Computational Logic

Certification of Nontermination Proofs using Strategies and Nonlooping Derivations

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University of Innsbruck

VSTTE, July 18, 2014

Outline

Motivation

Approach and Techniques

Forbidden Pattern Loops
- The Theory Side
- The IsaFoR Side
- The Tool and Certification Sides

Experiments and Conclusion

Why Certify Nontermination?

Example (Buggy map function)

term rewrite system (TRS) $\mathcal{R}$

$$\mathsf{map}\ f\ xs \to \mathsf{if}\ (\mathsf{empty}\ xs)\ \mathsf{nil}\ (:\ (f\ (\mathsf{hd}\ xs))\ (\mathsf{map}\ f\ xs))$$

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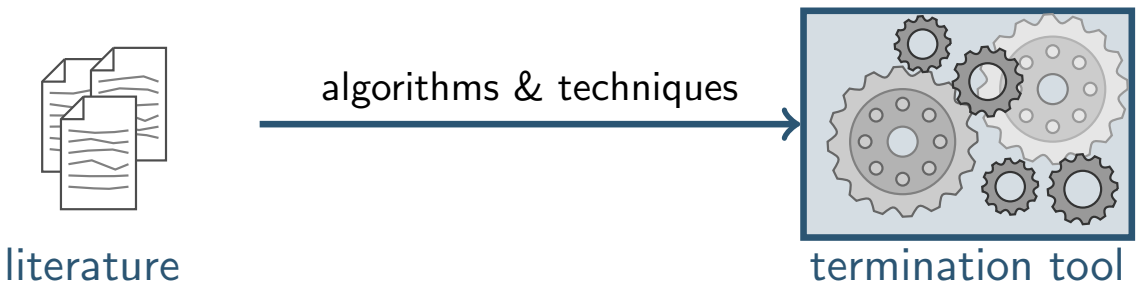
- but termination tools are complex and error-prone: certification can guarantee correctness of proofs

Approach Outline



Approach Outline

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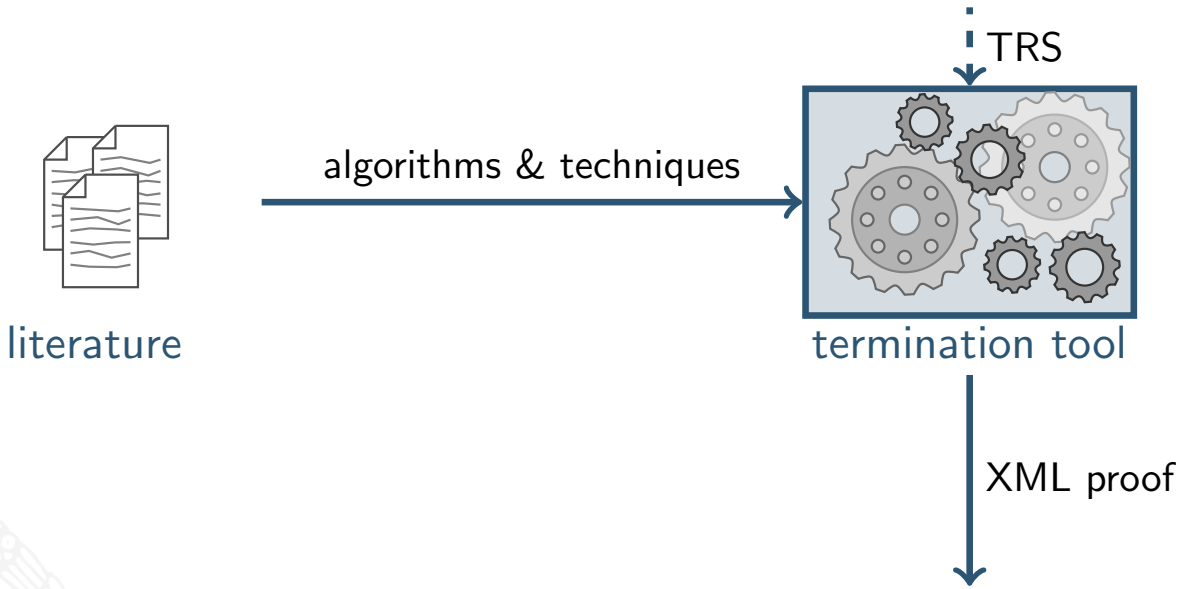

