Optimizing mkb_{TT}
System Description

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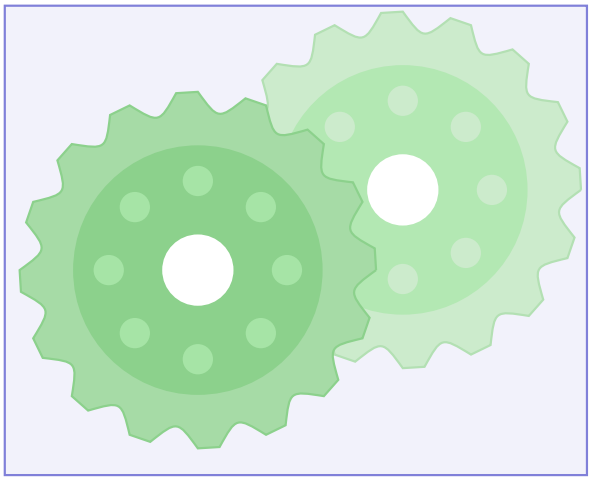
Content

mkb$_{TT}$



mkb$_{TT}$

mkb$_{TT}$ is Knuth-Bendix completion tool

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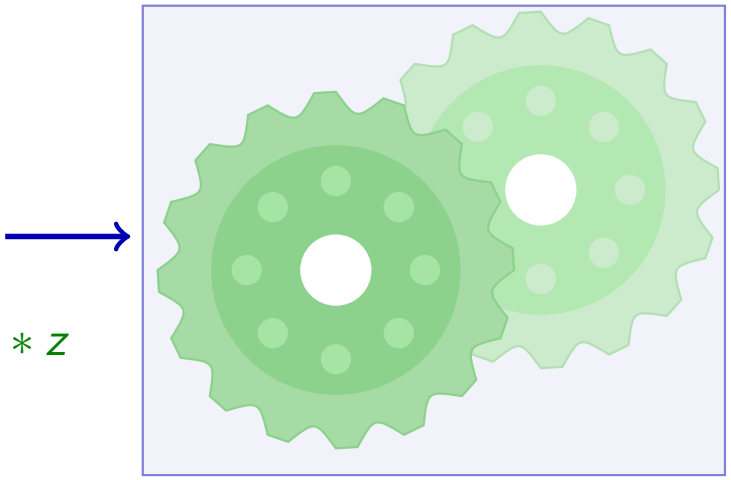
$$x * 1 \approx x$$
$$x * \mathrm{i}(x) \approx 1$$
$$x * (y * z) \approx (x * y) * z$$



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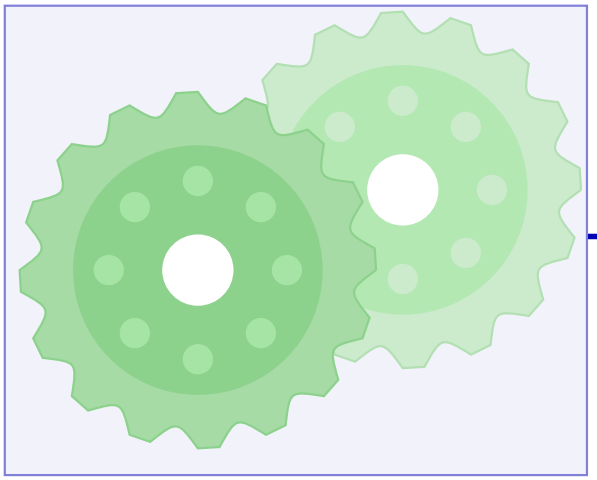

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$$\mathrm{i}(1) \to 1$$
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$$\mathrm{i}(x) * x \to 1$$
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$$(x * y) * z \to x * (y * z)$$
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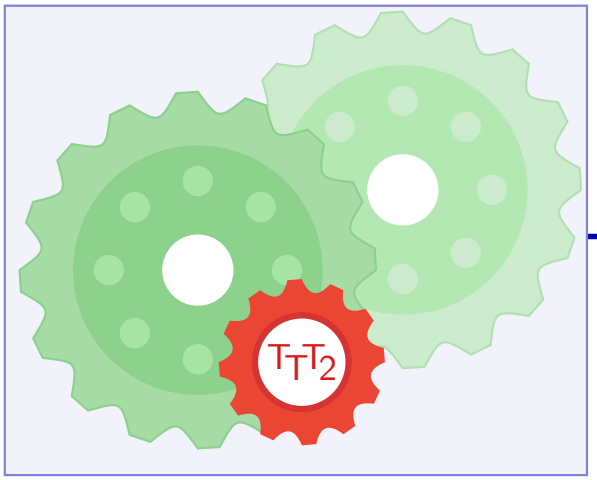

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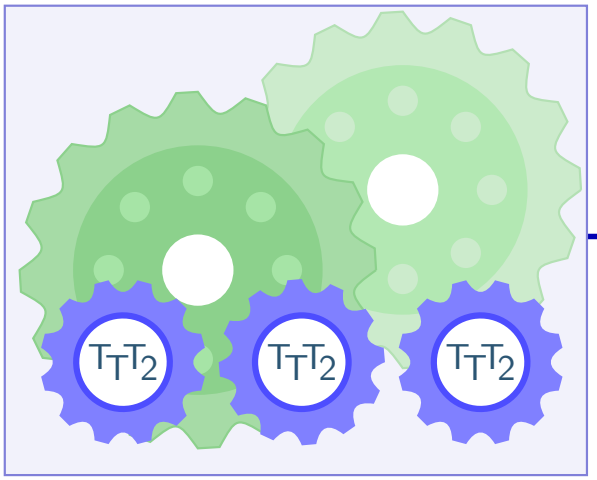

