(g(x)))) = f(f(y))

Example (CGE$_2$ in practice) iteration 7: 210 equalities

-(g(x)) = g(*(y,-(*(x,y)))), x = *(y,*(z,*(-(*(y,z)),x))), x = *(y,*(-(*(z,y)),*(z,x))), *(x,-(*(y,x))) = -(y), f(*(x,-(*(y,x)))) = f(-(y)), g(*(x,-(*(y,x)))) = g(-(y)), *(g(x),-(y)) =
-(*(y,g(-(x)))), *(g(*(x,y)),-(g(y))) = g(x), f(x) = f(*(y,-(*(-(x),y)))), x = *(y,-(*(-(x),y))), *(-(x),-(y)) = -(*(y,x)), *(-(x),y) = -(*(-(y),x)), -(g(f(f(x)))) = g(f(f(-(x)))), *(x,-(y)) =
-(*(y,-(x))), *(x,*(y,-(*(x,y)))) = 1, *(x,y) = -(*(-(y),-(x))), -(g(-(x))) = g(x), -(g(-(f(x)))) = g(f(x)), *(-(*(x,y)),x) = -(y), -(f(x)) = f(-(x)), -(g(x)) = g(-(x)), x = -(*(-(*(y,x)),y)),
-(f(f(x))) = f(f(-(x))), -(g(f(x))) = g(f(-(x))), f(-(g(-(x)))) = f(g(x)), *(-(*(x,f(y))),x) = f(-(y)), x = *(f(1),x), x = *(g(1),x), x = *(x,-(1)), -(f(-(x))) = f(x), x = *(f(y),*(f(-(y)),x)),
*(-(f(-(x))),y) = *(f(x),y), *(-(f(-(x))),f(y)) = f(*(x,y)), *(f(x),*(f(1),y)) = *(f(x),y), *(f(x),y) = *(f(x),*(f(1),y)), *(-(f(-(x))),f(*(y,z))) = f(*(x,*(y,z))), *(f(x),*(f(y),-(-(z)))) =
(f(x),(f(y),z)), 1 = g(1), 1 = f(1), x = -(-(x)), *(f(*(x,y)),-(f(y))) = f(x), *(f(*(x,y)),-(f(y))) = -(f(-(x))), *(f(x),y) = *(f(*(x,z)),*(-(f(z)),y)), *(f(*(x,y)),-(-(z))) = *(f(*(x,y)),z),
(f(x),(f(y),z)) = *(f(x),*(f(y),-(-(z)))), *(-(f(-(x))),y) = *(f(*(x,z)),*(-(f(z)),y)), 1 = -(1), 1 = -(-(1)), 1 = g(-(1)), 1 = -(f(1)), 1 = f(-(1)), x = *(-(*(y,-(1))),*(y,x)), *(f(x),y) =
(f((x,1)),y), *(-(f(-(-(x)))),f(x)) = f(1), *(-(f(1)),*(f(x),y)) = *(f(x),y), *(-(f(-(1))),*(f(x),y)) = *(f(x),y), *(-(f(-(x))),*(f(1),y)) = *(f(x),y), *(-(f(-(-(x)))),*(f(x),y)) = *(f(1),y), *(-
(f(-(x))),*(f(y),z)) = *(f(*(x,y)),z), *(-(f(-(-(x)))),*(f(*(x,y)),z)) = *(f(y),z), *(-(*(f(x),1)),*(f(*(x,y)),z)) = *(f(y),z), *(-(f(x)),*(f(*(x,y)),z)) = *(f(y),z), *(-(f(x)),*(g(y),f(*(x,z))))
= *(g(y),f(z)), *(-(g(-(1))),g(x)) = g(x), *(-(f(-(x))),f(1)) = f(x), *(-(g(-(x))),g(1)) = g(x), x = *(x,1), x = *(-(*(y,-(x))),y), *(-(*(-(*(x,-(y))),x)),y) = 1, *(x,y) = *(-(*(z,-
(x))),*(z,y)), x = *(-(y),*(-(*(z,-(y))),*(z,x))), *(x,1) = *(-(*(-(*(x,-(y))),1)),y), *(-(f(-(*(x,-(y))))),f(y)) = f(*(x,1)), *(-(g(-(*(x,-(y))))),g(y)) = g(*(x,1)), *(-(g(*(x,-(y)))),g(x))
= g(y), *(x,-(x)) = 1, *(-(*(x,1)),x) = 1, x = *(y,*(-(y),x)), x = *(-(*(-(x),1)),1), *(-(f(1)),f(x)) = f(x), *(-(g(1)),g(x)) = g(x), *(-(f(-(1))),f(x)) = f(x), *(-(g(x)),*(g(*(x,y)),z))
= *(g(y),z), *(f(x),*(g(y),z)) = *(g(y),*(f(x),z)), *(f(x),*(g(y),f(z))) = *(g(y),f(*(x,z))), *(x,*(-(g(y)),g(*(y,z)))) = *(x,g(z)), *(-(*(x,g(y))),*(x,g(*(y,z)))) = g(z), x = *(-(1),x),
x = *(-(-(x)),1), *(x,y) = *(-(-(x)),y), *(-(f(x)),f(*(x,y))) = f(y), *(-(g(x)),g(*(x,y))) = g(y), x = *(-(*(y,z)),*(y,*(z,x))), *(f(x),y) = *(f(1),*(f(x),y)), *(-(f(x)),*(g(y),f(x))) =
g(y), *(f(1),x) = *(f(-(y)),*(f(y),x)), *(x,y) = *(*(x,-(1)),y), *(x,*(-(1),y)) = *(x,y), x = *(*(-(*(y,1)),y),x), x = *(-(*(y,1)),*(y,x)), *(x,y) = *(*(x,-(-(y))),1), *(x,-(-(y))) =
*(x,y), *(x,*(y,1)) = *(x,y), *(x,*(-(-(y)),1)) = *(x,y), *(x,y) = *(x,-(-(y))), *(x,y) = *(x,*(y,1)), x = *(*(-(*(y,-(x))),y),1), x = *(-(*(y,-(x))),*(y,1)), *(*(-(g(x)),f(y)),g(x))
= f(y), *(*(*(-(1),-(*(x,y))),x),y) = 1, *(f(1),f(x)) = f(x), *(f(-(x)),f(x)) = f(1), *(x,y) = *(*(x,1),y), *(*(-(*(x,y)),x),y) = 1, *(x,1) = *(*(x,-(y)),y), *(f(g(x)),f(g(y))) =
f(g(*(x,y))), *(x,g(*(y,z))) = *(*(x,g(y)),g(z)), *(*(x,g(y)),f(z)) = *(*(x,f(z)),g(y)), *(f(f(x)),f(g(y))) = *(f(g(y)),f(f(x))), x = *(-(y),*(y,x)), *(f(x),*(f(y),z)) = *(f(*(x,y)),z),
(g(x),(g(y),z)) = *(g(*(x,y)),z), *(*(x,y),z) = *(x,*(y,z)), *(-(x),x) = 1, *(1,x) = x, *(f(x),f(y)) = f(*(x,y)), *(g(x),g(y)) = g(*(x,y)), *(f(x),g(y)) = *(g(y),f(x)), *(-(*(x,f(-