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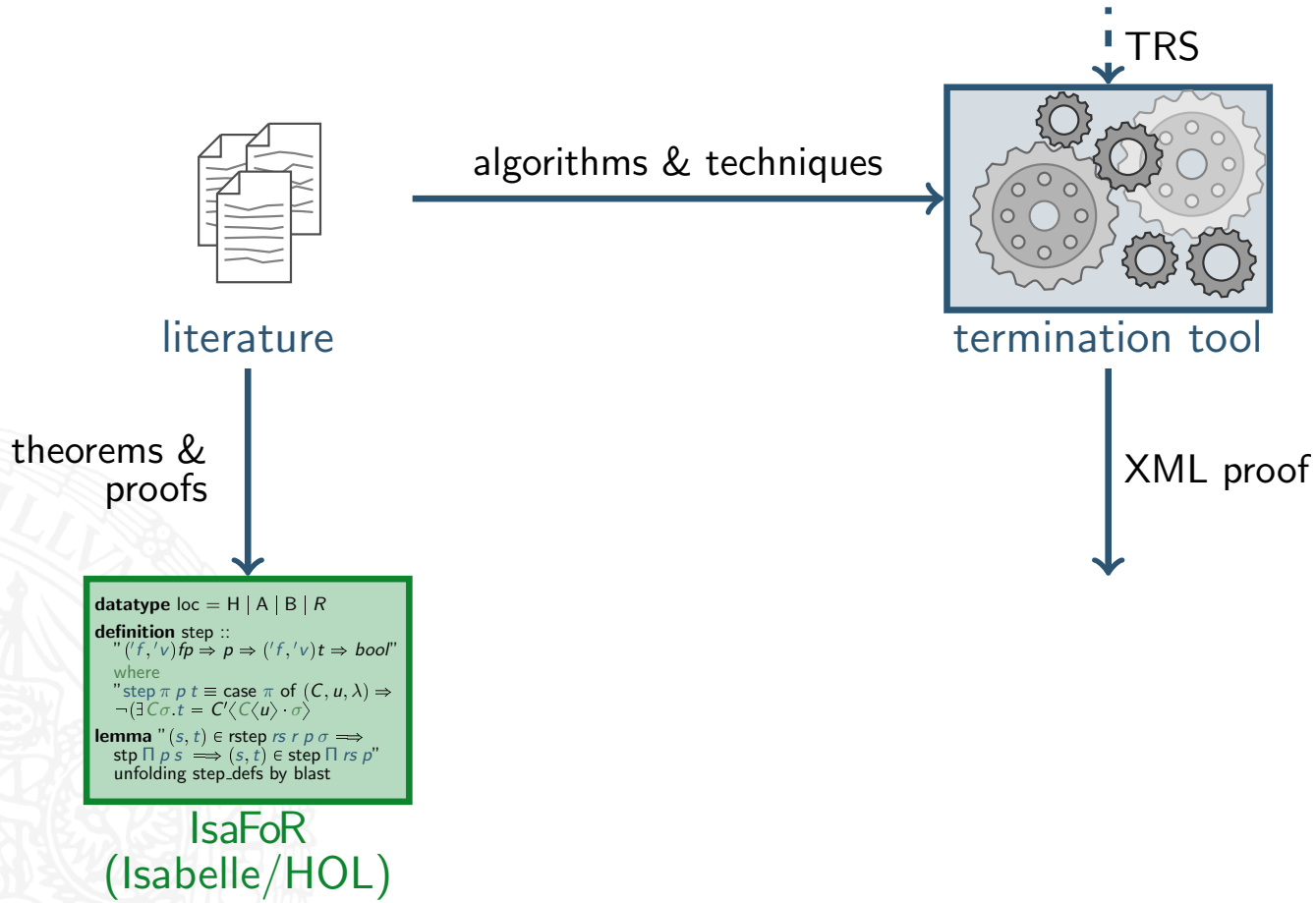