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mkb$_{TT}$ is Knuth-Bendix completion tool combining

- termination tools (*Wehrman, Stump, Westbrook '06*)
  instead of using reduction order
- multi-completion (*Kondo, Kurihara '99*)
  simulating multiple parallel processes

mkb_{TT} in a Nutshell

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- initial node set for equations $\mathcal{E}$ is
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- projection of node set $\mathcal{N}$ to process p yields equations $E_p(\mathcal{N})$, rules $R_p(\mathcal{N})$ and constraints $C_p(\mathcal{N})$

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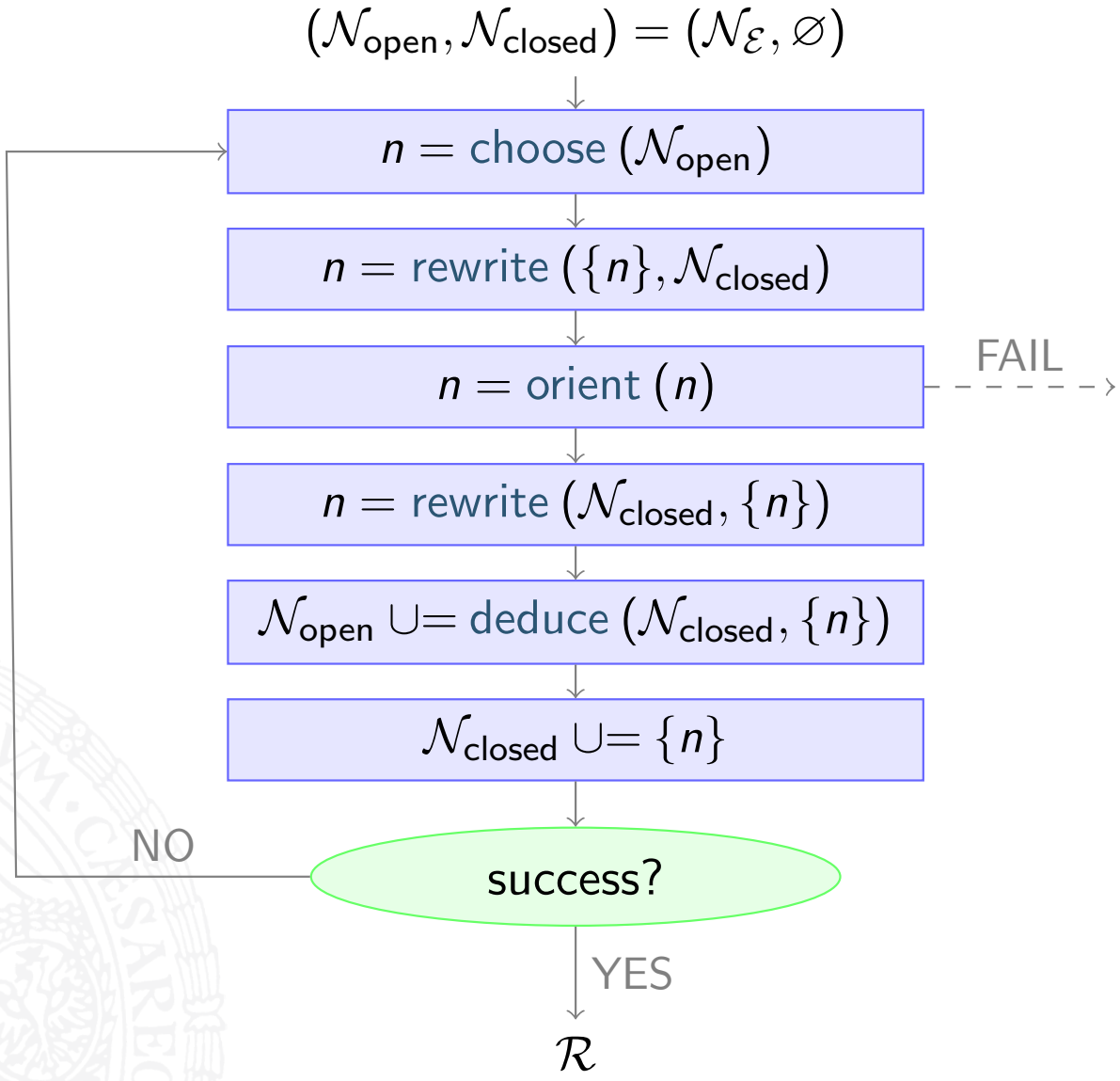
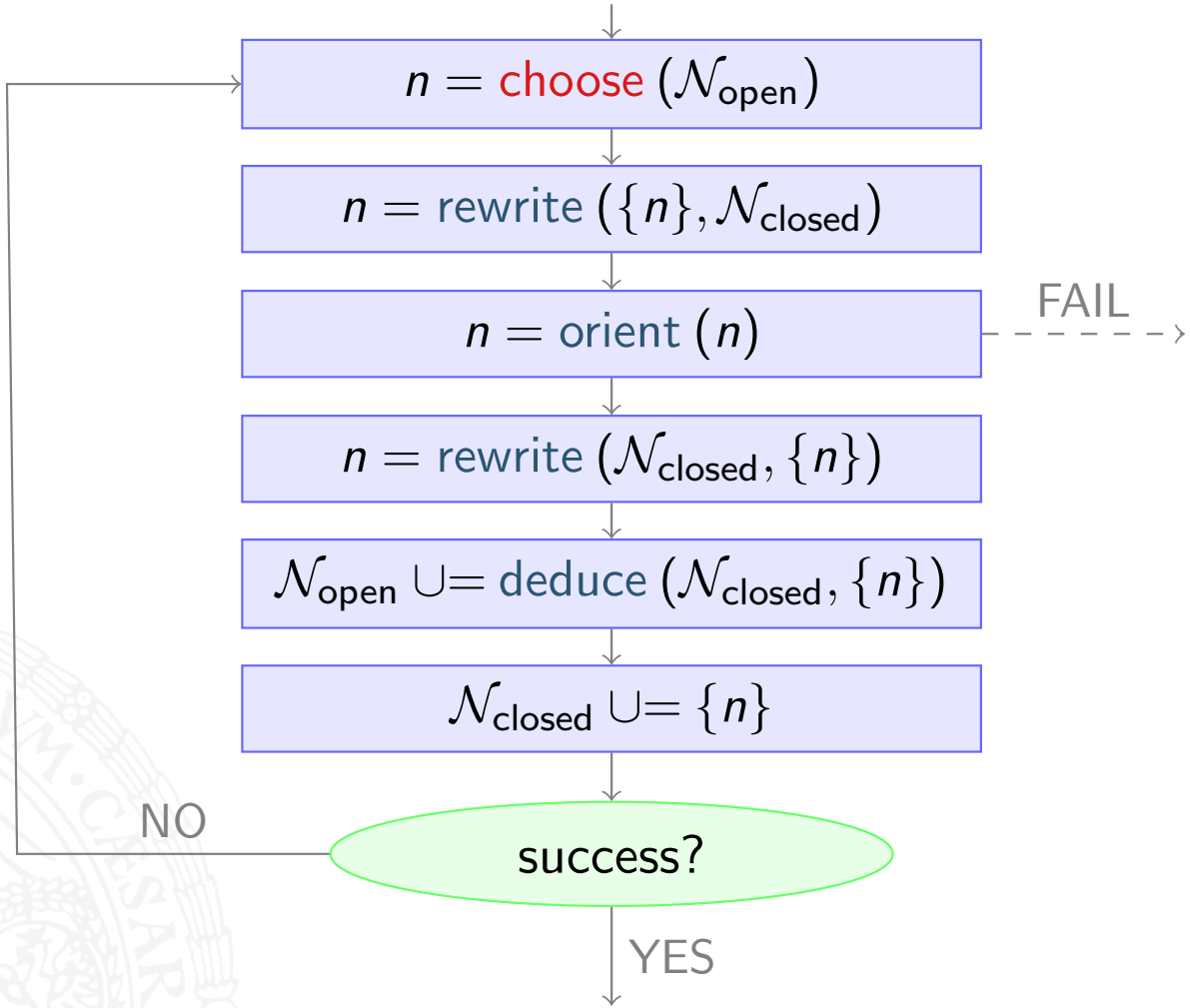
- projection of node set $\mathcal{N}$ to process p yields equations $E_p(\mathcal{N})$, rules $R_p(\mathcal{N})$ and constraints $C_p(\mathcal{N})$
- finite run is fair if all critical pairs with respect to $\mathcal{N}_\omega$ are deduced for some process p

