A Ground Joinability Criterion for Ordered Completion*

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Abstract

By Newman's Lemma and the Extended Critical Pair Lemma, ordered completion tools can establish ground confluence by ensuring ground joinability of all extended critical pairs. We present a new criterion for ground joinability over a given signature that extends the test by Martin and Nipkow (1990). The criterion is particularly useful when a suitable reduction order is to be discovered by means of a SAT/SMT-based search.

1 Introduction

Ordered completion [3] has been a highly influential calculus in automated deduction as it is refutationally complete for equational theorem proving. If no goal is supplied or the goal cannot be proven, ordered completion attempts to derive a presentation of the equational theory that is ground complete, i.e., terminating and ground confluent.

In theorem proving, ground completeness was mostly considered over a signature $\mathcal{F}^{\mathcal{C}}$ that extends the signature $\mathcal{F}$ of the (finite) input problem by infinitely many constants. Naturally ground completeness over $\mathcal{F}^{\mathcal{C}}$ implies ground completeness over $\mathcal{F}$, but the following example shows that the reverse does not hold.

Example 1. The rewrite system $\mathcal{R}$ consisting of the three rules

$$\mathsf{f}(\mathsf{g}(\mathsf{f}(x))) \to \mathsf{g}(\mathsf{f}(\mathsf{g}(x))) \qquad \mathsf{f}(\mathsf{a}) \to \mathsf{a} \qquad \mathsf{g}(\mathsf{a}) \to \mathsf{a}$$

is terminating and ground confluent over the signature $\mathcal{F} = \{\mathsf{f}, \mathsf{g}, \mathsf{a}\}$ as every ground term rewrites to a. But in presence of an additional constant c there is the non-joinable peak $\mathsf{g}(\mathsf{f}(\mathsf{g}(\mathsf{g}(\mathsf{f}(\mathsf{c}))))) \leftarrow \mathsf{f}(\mathsf{g}(\mathsf{f}(\mathsf{g}(\mathsf{f}(\mathsf{c}))))) \to \mathsf{f}(\mathsf{g}(\mathsf{g}(\mathsf{f}(\mathsf{g}(\mathsf{c}))))))$, so $\mathcal{R}$ is not ground confluent over $\mathcal{F}^{\mathcal{C}}$.

It is actually often sufficient if a system is ground complete for a fixed, finite signature $\mathcal{F}$, e.g., to witness satisfiability of a goal. However, this comes at the price of a profound difference with respect to computability: while ground confluence of ordered rewriting is decidable over $\mathcal{F}^{\mathcal{C}}$ for a large class of reduction orders [4] it is undecidable over $\mathcal{F}$, despite termination [5].

By Newman's Lemma and the Extended Critical Pair Lemma [3] ground confluence of a terminating system can be established by ensuring that all extended critical pairs are ground joinable. In this paper we propose a new ground joinability criterion for this setting that extends the test proposed by Martin and Nipkow [8]. It is strictly more powerful as far as ground completeness over the original signature $\mathcal{F}$ is concerned. In particular, our criterion produces constraints on the reduction order that are necessary for ground joinability. Using the fact that satisfiability of LPO and KBO constraints is in NP [9, 7], we exploit this test in the ordered completion tool MædMax, where maxSAT/maxSMT guides the search for a reduction order that admits a ground-complete presentation.

The remainder of this paper is organized as follows. In Section 2 we recall some relevant notions. In Section 3 the ground joinability criterion is explained. In Section 4 we add some remarks on our implementation and conclude.