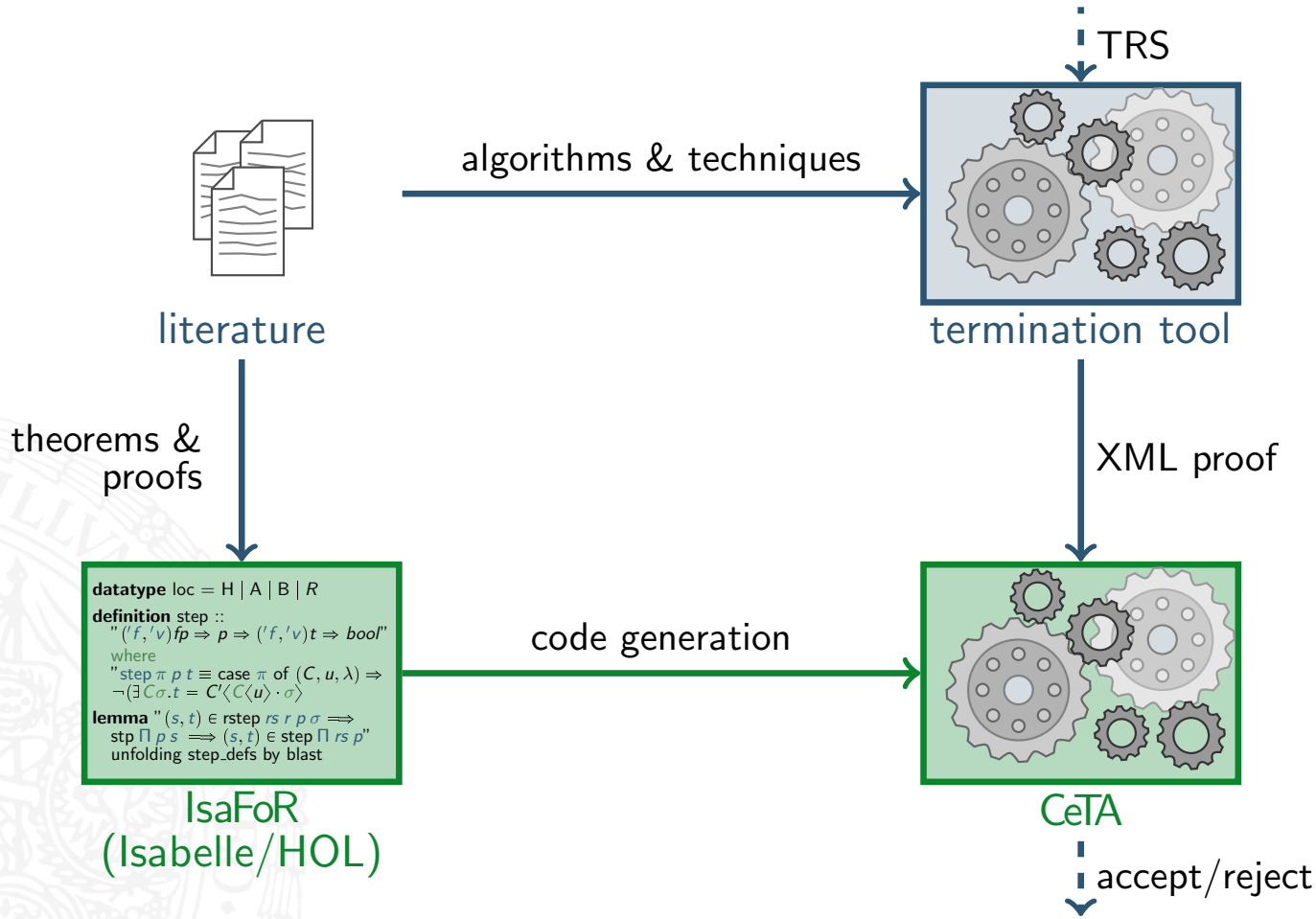
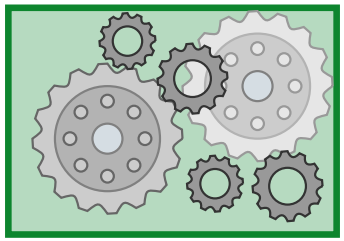