The Control Loop



Improvement 1: Selection Strategies



- mkb_{TT} 1.0:
  - select process for which $|E_p(\mathcal{N})| + |R_p(\mathcal{N})|$ is minimal
  - select mostly small, sometimes old node for process

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- mkb_{TT} 2.0: strategy language

$$\begin{aligned} \textit{strategy} ::= \quad & \texttt{?} \mid \texttt{(}\textit{node\_property}\texttt{,}\textit{strategy}\texttt{)} \\ & \mid \textit{float}\texttt{(}\textit{strategy}\texttt{:}\textit{strategy}\texttt{)} \end{aligned}$$

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- random

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- tuple of node property and strategy, compared lexicographically

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- $r(s_1, s_2)$ combines s_1 and s_2 according to ratio r

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\mathit{node\_property} &::= \ \texttt{*} \mid \texttt{data(}\mathit{termpair\_property}\texttt{)} \mid \texttt{el(}\mathit{pset\_property}\texttt{)}
\end{aligned}
$$

- age

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\mathit{node\_property} & ::= & \texttt{*} \mid \texttt{data(}\mathit{termpair\_property}\texttt{)} \mid \texttt{el(}\mathit{pset\_property}\texttt{)} \\
 & & \mid\texttt{-}\ \mathit{node\_property} \mid \mathit{node\_property}\ \texttt{+}\ \mathit{node\_property} \\
\mathit{pset\_property} & ::= & \texttt{\#} \mid \texttt{sum(}\mathit{process\_property}\texttt{)} \mid \texttt{min(}\mathit{process\_property}\texttt{)} \\
\mathit{process\_property} & ::= & \texttt{e(}\mathit{eqs\_property}\texttt{)} \mid \texttt{r(}\mathit{trs\_property}\texttt{)} \mid \texttt{c(}\mathit{trs\_property}\texttt{)} \\
 & & \mid \mathit{process\_property}\ \texttt{+}\ \mathit{process\_property} \\
\mathit{trs\_property} & ::= & \texttt{sum(}\mathit{termpair\_property}\texttt{)} \mid \texttt{cp(}\mathit{eqs\_property}\texttt{)} \mid \texttt{\#} \\
\mathit{eqs\_property} & ::= & \texttt{sum(}\mathit{termpair\_property}\texttt{)} \mid \texttt{\#} \\
\mathit{termpair\_property} & ::= & \texttt{sizemax} \mid \texttt{sizesum}
\end{array}
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\mathit{termpair\_property} & ::= & \texttt{sizemax} \mid \texttt{sizesum}
\end{array}
$$

Example (size-age ratio)

choose mostly small, sometimes old node:

```
0.9((data(sumsize),?):(*,?))
```

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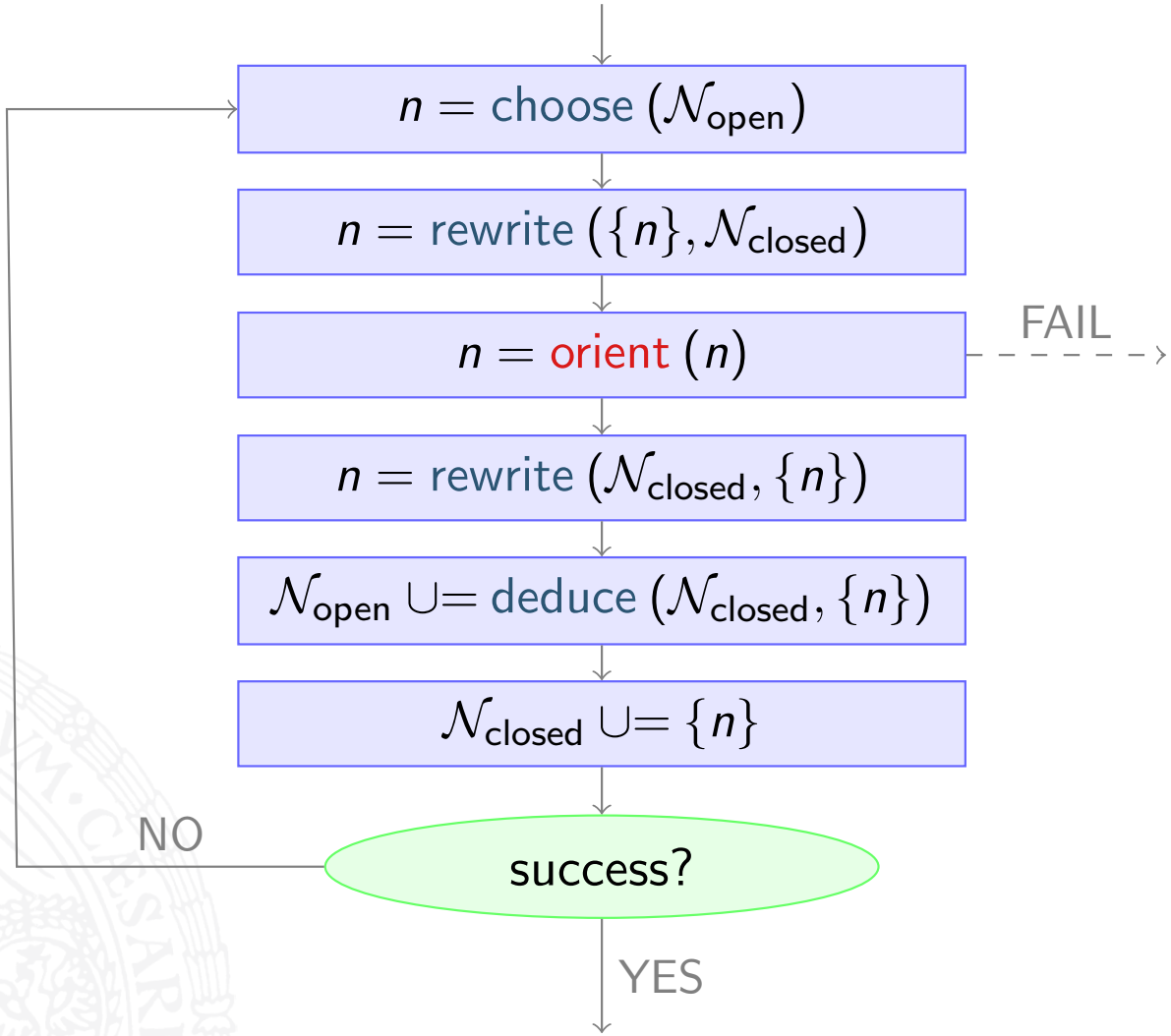
\textit{termpair\_property} & ::= & \texttt{sizemax} \mid \texttt{sizesum}
\end{array}
$$

Example (*sum*)

process with small $E_p(\mathcal{N})$ and $C_p(\mathcal{N})$, node with small terms and big $|E|$

```
(el(min(e(sum(sizesum))+c(sum(sizesum)))),(data(sizesum),(-el(#),?))))
```