(y)))),*(x,z)) = *(f(y),z), *(-(*(g(x),f(*(y,z)))),*(g(*(x,x39)),f(*(y,z)))) = g(x39), x = *(-(*(g(y),*(f(z),x39))),*(g(y),*(f(z),*(x39,x)))), *(-(*(x,f(y))),*(x,*(g(z),*(f(y),x39)))) =
*(g(z),x39), *(-(*(x,g(f(y)))),*(x,g(*(g(z),*(f(y),x39))))) = g(*(g(z),x39)), x = *(-(g(y)),*(-(*(z,g(x39))),*(z,*(g(*(x39,y)),x)))), *(-(*(x,g(y))),*(x,*(g(*(y,z)),x39))) = *(g(z),x39),
(-((x,g(g(y)))),*(x,g(*(g(*(y,z)),x39)))) = g(*(g(z),x39)), x = *(-(*(y,*(z,x39))),*(y,*(z,*(x39,x)))), x = *(-(*(y,*(z,-(x)))),*(y,z)), *(-(*(x,*(y,g(z)))),*(x,*(y,g(*(z,x39))))) =
g(x39), *(-(*(x,g(*(y,z)))),*(x,g(*(y,*(z,x39))))) = g(x39), *(-(*(x,f(y))),*(x,f(*(y,z)))) = f(z), *(-(*(x,g(f(y)))),*(x,g(f(*(y,z))))) = g(f(z)), *(-(*(x,g(g(y)))),*(x,g(g(*(y,z))))) =
g(g(z)), *(-(*(x,f(y))),*(x,*(g(z),f(y)))) = g(z), *(-(*(g(x),f(y))),*(g(*(x,z)),f(y))) = g(z), *(-(*(x,g(f(y)))),*(x,g(*(g(z),f(y))))) = g(g(z)), *(-(*(x,g(*(y,-(z))))),*(x,g(y))) = g(z),
(x,(y,z)) = *(-(*(x39,*(-(y),-(x)))),*(x39,z)), *(-(*(x,g(*(y,-(z))))),*(x,g(*(y,x39)))) = g(*(z,x39)), *(-(x),y) = *(-(*(z,x)),*(z,y)), *(f(*(x,-(*(y,x)))),z) = *(f(-(y)),z), *(-(g(x)),y)
= *(g(*(z,-(*(x,z)))),y), *(-(*(x,g(y))),*(x,g(z))) = g(*(-(y),z)), *(-(*(g(x),f(-(y)))),*(g(x),f(z))) = f(*(y,z)), *(-(*(x,g(f(*(y,-(z)))))),*(x,g(f(y)))) = g(f(z)), *(x,y) = *(-(*(z,*(x39,-
(x)))),*(z,*(x39,y))), *(-(*(x,g(f(y)))),*(x,g(*(f(*(y,z)),x39)))) = g(*(f(z),x39)), *(f(*(f(x),*(y,-(*(f(-(z)),y))))),x39) = *(f(f(*(x,z))),x39), *(-(*(x,f(*(y,z)))),*(x,*(f(y),x39))) = *(f(-
(z)),x39), f(*(f(x),*(y,-(*(f(-(z)),y))))) = f(f(*(x,z))), *(-(*(x,g(f(*(y,z))))),*(x,g(*(f(y),x39)))) = g(*(f(-(z)),x39)), *(f(*(x,-(y))),z) = *(f(*(x,*(x39,-(*(y,x39))))),z), f(*(x,-(y))) =
f(*(x,*(z,-(*(y,z))))), *(f(x),y) = *(f(*(z,-(*(-(x),z)))),y), *(f(*(x,-(*(y,*(z,x))))),x39) = *(f(-(*(y,z))),x39), f(*(x,-(*(y,*(z,x))))) = f(-(*(y,z))), *(-(*(x,f(y))),*(x,z)) = *(f(-(y)),z),
(-((x,f(*(y,z)))),*(x,f(y))) = f(-(z)), *(-(*(x,g(f(*(y,z))))),*(x,g(f(y)))) = -(g(f(z))), *(f(*(f(x),*(y,-(*(f(*(z,x)),y))))),x39) = *(f(f(-(z))),x39), f(*(f(x),*(y,-(*(f(*(z,x)),y)))))
= f(f(-(z))), *(f(*(g(x),*(f(y),-(*(g(x),f(*(z,y))))))),x39) = *(f(f(-(z))),x39), f(*(g(x),*(f(y),-(*(g(x),f(*(z,y))))))) = f(f(-(z))), *(f(*(g(x),*(y,-(*(g(*(z,x)),y))))),x39) = *(f(-
(g(z))),x39), f(*(g(x),*(y,-(*(g(*(z,x)),y))))) = f(-(g(z))), *(f(*(g(x),*(f(y),*(z,-(*(g(x),z)))))),x39) = *(f(f(y)),x39), f(*(g(x),*(f(y),*(z,-(*(g(x),z)))))) = f(f(y)), *(f(*(g(x),-
(*(y,g(*(z,x)))))),x39) = *(f(-(*(y,g(z)))),x39), f(*(g(x),-(*(y,g(*(z,x)))))) = f(-(*(y,g(z)))), *(f(f(*(x,-(*(y,x))))),z) = *(f(f(-(y))),z), f(f(*(x,-(*(y,x))))) = f(f(-(y))), *(f(-(g(x))),y) =
(f(g((z,-(*(x,z))))),y), f(-(g(x))) = f(g(*(y,-(*(x,y))))), *(f(*(g(x),-(*(g(x),f(y))))),z) = *(f(f(-(y))),z), f(*(g(x),-(*(g(x),f(y))))) = f(f(-(y))), *(f(*(f(*(x,y)),*(z,-(*(f(y),z))))),x39)
= *(f(f(x)),x39), f(*(f(*(x,y)),*(z,-(*(f(y),z))))) = f(f(x)), *(f(*(g(*(x,y)),*(z,-(*(g(y),z))))),x39) = *(f(g(x)),x39), f(*(g(*(x,y)),*(z,-(*(g(y),z))))) = f(g(x)), *(x,*(-(y),-(z))) = -
(*(z,*(y,-(x)))), *(-(x),-(*(y,z))) = -(*(y,*(z,x))), *(g(x),*(y,-(*(g(*(z,x)),y)))) = -(g(z)), *(g(x),*(f(y),*(z,-(*(g(x),z))))) = f(y), *(f(*(x,y)),*(z,-(*(f(y),z)))) = f(x), *(g(*(x,y)),*(z,-
(*(g(y),z)))) = g(x), *(x,*(y,*(z,-(*(x,*(y,z)))))) = 1, -(g(*(-(x),y))) = g(*(-(y),x)), *(x,*(-(*(g(y),x)),g(*(y,z)))) = g(z), *(-(*(-(*(g(*(x,y)),z)),g(x))),-(z)) = g(y), *(f(x),-(y))
= -(*(y,f(-(x)))), *(-(*(g(x),y)),-(g(z))) = -(*(g(*(z,x)),y)), *(g(x),*(f(y),*(z,-(*(g(x),*(f(y),z)))))) = 1, *(-(*(g(x),*(f(y),z))),f(y)) = -(*(g(x),z)), *(-(g(x)),-(*(y,g(z)))) = -
(*(y,g(*(z,x)))), -(g(*(x,y))) = g(*(-(y),-(x))), *(-(*(f(*(x,y)),z)),f(x)) = -(*(f(y),z)), *(-(*(g(*(x,y)),z)),g(x)) = -(*(g(y),z)), *(x,-(*(y,z))) = -(*(y,*(z,-(x)))), *(-(*(x,*(y,z))),*(x,y))
= -(z), *(-(*(g(x),f(y))),f(y)) = -(g(x))