Approach Outline

- termination tools provide evidence of nontermination
- CeTA automatically certifies correctness of evidence



CeTA's Nontermination Machinery 2013



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R. Thiemann and C. Sternagel.
Certification of Nontermination Proofs.
Proc. ITP 2012, pp. 266–282, 2012.

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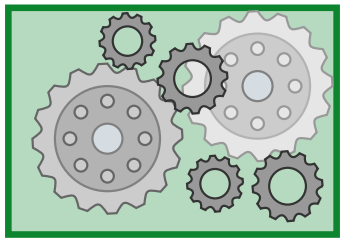

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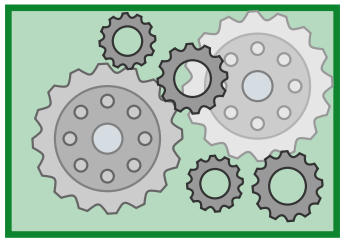
R. Thiemann, C. Sternagel, J. Giesl, and P. Schneider-Kamp.
Loops under Strategies ... Continued.
Proc. IWS 2010, pp. 51–65, 2010.

CeTA's Nontermination Machinery 2014



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B. Gramlich.

Abstract Relations between Restricted Termination and Confluence Properties of Rewrite Systems.
Fundamenta Informaticae, 24:3–23, 1995.

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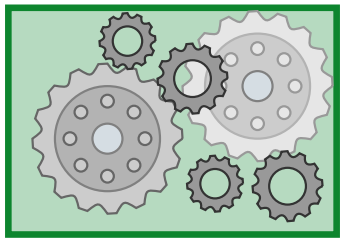

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J. Giesl, R. Thiemann, P. Schneider-Kamp, and S. Falke.
Mechanizing and Improving Dependency Pairs.
Journal of Automated Reasoning, 37(3):155–203, 2006.

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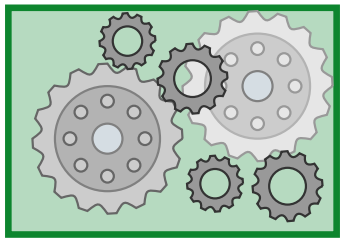

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F. Emmes, T. Enger, and J. Giesl.
Proving non-looping non-termination automatically.
Proc. IJCAR 2012, pp. 225–240, 2012.

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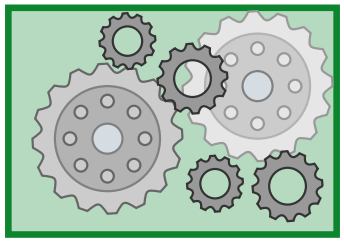

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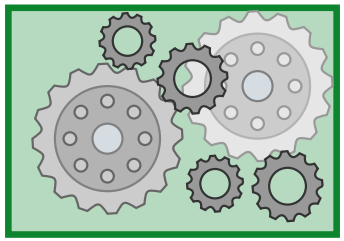

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We want to check loops in presence of strategies!

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intended evaluation strategy for if is obtained by

$$\Pi = \{(\text{if } b\ t\ e, p, \lambda) \mid p \in \{12, 2\} \text{ and } \lambda \in \{\mathsf{H}, \mathsf{B}\}$$

Looking At Loops

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Rewrite sequence $t_1 \to_{q_1} t_2 \to_{q_2} \cdots \to_{q_{l-1}} t_l$ is loop if $t_l = C[t_1\mu]$.

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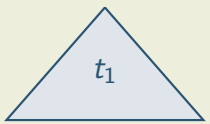

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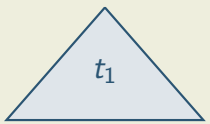

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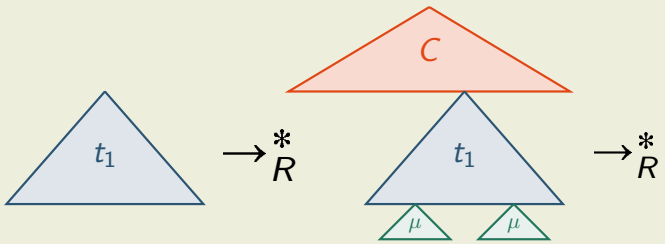