Improvement 2: Process Isomorphisms



Example (Renaming Isomorphism)

- mkb_{TT} run on system of commuting group endomorphisms CGE_2

$\mathcal{N}_0$

$$E_\epsilon = \begin{cases} (x * y) * z \approx x * (y * z) & \mathsf{f}(x * y) \approx \mathsf{f}(x) * \mathsf{f}(y) \\ x * 1 \approx x & \mathsf{g}(x * y) \approx \mathsf{g}(x) * \mathsf{g}(y) \\ \mathsf{i}(x) * x \approx 1 & \mathsf{g}(x) * \mathsf{f}(y) \approx \mathsf{f}(y) * \mathsf{g}(x) \end{cases} \qquad R_\epsilon = C_\epsilon = \varnothing$$

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$$\mathcal{N}_0 \vdash \mathcal{N}_1$$

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$$\mathcal{N}_0 \vdash \mathcal{N}_1 \vdash \mathcal{N}_2 \vdash \quad \cdots \quad \vdash \mathcal{N}_i$$

$$E_p = \begin{cases} (x * y) * z \approx x * (y * z) \\ x * 1 \;\; x \\ \mathrm{f}(1) \approx 1 \\ \mathrm{g}(1) \approx 1 \\ \mathrm{g}(x) * \mathrm{f}(y) \approx \mathrm{f}(y) * \mathrm{g}(x) \end{cases} \qquad R_p = C_p = \begin{cases} \mathrm{i}(x) * x \to 1 \\ \mathrm{f}(x * y) \to \mathrm{f}(x) * \mathrm{f}(y) \\ \mathrm{g}(x * y) \to \mathrm{g}(x) * \mathrm{g}(y) \end{cases}$$

Example (Renaming Isomorphism)

- mkb_{TT} run on system of commuting group endomorphisms CGE_2

$$\text{orient}\quad \frac{\langle \mathrm{g}(x) * \mathrm{f}(y) : \mathrm{f}(y) * \mathrm{g}(x), \varnothing, \varnothing, \{p\}, \varnothing, \varnothing \rangle}{}$$

$$\mathcal{N}_0 \vdash \mathcal{N}_1 \vdash \mathcal{N}_2 \vdash \quad \cdots \quad \vdash \mathcal{N}_i \vdash$$

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$$\mathcal{N}_0 \vdash \mathcal{N}_1 \vdash \mathcal{N}_2 \vdash \quad \cdots \quad \vdash \mathcal{N}_i \vdash$$

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orient $\dfrac{\langle \mathsf{g}(x) * \mathsf{f}(y) : \mathsf{f}(y) * \mathsf{g}(x), \varnothing, \varnothing, \{p\}, \varnothing, \varnothing\rangle}{\langle \mathsf{g}(x) * \mathsf{f}(y) : \mathsf{f}(y) * \mathsf{g}(x), \{p0\}, \{p1\}, \varnothing, \varnothing, \varnothing\rangle}$

$$\mathcal{N}_0 \vdash \mathcal{N}_1 \vdash \mathcal{N}_2 \vdash \quad \cdots \quad \vdash \mathcal{N}_i \vdash \mathcal{N}_{i+1}$$

$$E_{p0} = \begin{cases} (x * y) * z \approx x * (y * z) \\ x * 1 \approx x \\ \mathsf{f}(1) \approx 1 \\ \mathsf{g}(1) \approx 1 \end{cases} \qquad R_{p0} = C_{p0} = \begin{cases} \mathsf{i}(x) * x \to 1 \\ \mathsf{f}(x * y) \to \mathsf{f}(x) * \mathsf{f}(y) \\ \mathsf{g}(x * y) \to \mathsf{g}(x) * \mathsf{g}(y) \\ \mathsf{g}(x) * \mathsf{f}(y) \to \mathsf{f}(y) * \mathsf{g}(x) \end{cases}$$

$$E_{p1} = \begin{cases} (x * y) * z \approx x * (y * z) \\ x * 1 \approx x \\ \mathsf{f}(1) \approx 1 \\ \mathsf{g}(1) \approx 1 \end{cases} \qquad R_{p1} = C_{p1} = \begin{cases} \mathsf{i}(x) * x \to 1 \\ \mathsf{f}(x * y) \to \mathsf{f}(x) * \mathsf{f}(y) \\ \mathsf{g}(x * y) \to \mathsf{g}(x) * \mathsf{g}(y) \\ \mathsf{f}(y) * \mathsf{g}(x) \to \mathsf{g}(x) * \mathsf{f}(y) \end{cases}$$

Example (Renaming Isomorphism)

- mkb_{TT} run on system of commuting group endomorphisms CGE_2

$$\mathcal{N}_0 \vdash \mathcal{N}_1 \vdash \mathcal{N}_2 \vdash \quad \cdots \quad \vdash \mathcal{N}_i \vdash \mathcal{N}_{i+1}$$

$$E_{p0} = \begin{cases} (x * y) * z \approx x * (y * z) \\ x * 1 \approx x \\ \mathrm{f}(1) \approx 1 \\ \mathrm{g}(1) \approx 1 \end{cases} \qquad R_{p0} = C_{p0} = \begin{cases} \mathrm{i}(x) * x \to 1 \\ \mathrm{f}(x * y) \to \mathrm{f}(x) * \mathrm{f}(y) \\ \mathrm{g}(x * y) \to \mathrm{g}(x) * \mathrm{g}(y) \\ \mathrm{g}(x) * \mathrm{f}(y) \to \mathrm{f}(y) * \mathrm{g}(x) \end{cases}$$