Example (CGE$_2$ in practice) iteration 8: 282 equalities

(g(x),-(g(y))) = g((x,-(y))), x = *(-(*(y,*(z,-(x)))),*(y,z)), *(-(x),y) = *(-(*(z,x)),*(z,y)), *(f(x),-(y)) = -(*(y,f(-(x)))), *(-(g(*(x,y))),g(x)) = -(g(y)), *(g(x),-(g(*(y,x)))) = -(g(y)), *(g(x),*(f(y),-(g(x)))) = f(y), -(g(*(-(x),y))) = g(*(-(y),x)), *(g(x),-(g(*(-(y),x)))) = g(y), *(-(x),f(y)) = -(*(f(-(y)),x)), *(f(-(x)),-(y)) = -(*(y,f(x))), *(-(x),f(-(y))) = -(*(f(y),x)), -(g(x)) = g(*(y,-(*(x,y)))), x = *(y,*(z,*(-(*(y,z)),x))), x = *(y,*(-(*(z,y)),*(z,x))), *(x,-(*(y,x))) = -(y), f(*(x,-(*(y,x)))) = f(-(y)), g(*(x,-(*(y,x)))) = g(-(y)), *(g(x),-(y)) = -(*(y,g(-(x)))), *(g(*(x,y)),-(g(y))) = g(x), f(x) = f(*(y,-(*(-(x),y)))), x = *(y,-(*(-(x),y))), *(-(x),-(y)) = -(*(y,x)), *(-(x),y) = -(*(-(y),x)), -(g(f(f(x)))) = g(f(f(-(x)))), *(x,-(y)) = -(*(y,-(x))), *(x,*(y,-(*(x,y)))) = 1, *(x,y) = -(*(-(y),-(x))), -(g(-(x))) = g(x), -(g(-(f(x)))) = g(f(x)), *(-(*(x,y)),x) = -(y), -(f(x)) = f(-(x)), -(g(x)) = g(-(x)), x = -(*(-(*(y,x)),y)), -(f(f(x))) = f(f(-(x))), -(g(f(x))) = g(f(-(x))), f(-(g(-(x)))) = f(g(x)), *(-(*(x,f(y))),x) = f(-(y)), x = *(f(1),x), x = *(g(1),x), x = *(x,-(1)), -(f(-(x))) = f(x), x = *(f(y),*(f(-(y)),x)), *(-(f(-(x))),y) = *(f(x),y), *(-(f(-(x))),f(y)) = f(*(x,y)), *(f(x),*(f(1),y)) = *(f(x),y), *(f(x),y) = *(f(x),*(f(1),y)), *(-(f(-(x))),f(*(y,z))) = f(*(x,*(y,z))), *(f(x),*(f(y),-(-(z)))) = *(f(x),*(f(y),z)), 1 = g(1), 1 = f(1), x = -(-(x)), *(f(*(x,y)),-(f(y))) = f(x), *(f(*(x,y)),-(f(y))) = -(f(-(x))), *(f(x),y) = *(f(*(x,z)),*(-(f(z)),y)), *(f(*(x,y)),-(-(z))) = *(f(*(x,y)),z), *(f(x),*(f(y),z)) = *(f(x),*(f(y),-(-(z)))), *(-(f(-(x))),y) = *(f(*(x,z)),*(-(f(z)),y)), 1 = -(1), 1 = -(-(1)), 1 = g(-(1)), 1 = -(f(1)), 1 = f(-(1)), x = *(-(*(y,-(1))),*(y,x)), *(f(x),y) = *(f(*(x,1)),y), *(-(f(-(-(x)))),f(x)) = f(1), *(-(f(1)),*(f(x),y)) = *(f(x),y), *(-(f(-(1))),*(f(x),y)) = *(f(x),y), *(-(f(-(x))),*(f(1),y)) = *(f(x),y), *(-(f(-(-(x)))),*(f(x),y)) = *(f(1),y), *(-(f(-(x))),*(f(y),z)) = *(f(*(x,y)),z), *(-(f(-(-(x)))),*(f(*(x,y)),z)) = *(f(y),z), *(-(*(f(x),1)),*(f(*(x,y)),z)) = *(f(y),z), *(-(f(x)),*(f(*(x,y)),z)) = *(f(y),z), *(-(f(x)),*(g(y),f(*(x,z)))) = *(g(y),f(z)), *(-(g(-(1))),g(x)) = g(x), *(-(f(-(x))),f(1)) = f(x), *(-(g(-(x))),g(1)) = g(x), x = *(x,1), x = *(-(*(y,-(x))),y), *(-(*(-(*(x,-(y))),x)),y) = 1, *(x,y) = *(-(*(z,-(x))),*(z,y)), x = *(-(y),*(-(*(z,-(y))),*(z,x))), *(x,1) = *(-(*(-(*(x,-(y))),1)),y), *(-(f(-(*(x,-(y))))),f(y)) = f(*(x,1)), *(-(g(-(*(x,-(y))))),g(y)) = g(*(x,1)), *(-(g(*(x,-(y))))),g(x)) = g(y), *(x,-(x)) = 1, *(-(*(x,1)),x) = 1, x = *(y,*(-(y),x)), x = *(-(*(-(x),1)),1), *(-(f(1)),f(x)) = f(x), *(-(g(1)),g(x)) = g(x), *(-(f(-(1))),f(x)) = f(x), *(-(g(x)),*(g(*(x,y)),z)) = *(g(y),z), *(f(x),*(g(y),z)) = *(g(y),*(f(x),z)), *(f(x),*(g(y),f(z))) = *(g(y),f(*(x,z))), *(x,*(-(g(y)),g(*(y,z)))) = *(x,g(z)), *(-(*(x,g(y))),*(x,g(*(y,z)))) = g(z), x = *(-(1),x), x = *(-(-(x)),1), *(x,y) = *(-(-(x)),y), *(-(f(x)),f(*(x,y))) = f(y), *(-(g(x)),g(*(x,y))) = g(y), x = *(-(*(y,z)),*(y,*(z,x))), *(f(x),y) = *(f(1),*(f(x),y)), *(-(f(x)),*(g(y),f(x))) = g(y), *(f(1),x) = *(f(-(y)),*(f(y),x)), *(x,y) = *(*(x,-(1)),y), *(x,*(-(1),y)) = *(x,y), x = *(*(-(*(y,1)),y),x), x = *(-(*(y,1)),*(y,x)), *(x,y) = *(*(x,-(-(y))),1), *(x,-(-(y))) = *(x,y), *(x,*(y,1)) = *(x,y), *(x,*(-(-(y)),1)) = *(x,y), *(x,y) = *(x,-(-(y))), *(x,y) = *(x,*(y,1)), x = *(*(-(*(y,-(x))),y),1), x = *(-(*(y,-(x))),*(y,1)), *(*(-(g(x)),f(y)),g(x)) = f(y), *(*(*(-(1),-(*(x,y))),x),y) = 