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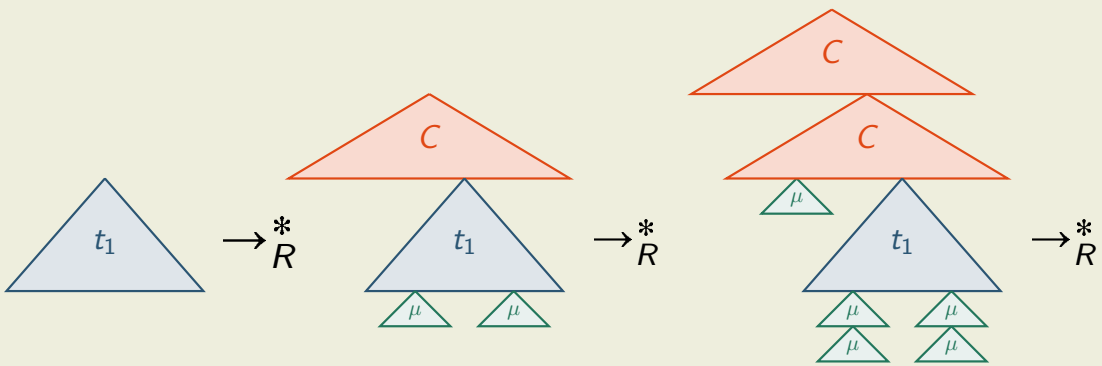

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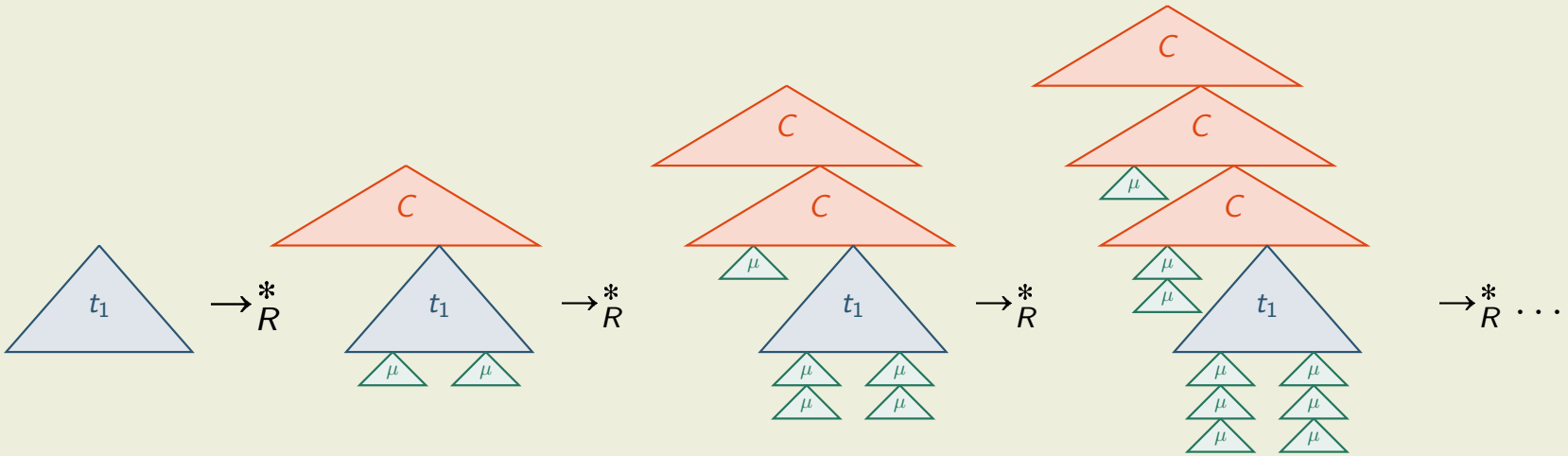