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- states of $p0$ and $p1$ are identical up to renaming function symbols

Definition

$$\mathcal{R} \cong_\theta \mathcal{R}'$$

i.e., rewrite systems $\mathcal{R}, \mathcal{R}'$ are isomorphic via $\theta \colon \mathcal{T}(\mathcal{F}, \mathcal{V}) \to \mathcal{T}(\mathcal{F}, \mathcal{V})$ if

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Lemma

Assume p, q are isomorphic in $\mathcal{N}$

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Lemma

Assume p, q are isomorphic in $\mathcal{N}$ and $\exists$ fair $\mathcal{N} \vdash^* \mathcal{N}'$ with $E_p(\mathcal{N}') = \varnothing$.

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Lemma

Assume p, q are isomorphic in $\mathcal{N}$ and $\exists$ fair $\mathcal{N} \vdash^* \mathcal{N}'$ with $E_p(\mathcal{N}') = \varnothing$.
Then $\exists\, \mathcal{N}''$ such that $\mathcal{N} \vdash^* \mathcal{N}''$ is fair and $E_q(\mathcal{N}'') = \varnothing$.

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- for $f \in \mathcal{F}$ with arity $n > 0$ choose permutation π_f of $\{1, \ldots, n\}$

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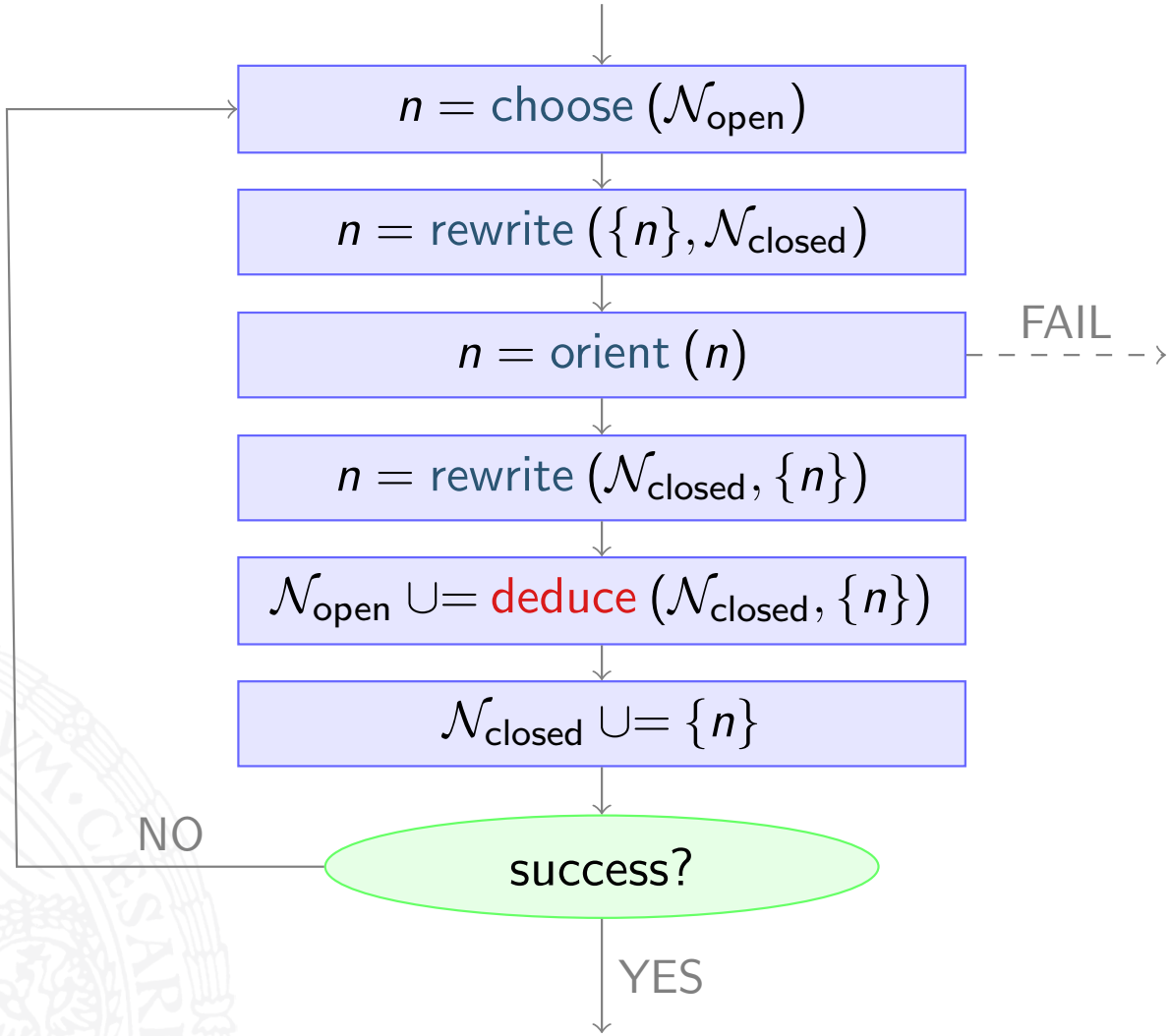
Implementation