1, *(f(1),f(x)) = f(x), *(f(-(x)),f(x)) = f(1), *(x,y) = *(*(x,1),y), *(*(-(*(x,y)),x),y) = 1, *(x,1) = *(*(x,-(y)),y), *(f(g(x)),f(g(y))) = f(g(*(x,y))), *(x,g(*(y,z))) = *(*(x,g(y)),g(z)), *(*(x,g(y)),f(z)) = *(*(x,f(z)),g(y)), *(f(f(x)),f(g(y))) = *(f(g(y)),f(f(x))), x = *(-(y),*(y,x)), *(f(x),*(f(y),z)) = *(f(*(x,y)),z), *(g(x),*(g(y),z)) = *(g(*(x,y)),z), *(*(x,y),z) = *(x,*(y,z)), *(-(x),x) = 1, *(1,x) = x, *(f(x),f(y)) = f(*(x,y)), *(g(x),g(y)) = g(*(x,y)), *(f(x),g(y)) = *(g(y),f(x)), *(-(*(x,f(-(y)))),*(x,z)) = *(f(y),z), *(-(*(g(x),f(*(y,z)))),*(g(*(x,x39)),f(*(y,z)))) = g(x39), x = *(-(*(g(y),*(f(z),x39))),*(g(y),*(f(z),*(x39,x)))), *(-(*(x,f(y))),*(x,*(g(z),*(f(y),x39)))) = *(g(z),x39), *(-(*(x,g(f(y)))),*(x,g(*(g(z),*(f(y),x39))))) = g(*(g(z),x39)), x = *(-(g(y)),*(-(*(z,g(x39))),*(z,*(g(*(x39,y)),x)))), *(-(*(x,g(y))),*(x,*(g(*(y,z)),x39))) = *(g(z),x39), *(-(*(x,g(g(y)))),*(x,g(*(g(*(y,z)),x39)))) = g(*(g(z),x39)), x = *(-(*(y,*(z,x39))),*(y,*(z,*(x39,x)))), *(-(*(x,*(y,g(z)))),*(x,*(y,g(*(z,x39))))) = g(x39), *(-(*(x,g(*(y,z)))),*(x,g(*(y,*(z,x39))))) = g(x39), *(-(*(x,f(y))),*(x,f(*(y,z)))) = f(z), *(-(*(x,g(f(y)))),*(x,g(f(*(y,z))))) = g(f(z)), *(-(*(x,g(g(y)))),*(x,g(g(*(y,z))))) = g(g(z)), *(-(*(x,f(y))),*(x,*(g(z),f(y)))) = g(z), *(-(*(g(x),f(y))),*(g(*(x,z)),f(y))) = g(z), *(-(*(x,g(f(y)))),*(x,g(*(g(z),f(y))))) = g(g(z)), *(-(*(x,g(*(y,-(z))))),*(x,g(y))) = g(z), *(-(*(x,g(*(y,-(z))))),*(x,g(*(y,x39)))) = g(*(z,x39)), *(-(*(x,g(y))),*(x,g(z))) = g(*(-(y),z)), *(-(*(g(x),f(-(y)))),*(g(x),f(z))) = f(*(y,z)), *(-(*(x,g(f(*(y,-(z)))))),*(x,g(f(y)))) = g(f(z)), *(x,y) = *(-(*(z,*(x39,-(x)))),*(z,*(x39,y))), *(-(*(x,g(f(y)))),*(x,g(*(f(*(y,z)),x39)))) = g(*(f(z),x39)), *(-(*(x,f(*(y,z)))),*(x,*(f(y),x39))) = *(f(-(z)),x39), *(-(*(x,g(f(*(y,z))))),*(x,g(*(f(y),x39)))) = g(*(f(-(z)),x39)), *(f(*(x,-(*(y,*(z,x))))),x39) = *(f(-(*(y,z))),x39), f(*(x,-(*(y,*(z,x))))) = f(-(*(y,z))), *(-(*(x,f(y))),*(x,z)) = *(f(-(y)),z), *(-(*(x,f(*(y,z)))),*(x,f(y))) = f(-(z)), *(-(*(x,g(f(*(y,z))))),*(x,g(f(y)))) = -(g(f(z))), *(f(*(g(x),*(f(y),-(*(g(x),f(*(z,y))))))),x39) = *(f(f(-(z))),x39), f(*(g(x),*(f(y),-(*(g(x),f(*(z,y))))))) = f(f(-(z))), *(f(*(g(x),-(g(*(y,x))))),z) = *(f(-(g(y))),z), f(*(g(x),-(g(*(y,x))))) = f(-(g(y))), *(f(*(g(x),*(f(y),-(g(x))))),z) = *(f(f(y)),z), f(*(g(x),*(f(y),-(g(x))))) = f(f(y)), *(f(*(g(x),-(*(y,g(*(z,x)))))),x39) = *(f(-(*(y,g(z)))),x39), f(*(g(x),-(*(y,g(*(z,x)))))) = f(-(*(y,g(z)))), *(f(*(g(x),-(*(g(x),f(y))))),z) = *(f(f(-(y))),z), f(*(g(x),-(*(g(x),f(y))))) = f(f(-(y))), *(x,-(*(y,z))) = -(*(y,*(z,-(x)))), *(x,*(y,*(z,-(*(x,*(y,z)))))) = 1, *(x,*(-(*(g(y),x)),g(*(y,z)))) = g(z), *(g(x),*(f(y),*(z,-(*(g(x),*(f(y),z)))))) = 1, *(-(*(g(x),*(f(y),z))),f(y)) = -(*(g(x),z)), *(-(*(f(*(x,y)),z)),f(x)) = -(*(f(y),z)), *(-(*(g(*(x,y)),z)),g(x)) = -(*(g(y),z)), *(-(*(x,*(y,z))),*(x,y)) = -(z), *(-(*(g(x),f(y))),f(y)) = -(g(x)), *(x,*(-(*(y,x)),z)) = *(x39,*(-(*(y,x39)),z)), *(x,*(-(*(y,x)),z)) = *(-(*(x39,y)),*(x39,z)), *(x,*(-(*(y,x)),z)) = *(-(y),z), *(g(x),-(g(*(y,*(-(*(z,y)),x))))) = g(z), *(-(x),y) = -(*(z,*(-(*(y,z)),x))), *(-(g(x)),y) = *(g(z),*(x39,*(-(*(g(*(x,z)),x39)),y)))), *(-(g(x)),*(g(y),z)) = *(g(*(x39,*(-(*(x,x39)),y))),z), *(f(x),y) = *(g(z),*(f(x),*(x39,*(-(*(g(z),x39)),y)))), *(-(*(x,g(y))),*(x,g(z))) = g(*(x39,*(-(*(y,x39)),z))), *(-(g(x)),g(y)) = g(*(z,*(-(*(x,z)),y))), *(f(x),y) = *(f(*(x,z)),*(x39,*(-(*(f(z),x39)),y))), *(g(x),y) = *(g(*(x,z)),*(x39,*(-(*(g(z),x39)),y))), x = *(y,*(z,*(x39,*(-(*(y,*(z,x39))),x)))), *(-(*(x,y)),*(x,z)) = *(-(*(x39,y)),*(x39,z)), *(g(x),-(g(*(-(*(y,z)),*(y,x))))) = g(z), *(-(g(x)),y) = *(g(z),*(-(*(x39,g(*(x,z)))),*(x39,y))), *(-(g(x)),*(g(y),z)) = *(g(*(-(*(x39,x)),*(x39,y))),z), *(f(x),y) = *(g(z),*(f(x),*(-(*(x39,g(z))),*(x39,y)))), *(-(*(x,g(y))),*(x,g(z))) = g(*(-(*(x39,y)),*(x39,z))), *(-(g(x)),g(y)) = g(*(-(*(z,x)),*(z,y))), *(f(x),y) = *(f(*(x,z)),*(-(*(x39,f(z))),*(x39,y))), *(g(x),y) = *(g(*(x,z)),*(-(*(x39,g(z))),*(x39,y))), x = *(y,*(z,*(-(*(x39,*(y,z))),*(x39,x)))), *(-(x),*(-(y),z)) = *(-(*(y,x)),z), -(g(f(f(*(-(x),y))))) = g(f(f(*(-(y),x)))), *(-(x),*(y,-(z))) = -(*(z,*(-(y),x))), -(g(f(*(-(x),y)))) = g(f(*(-(y),x))), *(-(g(*(x,*(-(y),z)))),g(x)) = g(*(-(z),y)), *(-(g(f(f(x)))),*(f(y),z)) = *(f(y),*(-(g(f(f(x)))),z)), *(-(g(f(f(x)))),*(g(y),z)) = *(g(*(f(f(-(x))),y)),z), *(g(x),*(-(g(f(f(y)))),z)) = *(g(*(x,f(f(-(y))))),z), *(-(g(f(f(x)))),g(y)) = g(*(f(f(-(x))),y)), *(g(x),-(g(f(f(y))))) = g(*(x,f(f(-(y))))), *(-(g(f(f(x)))),f(y)) = *(f(y),-(g(f(f(x))))), -(g(f(f(*(x,-(y)))))) = g(f(f(*(y,-(x))))), *(-(g(*(x,*(y,-(z))))),g(x)) = g(*(z,-(y))), *(g(x),-(g(*(y,*(z,x))))) = -(g(*(y,z))), *(x,-(*(y,*(z,x)))) = -(*(y,z)), *(-(*(x,y)),*(x,-(z))) = -(*(z,y)), *(g(x),-(*(y,g(*(z,x))))) = -(*(y,g(z))), *(-(g(x)),*(-(g(y)),z)) = *(-(g(*(y,x))),z), *(f(x),-(y)) = *(g(z),*(f(x),-(*(y,g(z))))), *(-(*(x,g(y))),*(x,-(g(z)))) = -(g(*(z,y))), *(f(x),-(y)) = *(f(*(x,z)),-(*(y,f(z)))), *(g(x),-(y)) = *(g(*(x,z)),-(*(y,g(z)))), *(x,*(y,-(*(z,*(x,y))))) = -(z), x = *(g(*(y,z)),*(-(g(z)),*(-(g(y)),x))), *(x,*(-(*(g(*(y,z)),x)),g(y))) = -(g(z)), *(-(*(x,g(*(y,z)))),*(x,g(y))) = -(g(z)), *(-(g(g(x))),g(g(*(x,y)))) = g(g(y)), *(-(g(g(x))),y) = *(-(g(g(*(z,x)))),*(g(g(z)),y)), *(g(x),f(y)) = *(g(*(x,z)),*(f(y),-(g(z)))), *(-(*(x,g(g(*(y,z))))),*(x,g(g(y)))) = -(g(g(z))), *(-(g(g(*(x,y)))),g(g(x))) = -(g(g(y))), *(g(*(x,*(y,z))),-(g(z))) = g(*(x,y)), *(g(x),y) = *(g(*(x,z)),*(-(g(z)),y)), *(-(g(x)),y) = *(g(z),*(-(g(*(x,z))),y)), -(g(f(f(f(x))))) = g(f(f(f(-(x))))), *(-(g(*(x,f(-(y))))),g(x)) = g(f(y)), x = *(-(g(*(y,-(z)))),*(g(y),*(-(g(z)),x))), x = *(g(y),*(-(g(z)),*(-(g(*(y,-(z)))),x))), x = *(-(g(y)),*(-(g(*(z,-(y)))),*(g(z),x))), *(g(g(x)),-(g(g(y)))) = -(g(g(*(y,-(x))))), *(-(g(x)),*(-(g(*(y,-(x)))),g(*(y,z)))) = g(z), *(-(g(*(x,-(y)))),*(g(x),z)) = *(g(y),z), x = *(g(*(y,z)),*(x39,*(-(*(g(z),x39)),*(-(g(y)),x)))), *(g(*(g(x),y)),-(g(*(g(*(z,x)),y)))) = -(g(g(z))), x = *(g(y),*(f(z),*(x39,*(-(*(g(y),*(f(z),x39))),x)))), x = *(g(y),*(z,*(-(*(g(y),*(f(x39),z))),*(f(x39),x)))), *(x,*(-(*(f(y),x)),*(g(z),*(f(y),x39)))) = *(g(z),x39), *(g(x),*(y,-(*(g(x),*(f(z),y))))) = f(-(z)), *(g(*(g(x),*(f(y),z))),-(g(*(g(x),z)))) = g(f(y)), x = *(-(*(y,g(z))),*(y,*(g(*(z,x39)),*(-(g(x39)),x)))), x = *(y,*(g(*(z,x39)),*(-(g(x39)),*(-(*(y,g(z))),x)))), *(g(g(x)),-(g(*(y,g(*(z,x)))))) = -(g(*(y,g(z)))), *(g(g(x)),-(g(g(*(y,x))))) = -(g(g(y))), x = *(f(y),*(z,*(-(*(f(*(x39,y)),z)),*(f(x39),x)))), *(x,*(-(*(f(y),x)),*(f(*(y,z)),x39))) = *(f(z),x39), *(g(*(f(*(x,y)),z)),-(g(*(f(y),z)))) = g(f(x)), x = *(g(y),*(z,*(-(*(g(*(x39,y)),z)),*(g(x39),x)))), *(x,*(-(*(g(y),x)),*(g(*(y,z)),x39))) = *(g(z),x39), *(g(*(g(*(x,y)),z)),-(g(*(g(y),z)))) = g(g(x)), x = *(y,*(-(*(z,*(x39,y))),*(z,*(x39,x)))), *(-(g(*(x,*(y,-(z))))),g(*(x,y))) = g(z), *(x,*(-(*(f(y),x)),f(*(y,z)))) = f(z), *(g(f(*(x,y))),-(g(f(y)))) = g(f(x)), x = *(g(y),*(-(g(*(z,y))),*(g(z),x))), *(g(g(*(x,y))),-(g(g(y)))) = g(g(x)), x = *(g(y),*(-(*(g(y),f(z))),*(f(z),x))), *(x,*(-(*(f(y),x)),*(g(z),f(y)))) = g(z), *(g(x),-(*(g(x),f(y)))) = f(-(y)), *(g(*(g(x),f(y))),-(g(g(x)))) = g(f(y))