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2 Preliminaries

In the sequel standard no(ta)tions from term rewriting are used [2]. We consider the set of (ground) terms $\mathcal{T}(\mathcal{F},\mathcal{V})$ ($\mathcal{T}(\mathcal{F})$) over a finite signature $\mathcal{F}$ that is assumed to contain a constant. The signature $\mathcal{F}^{\mathcal{C}}$ extends $\mathcal{F}$ by an infinite set of constants $\mathcal{C}$. A substitution σ is *$\mathcal{F}$-grounding* for a set of variables V if $\sigma(x) \in \mathcal{T}(\mathcal{F})$ for all $x \in V$, and $\mathcal{F}$-grounding for a term t if it is $\mathcal{F}$-grounding for $\mathcal{V}\mathrm{ar}(t)$. Given a TRS $\mathcal{R}$, terms s and t are *ground joinable on $\mathcal{F}$*, denoted $s \Downarrow_{\mathcal{F}} t$, if $s\sigma \downarrow_{\mathcal{R}} t\sigma$ holds for all $\mathcal{F}$-grounding substitutions σ. A substitution σ is $\mathcal{R}$-reducible if $\sigma(x)$ is $\mathcal{R}$-reducible for some $x \in \mathcal{D}\mathrm{om}(\sigma)$.

We consider simplification orders $>$ that are ground total. (Any ground-total reduction order can be extended to enjoy the subterm property [2]). These properties are for instance satisfied by lexicographic path orders (LPO) and Knuth-Bendix orders (KBO) with a total precedence [2]. *Ordered rewriting* is a key notion for ordered completion: For a set of equations $\mathcal{E}$ and a reduction order $>$, the (infinite) TRS $\mathcal{E}^{>}$ consists of all rewrite rules $\ell\sigma \to r\sigma$ such that $\ell \approx r \in \mathcal{E} \cup \mathcal{E}^{-1}$ and $\ell\sigma > r\sigma$. A TRS $\mathcal{R}$ is *ground confluent* with respect to $\mathcal{F}$ if $s \;{}_{\mathcal{R}}^{*}\!\!\leftarrow \cdot \to_{\mathcal{R}}^{*} t$ implies $s \to_{\mathcal{R}}^{*} \cdot {}_{\mathcal{R}}^{*}\!\!\leftarrow t$ for all $s,t \in \mathcal{T}(\mathcal{F})$. A terminating TRS $\mathcal{R}$ is ground confluent if the set of extended critical pairs $\mathrm{CP}_{>}(\mathcal{R})$ are ground joinable [3].

3 A Criterion for Ground Joinability

We build on the idea by Martin and Nipkow [8] to perform a case distinction by considering ordered rewriting using all extensions of $>$ that reflect possible combinations of $\sigma(x) > \sigma(y)$, $\sigma(y) > \sigma(x)$ or $\sigma(x) = \sigma(y)$ for $x, y \in \mathcal{V}$ and a grounding substitution σ.

Suppose $\mathcal{F}$ contains the signature of the input problem, and $>$ is an $\mathcal{F}$-ground-total simplification order. Let O be a set of *ordering constraints*, i.e., a set of variable pairs $x >_{\mathsf{O}} y$ or $x =_{\mathsf{O}} y$. Let $\succeq_{\mathsf{O}}$ be a quasi-order on $\mathcal{T}(\mathcal{F},\mathcal{V})$ that contains $>$ and O, and $\succ_{\mathsf{O}}$ be its strict part. For a substitution σ that is $\mathcal{F}$-grounding on $\mathcal{V}\mathrm{ar}(\mathsf{O})$ and a set of ordering constraints O, the quasi-order $\succeq_{\mathsf{O}}$ *covers* σ if $\sigma(x) > \sigma(y)$ ($\sigma(x) = \sigma(y)$) holds for all $x >_{\mathsf{O}} y$ ($x =_{\mathsf{O}} y$) in O. Moreover, $\succ_{\mathsf{O}}$ is *compatible* with $>$ if $s \succ_{\mathsf{O}} t$ implies $s\sigma > t\sigma$ for all $s, t \in \mathcal{T}(\mathcal{F},\mathcal{V})$ and all substitutions σ that are $\mathcal{F}$-grounding for s and t and covered by $\succeq_{\mathsf{O}}$.