- check before process splits in orient, or repeatedly

Improvement 3: Critical Pair Criteria



Definition (Deduce in mkb_{TT})

$$\text{deduce} \quad \frac{\mathcal{N}}{\text{if } \langle l : r, R, \ldots \rangle, \langle l' : r', R', \ldots \rangle \in \mathcal{N}}$$

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deduce $\dfrac{\mathcal{N}}{}$

if $\langle l : r, R, \ldots \rangle, \langle l' : r', R', \ldots \rangle \in \mathcal{N}$
such that $s \xleftarrow{l \to r} u \xrightarrow{l' \to r'} t$ and $s \approx t$ is critical pair

Definition (Deduce in mkb_{TT})

deduce
$$\frac{\mathcal{N}}{\mathcal{N} \cup \langle s : t, \varnothing, \varnothing, R \cap R', \varnothing, \varnothing, \varnothing \rangle}$$

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Example

$\mathcal{N}$:

$$\langle \sqrt{-x+x} : 0,\ \{0,1\}, \ldots \rangle \quad (1)$$
$$\langle -0+0 : 0,\ \{0,1\}, \ldots \rangle \quad (2)$$
$$\langle -0 : 0,\ \{0,1\}, \ldots \rangle \quad (3)$$

$\mathrm{CP}(\mathcal{N})$:

$$\langle \sqrt{0} : 0,\ \varnothing, \varnothing,\ \{0,1\}, \ldots \rangle \quad \text{from } \langle (1), 1, (2) \rangle$$
$$\langle \sqrt{0+0} : 0,\ \varnothing, \varnothing,\ \{0,1\}, \ldots \rangle \quad \text{from } \langle (1), 11, (3) \rangle$$
$$\langle \sqrt{0} : 0,\ \varnothing, \varnothing,\ \{0,1\}, \ldots \rangle \quad \text{from } \langle (2), 1, (3) \rangle$$

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Example

all critical pairs required?

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Critical Pair Criteria in mkb_{TT}

- primality criterion PCP — Kapur et al '88
- blocking criterion BCP — Bachmair/Dershowitz '88
- connectedness criterion CCP — Küchlin '85

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if $\langle l : r, R, \ldots \rangle, \langle l' : r', R', \ldots \rangle \in \mathcal{N}$
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Critical Pair Criteria in mkb_{TT}

exploit sharing

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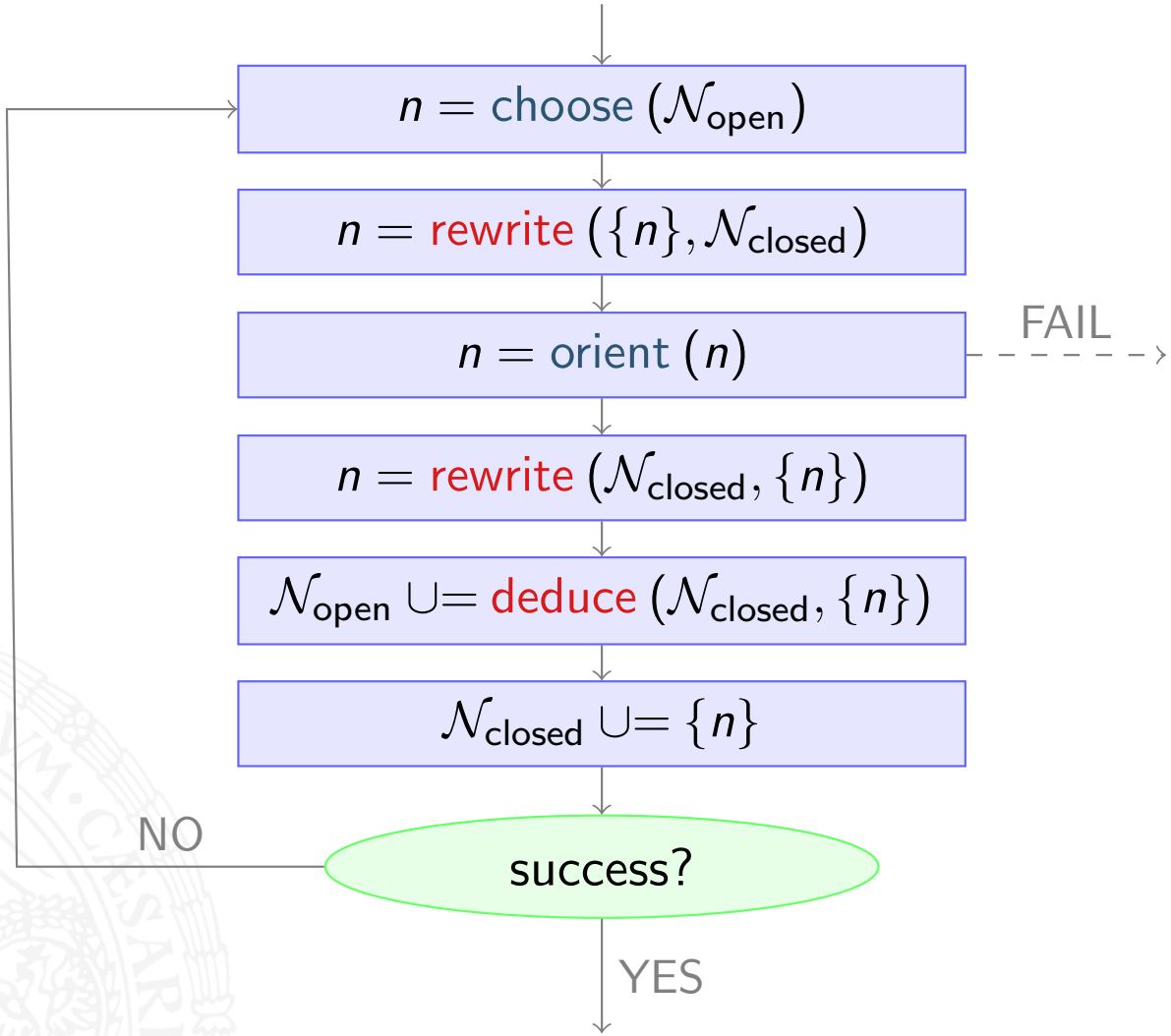
- blocking criterion BCP — Bachmair/Dershowitz '88
- connectedness criterion CCP — Küchlin '85