Example (CGE_2 in practice) iteration 8: 282 equalities

$\mathcal{R}_{8,2}$ consists of 158 rules:

$f(*(-(x),x)) \to 1$
$f(f(*(-(x),x))) \to 1$
$-(*(f(-(x)),y)) \to *(-(y),f(x))$
$*(x,*(-(*(y,x)),*(y,z))) \to z$
$-(*(x,y)) \to *(-(y),-(x))$
$-(*(-(*(x,y)),x)) \to y$
$*(f(x),*(f(-(x)),y)) \to y$
$-(-(x)) \to x$
$-(1) \to 1$
$f(-(1)) \to 1$
$*(x,1) \to x$
$*(-(*(-(*(x,-(y))),1)),y) \to *(x,1)$
$*(g(x),*(f(y),z)) \to *(f(y),*(g(x),z))$
$*(-(*(x,y)),*(x,*(y,z))) \to z$
$*(*(-(*(x,1)),x),y) \to y$
$*(x,*(y,1)) \to *(x,y)$
$*(*(x,-(y)),y) \to *(x,1)$
$*(x,*(y,-(*(z,*(x,y))))) \to -(z)$
$*(x,*(-(*(y,x)),z)) \to *(-(y),z)$
$-(g(*(x,-(y)))) \to g(*(y,-(x)))$
$-(g(*(f(-(x)),y))) \to g(*(-(y),f(x)))$
$*(g(x),*(f(y),-(g(x)))) \to f(y)$
$f(*(x,-(*(y,x)))) \to f(-(y))$
$*(x,*(y,-(*(x,y)))) \to 1$
$-(f(x)) \to f(-(x))$
$f(-(g(-(x)))) \to f(g(x))$
$*(-(f(-(x))),f(y)) \to f(*(x,y))$
$*(f(*(x,y)),-(f(y))) \to f(x)$
$*(-(f(1)),*(f(x),y)) \to *(f(x),y)$
$*(-(f(-(x))),*(f(y),z)) \to *(f(*(x,y)),z)$
$*(-(f(x)),*(g(y),f(*(x,z)))) \to *(g(y),f(z))$
$*(-(*(-(*(x,-(y))),x)),y) \to 1$
$*(x,-(x)) \to 1$
$*(-(f(-(1))),f(x)) \to f(x)$
$*(-(*(x,g(y))),*(x,g(*(y,z)))) \to g(z)$
$*(g(x),f(*(-(y),y))) \to g(x)$
$*(x,*(-(-(y)),1)) \to *(x,y)$
$*(f(-(x)),f(x)) \to f(1)$
$*(f(x),*(f(y),z)) \to *(f(*(x,y)),z)$
$*(f(x),f(y)) \to f(*(x,y)) * (g(x),g(y))$