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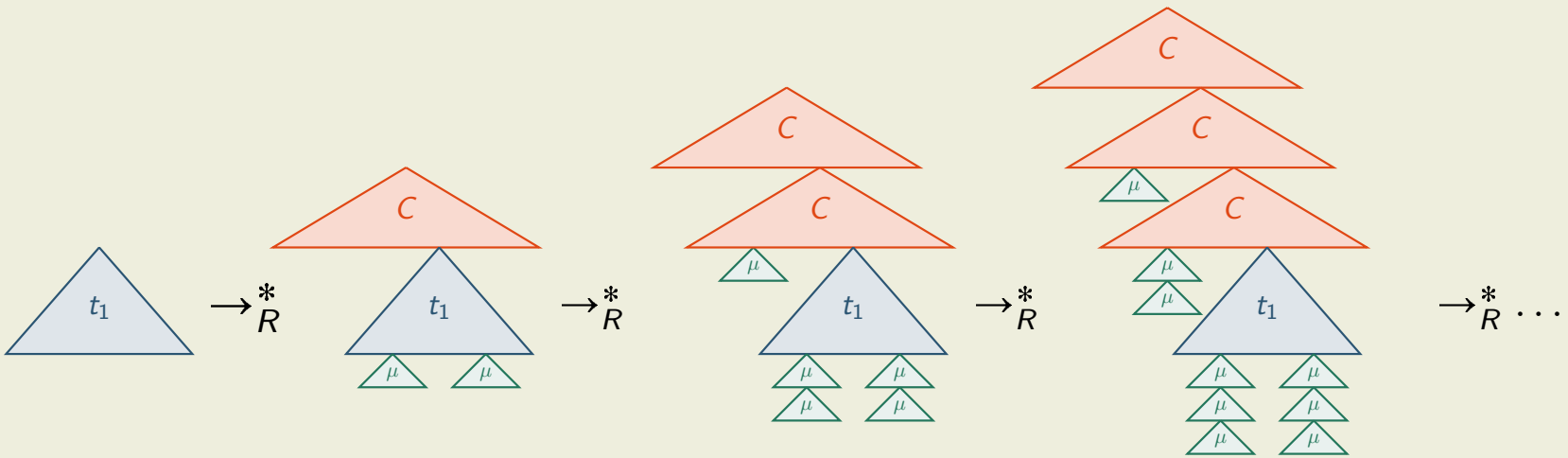

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Definition (Π-Loop)

Loop is Π-loop if every step in infinite rewrite sequence is valid Π-step.

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Rewrite sequence $t_1 \to_{q_1} t_2 \to_{q_2} \cdots \to_{q_{l-1}} t_l$ is loop if $t_l = C[t_1\mu]$.

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Loop gives rise to infinite rewrite sequence

$$t_1 \to_R^* t_1(C,\mu)^1 \to_R^* t_1(C,\mu)^2 \to_R^* t_1(C,\mu)^3 \to_R^* \ldots$$

Definition (Π-Loop)

Loop is Π-loop if every step in infinite rewrite sequence is valid Π-step.

... to check whether loop constitutes Π-loop:
check every step $t_i \to t_{i+1}$ against every $\pi \in \Pi$

Decision Problem (Case H)

($\star$) Given loop step $t_i \to_q t_{i+1}$ with $C|_p = \Box$ and pattern (ℓ, o, H):
are there n, o', σ such that $t_i(C, \mu)^n|_{o'} = \ell\sigma$ and $p^n q = o' o$?

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Solvability of mat

similarly, cases for A, B, and R can be decided
using appropriate sets of (extended) matching problems

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- forbidden pattern loops cover in particular loops for innermost, outermost, context-sensitive and leftmost strategies
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3 Phases of Solving Procedure

1. transform to solved form
2. build respective (extended) identity problems
3. solve (extended) identity problems

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Phase 1: Transformation to Solved Form

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In IsaFoR: Using Isabelle's `function` Package

challenge: get both termination and efficiency

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i.e., do not recompute $\mathcal{V}_{\mathsf{incr},\mu}$ in every iteration

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In IsaFoR: Using Isabelle's `function` Package

challenge: get both termination and efficiency

- previously: two functions
- new: pass (μ, V_i) as argument, encapsulated in type with invariant $V_i = \mathcal{V}_{\mathrm{incr},\mu}$, and provide selectors to both μ and V_i
  quotient construction with `lifting & transfer` package

The Tool Side

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The Certification Side

Demo

Outline

Motivation

Approach and Techniques

Forbidden Pattern Loops
- The Theory Side
- The IsaFoR Side
- The Tool and Certification Sides

Experiments and Conclusion

Experiments

Testbed

all 596 first-order TRSs from TPDB 8.0.7 where at least one tool generated nontermination proof in termination competition 2013

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- AProVE'13 and T$_T$T$_2$'13 are versions restricted to (non)termination techniques that could be certified by CeTA in 2013
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Results

	AProVE'13	AProVE'14
# nontermination proofs	276	575
# certified proofs	276	563

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# nontermination proofs	276	575	221	417
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Results

certify all steps which apply known techniques

			T$_T$T$_2$'13	T$_T$T$_2$'14
# nontermination pr			221	417
# certified proofs	276	563	221	417
# partially certified proofs	–	12	–	–

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Formalization & Details

`http://cl-informatik.uibk.ac.at/software/ntcert/`