Improvement 4: Term Indexing



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Given

- set of terms L
- binary relation R on terms
- term t

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identify all $s \in L$ with $s \; R \; t$

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- term t — query term

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- retrieval of variants and encompassments in rewrite_1 and rewrite_2

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Example

mkb_{TT} faces term indexing problem for

- retrieval of variants and encompassments in rewrite_1 and rewrite_2
- retrieval of unifiable terms in deduce

Implementation

- path indexing and discrimination trees for unifiable terms
- additionally also code trees for encompassments and variants

Experiments

hardware:	AMD Opteron® 885, 2.6GHz, 64GB memory
database:	101 systems collected from various papers
settings:	600 seconds timeout, termination checks with $T_T T_2$

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New!

First automatical completion of CGE_4 system

$$\begin{aligned} \mathrm{e} \cdot x &\approx x \\ x^{-} \cdot x &\approx \mathrm{e} \\ (x \cdot y) \cdot z &\approx x \cdot (y \cdot z) \end{aligned} \qquad \begin{aligned} \mathrm{f_i}(x \cdot y) &\approx \mathrm{f_i}(x) \cdot \mathrm{f_i}(y) \quad 1 \leq i \leq 4 \\ \mathrm{f_i}(x) \cdot \mathrm{f_j}(y) &\approx \mathrm{f_j}(y) \cdot \mathrm{f_i}(x) \quad 1 \leq i < j \leq 4 \end{aligned}$$

into a 38 rule convergent TRS in 622 seconds

Selection Strategies

	sum	max	slothrop	old
CGE_2	138	9	16	8

time in seconds

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SK3.4	75	2	3	38
GRP484-1	252	∞	∞	∞

time in seconds

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Isomorphisms

- renaming isomorphisms

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Isomorphisms

- renaming isomorphisms
  - CGE_2: 4 instead of 138 seconds, CGE_3: 30 instead of 192 seconds

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time in seconds

Isomorphisms

- renaming isomorphisms
  - CGE_2: 4 instead of 138 seconds, CGE_3: 30 instead of 192 seconds
  - on database: number of processes and time decreased by 15%
- argument permutations
  - no improvement

Critical Pair Criteria

	none	PCP		BCP		CCP		all	
	(1)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
Chr89-A_2	126	133	70	134	51	168	25	137	75
GRP463-1	8	5	24	7	24	9	9	6	27

(1) time in seconds (2) redundant critical pairs for successful process

Critical Pair Criteria

	none	PCP		BCP		CCP		all	
	(1)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
Chr89-A_2	126	133	70	134	51	168	25	137	75
GRP463-1	8	5	24	7	24	9	9	6	27
BGK94-D_8	∞	550	28	550	28	∞		549	28

(1) time in seconds (2) redundant critical pairs for successful process

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WS06-1	138	139	0	140	0	139	0	138	0

(1) time in seconds (2) redundant critical pairs for successful process

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WS06-1	138	139	0	140	0	139	0	138	0
⋮	⋮	⋮		⋮		⋮		⋮	
successes	70	71		71		70		71	

(1) time in seconds (2) redundant critical pairs for successful process

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(1) time in seconds (2) redundant critical pairs for successful process

Term Indexing

percentage of retrieval time compared to naive search

	path indexing	discrimination trees	code trees
encompassments	89	39	27
variants	19	6	6
unifiable terms	90	30	

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⋮	⋮	⋮		⋮		⋮		⋮	
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(1) time in seconds (2) redundant critical pairs for successful process

Term Indexing

percentage of retrieval time compared to naive search

	path indexing	discrimination trees	code trees
encompassments	89	39	27
variants	19	6	6
unifiable terms	90	30	
execution time	95	83	78

Conclusion

mkb_{TT} is automatic completion tool with

- indexing techniques (pay off)
- selection strategies (considerable impact – optimal one?)
- critical pair criteria (tiny improvements)
- isomorphisms (renamings are useful for special systems)

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mkb_{TT} Online

various options can be controlled via web interface:
http://cl-informatik.uibk.ac.at/software/mkbtt