$g(*(-(x),x)) \to 1$
$*(-(*(x,*(y,-(z)))),*(x,y)) \to z$
$-(*(x,f(y))) \to *(f(-(y)),-(x))$
$-(*(x,g(-(y)))) \to *(g(y),-(x))$
$-(*(-(x),y)) \to *(-(y),x)$
$*(f(1),x) \to x$
$*(f(x),*(f(1),y)) \to *(f(x),y)$
$*(f(*(x,y)),*(-(f(y)),z)) \to *(f(x),z)$
$-(-(1)) \to 1$
$*(-(*(x,-(1))),*(x,y)) \to y$
$*(-(*(x,-(y))),x) \to y$
$f(*(x,*(-(y),y))) \to f(x)$
$*(-(1),x) \to x$
$*(f(1),*(f(x),y)) \to *(f(x),y)$
$*(-(*(x,1)),*(x,y)) \to y$
$*(*(-(*(x,-(y))),x),1) \to y$
$*(*(x,g(y)),g(z)) \to *(x,g(*(y,z)))$
$-(g(f(f(f(x))))) \to g(f(f(f(-(x)))))$
$*(x,-(*(y,*(z,x)))) \to -(*(y,z))$
$*(-(*(g(x),f(y))),f(y)) \to -(g(x))$
$*(g(x),-(g(y))) \to g(*(x,-(y)))$
$-(g(*(-(x),y))) \to g(*(-(y),x))$
$g(*(x,-(*(y,x)))) \to g(-(y))$
$-(g(-(x))) \to g(x)$
$-(g(x)) \to g(-(x))$
$*(-(*(x,f(y))),x) \to f(-(y))$
$*(f(x),*(f(1),y)) \to *(f(x),y)$
$*(f(*(x,y)),-(f(y))) \to -(f(-(x)))$
$*(-(f(-(1))),*(f(x),y)) \to *(f(x),y)$
$*(-(f(-(-(x)))),*(f(*(x,y)),z)) \to *(f(y),z)$
$*(-(g(-(1))),g(x)) \to g(x)$
$*(-(f(-(*(x,-(y))))),f(y)) \to f(*(x,1))$
$*(-(*(x,1)),x) \to 1$
$*(-(g(x)),*(g(*(x,y)),z)) \to *(g(y),z)$
$*(-(f(x)),f(*(x,y))) \to f(y)$
$*(x,*(-(1),y)) \to *(x,y)$
$*(*(-(g(x)),f(y)),g(x)) \to f(y)$
$*(*(-(*(x,y)),x),y) \to 1$
$*(g(x),*(g(y),z)) \to *(g(*(x,y)),z)$
$\to g(*(x,y))$

$*(x,*(-(y),y)) \to x$
$*(-(*(x,y)),*(x,z)) \to *(-(y),z)$
$-(*(f(x),y)) \to *(-(y),f(-(x)))$
$f(*(x,-(*(-(y),x)))) \to f(y)$
$-(*(x,-(y))) \to *(y,-(x))$
$*(g(1),x) \to x$
$g(1) \to 1$
$*(f(x),*(f(y),-(-(z)))) \to *(f(x),*(f(y),z))$
$g(-(1)) \to 1$
$*(f(*(x,1)),y) \to *(f(x),y)$
$*(-(*(x,-(y))),*(x,z)) \to *(y,z)$
$*(x,*(-(x),y)) \to y$
$*(-(-(x)),1) \to x$
$*(f(-(x)),*(f(x),y)) \to *(f(1),y)$
$*(*(x,-(-(y))),1) \to *(x,y)$
$*(-(*(x,-(y))),*(x,1)) \to y$
$*(-(x),*(x,y)) \to y$
$*(g(x),-(*(g(x),f(y)))) \to f(-(y))$
$*(-(x),x) \to 1$
$*(-(g(x)),f(y)) \to *(f(y),-(g(x)))$
$*(-(g(*(x,y))),g(x)) \to -(g(y))$
$*(g(x),-(g(*(-(y),x)))) \to g(y)$
$*(g(*(x,y)),-(g(y))) \to g(x)$
$-(g(-(f(x)))) \to g(f(x))$
$-(f(f(x))) \to f(f(-(x)))$
$-(f(-(x))) \to f(x)$
$*(-(f(-(x))),f(*(y,z))) \to f(*(x,*(y,z)))$
$*(f(*(x,y)),-(-(z))) \to *(f(*(x,y)),z)$
$*(-(f(-(x))),*(f(1),y)) \to *(f(x),y)$
$*(-(*(f(x),1)),*(f(*(x,y)),z)) \to *(f(y),z)$
$*(-(f(-(x))),f(1)) \to f(x)$
$*(-(g(-(*(x,-(y))))),g(y)) \to g(*(x,1))$
$*(-(f(1)),f(x)) \to f(x)$
$*(f(x),*(g(y),f(z))) \to *(g(y),f(*(x,z)))$
$*(-(g(x)),g(*(x,y))) \to g(y)$
$*(x,-(-(y))) \to *(x,y)$
$*(*(*(-(1),-(*(x,y))),x),y) \to 1$
$*(f(g(x)),f(g(y))) \to f(g(*(x,y)))$
$*(*(x,y),z) \to *(x,*(y,z))$

$f(g(*(-(x),x))) \to 1$
$-(*(x,f(-(y)))) \to *(f(y),-(x))$
$*(x,*(y,*(-(*(x,y)),z))) \to z$
$*(x,-(*(-(y),x))) \to y$
$-(*(-(x),-(y))) \to *(y,x)$
$*(x,-(1)) \to x$
$f(1) \to 1$
$*(f(*(x,y)),*(-(f(y)),z)) \to *(-(f(-(x))),z)$
$-(f(1)) \to 1$
$*(f(*(-(x),x)),y) \to y$
$*(-(x),*(-(*(y,-(x))),*(y,z))) \to z$
$*(-(*(-(x),1)),1) \to x$
$*(-(-(x)),y) \to *(x,y)$
$*(*(x,-(1)),y) \to *(x,y)$
$*(x,-(-(y))) \to *(x,y)$
$*(*(x,1),y) \to *(x,y)$
$*(g(x),f(y)) \to *(f(y),g(x))$
$*(-(g(x)),g(y)) \to g(*(-(x),y))$
$*(-(g(x)),g(x)) \to 1$
$-(g(*(x,f(-(y))))) \to g(*(f(y),-(x)))$
$*(g(x),-(g(*(y,x)))) \to -(g(y))$
$*(x,-(*(y,x))) \to -(y)$
$-(g(f(f(x)))) \to g(f(f(-(x))))$
$*(-(*(x,y)),x) \to -(y)$
$-(g(f(x))) \to g(f(-(x)))$
$*(-(f(-(x))),y) \to *(f(x),y)$
$*(f(x),*(f(y),-(-(z)))) \to *(f(x),*(f(y),z))$
$*(-(f(-(-(x)))),f(x)) \to f(1)$
$*(-(f(-(-(x)))),*(f(x),y)) \to *(f(1),y)$
$*(-(f(x)),*(f(*(x,y)),z)) \to *(f(y),z)$
$*(-(g(-(x))),g(1)) \to g(x)$
$*(-(g(*(x,-(y)))),g(x)) \to g(y)$
$*(-(g(1)),g(x)) \to g(x)$
$*(x,*(-(g(y)),g(*(y,z)))) \to *(x,g(z))$
$*(-(f(x)),*(g(y),f(x))) \to g(y)$
$*(x,*(y,1)) \to *(x,y)$
$*(f(1),f(x)) \to f(x)$
$*(*(x,g(y)),f(z)) \to *(*(x,f(z)),g(y))$
$*(1,x) \to x$

Example (CGE_2 in practice) iteration 8: 282 equalities

$\mathcal{R}_{8,2}$ reduced:

$$
\begin{array}{rcl}
1 \cdot x \to x & f(x) \cdot f(y) \to f(x \cdot y) & x \cdot (y \cdot z) \to (x \cdot y) \cdot z \\
x \cdot 1 \to x & f(1) \to 1 & (x \cdot y) \cdot y^{-} \to x \\
x^{-} \cdot x \to 1 & f(x)^{-} \to f(x^{-}) & (x \cdot y^{-}) \cdot y \to x \\
x \cdot x^{-} \to 1 & g(x) \cdot g(y) \to g(x \cdot y) & f(x) \cdot (f(y) \cdot z) \to f(x \cdot y) \cdot z \\
1^{-} \to 1 & (x \cdot y)^{-} \to y^{-} \cdot x^{-} & g(x) \cdot (g(y) \cdot z) \to g(x \cdot y) \cdot z \\
x^{--} \to x & g(x)^{-} \to g(x^{-}) & g(x) \cdot (f(y) \cdot z) \to f(x) \cdot (g(y) \cdot z) \\
g(1) \to 1 & g(x) \cdot f(y) \to f(y) \cdot g(x) &
\end{array}
$$

Example (CGE_2 in practice) iteration 8: 282 equalities

$\mathcal{R}_{8,2}$ reduced:

$$
\begin{array}{rcc}
1 \cdot x \to x & \mathsf{f}(x) \cdot \mathsf{f}(y) \to \mathsf{f}(x \cdot y) & x \cdot (y \cdot z) \to (x \cdot y) \cdot z \\
x \cdot 1 \to x & \mathsf{f}(1) \to 1 & (x \cdot y) \cdot y^- \to x \\
x^- \cdot x \to 1 & \mathsf{f}(x)^- \to \mathsf{f}(x^-) & (x \cdot y^-) \cdot y \to x \\
x \cdot x^- \to 1 & \mathsf{g}(x) \cdot \mathsf{g}(y) \to \mathsf{g}(x \cdot y) & \mathsf{f}(x) \cdot (\mathsf{f}(y) \cdot z) \to \mathsf{f}(x \cdot y) \cdot z \\
1^- \to 1 & (x \cdot y)^- \to y^- \cdot x^- & \mathsf{g}(x) \cdot (\mathsf{g}(y) \cdot z) \to \mathsf{g}(x \cdot y) \cdot z \\
x^{--} \to x & \mathsf{g}(x)^- \to \mathsf{g}(x^-) & \mathsf{g}(x) \cdot (\mathsf{f}(y) \cdot z) \to \mathsf{f}(x) \cdot (\mathsf{g}(y) \cdot z) \\
\mathsf{g}(1) \to 1 & \mathsf{g}(x) \cdot \mathsf{f}(y) \to \mathsf{f}(y) \cdot \mathsf{g}(x) &
\end{array}
$$

50 arithmetic and 16K boolean variables, 86K clauses ... 5.6 seconds

Outline

Implementation in Maxcomp

Supported Termination Strategies

- base orders: linear polynomials, KBO, LPO with argument filterings
- DPs, optionally DGs (cycle and SCC variants), followed by arbitrary sequence of reduction pair processors

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Supported Termination Strategies

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Optimizations

- avoid re-submitting same clauses: maintain state of SMT solver throughout completion run, pushing and popping clauses
- to extend active equation set A, add n new critical pairs and m reduced passive equations, where n and m depend on $|A|$
- restarts
- combined auto strategy: start with both LPO and DP strategy, but after first iteration keep only one strategy

Experiments

115 systems from Maxcomp distribution, $\infty := 180$ seconds

	method	#	avg. time	CGE_2	CGE_3	pr	equiv_proofs
(1)	Maxcomp	85	3.8	∞	∞	∞	∞
(2)	DPs	83	14.7	6.4	∞	∞	1.6
(3)	DG	40	2.8	∞	∞	∞	∞
(4)	DG/2SCCs	25	2.5	∞	∞	∞	∞
(5)	auto	92	10.2	5.6	135.1	150.2	1.5
(6)	KBCV	88	4.4	∞	∞	∞	∞
(7)	mkbTT	83	6.1	6.7	45.17	∞	2.0

(1) Maxcomp using LPO
(2) DPs, linear polynomials * 3, LPO + AF
(3) DPs, DG, linear polynomials * 3, LPO + AF
(4) DPs, DG/2 SCCs, linear polynomials * 3, LPO + AF
(5) start with LPO and (2), abandon less successful branch

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`http://cl-informatik.uibk.ac.at/software/maxcompdp/`

Summary

- Maxcomp can complete CGE_2, CGE_3, equiv_proofs, proofreduction
- lightweight DP techniques are feasible, but graphs (currently) not
- with some tweaking, Maxcomp wins some problems over other tools

Future Work

- WPO
- (try to) improve graph encoding
- certifiable output
- more input systems?