For instance, if $>$ is an LPO and $x >_{\mathsf{O}} y \in \mathsf{O}$ then $\succeq_{\mathsf{O}}$ covers $\sigma = \{x \mapsto \mathsf{g}(\mathsf{a}), y \mapsto \mathsf{a}\}$. If $\succ_{\mathsf{O}}$ is compatible with $>$ then $\mathsf{f}(\mathsf{f}(x,z),y) \succ_{\mathsf{O}} \mathsf{f}(y,\mathsf{f}(x,z))$ may hold, but $\mathsf{f}(y,x) \succ_{\mathsf{O}} \mathsf{f}(x,y)$ cannot.

Lemma 2. *[8, 1] Suppose $\succ_{\mathsf{O}}$ is compatible with $>$ and $\succeq_{\mathsf{O}}$ covers a substitution σ that is $\mathcal{F}$-grounding for s and t. Then $s \to_{\mathcal{E}^{\succ_{\mathsf{O}}}} t$ implies $s\sigma \to_{\mathcal{E}^{>}} t\sigma$.* □

In the remainder of this section we assume that $\succ_{\mathsf{O}}$ is compatible with $>$. Next, we present the inference system GJ which is used to conclude ground joinability of a given equation.

Definition 3. Let $\mathcal{E}$ be a given equational system. The inference system GJ operates on a set $\mathcal{G}$ of tuples $(s \approx t, \mathsf{O}, \sigma)$ where $s \approx t$ is an equation over $\mathcal{T}(\mathcal{F},\mathcal{V})$, O is a set of ordering constraints, and σ is a substitution. It consists of the following seven inference rules:

delete	$\mathcal{G} \uplus \{(s \approx s, \mathsf{O}, \sigma)\} \Rightarrow \mathcal{G}$
rewrite	$\mathcal{G} \uplus \{(s \approx t, \mathsf{O}, \sigma)\} \Rightarrow \mathcal{G} \cup \{(u \approx t, \mathsf{O}, \sigma)\}$
	$\mathcal{G} \uplus \{(t \approx s, \mathsf{O}, \sigma)\} \Rightarrow \mathcal{G} \cup \{(t \approx u, \mathsf{O}, \sigma)\}$
	if $s \to_{\mathcal{E}^{\succ_{\mathsf{O}}}} u$
equation	$\mathcal{G} \uplus \{(s \approx t, \mathsf{O}, \sigma)\} \Rightarrow \mathcal{G}$
	if $s \leftrightarrow_{\mathcal{E}} t$

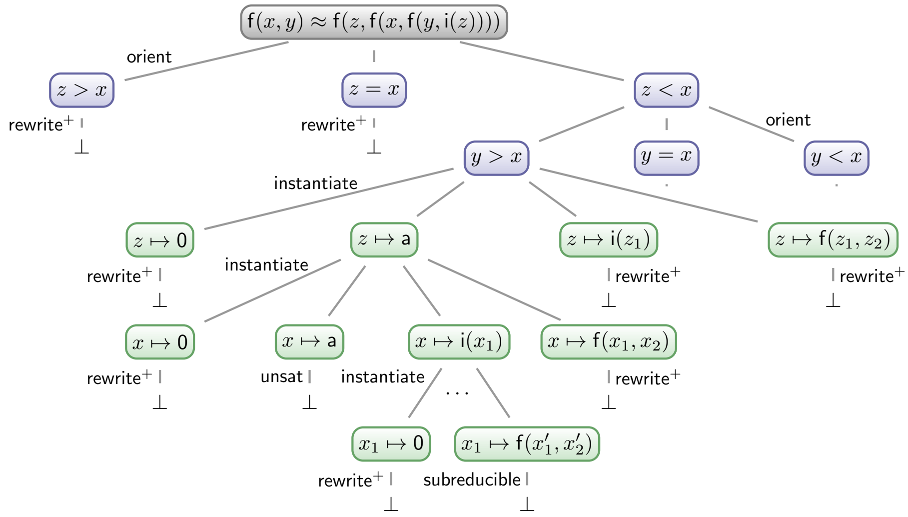

Figure 1: Proving ground joinability of $\mathsf{f}(x,y) \approx \mathsf{f}(z,\mathsf{f}(x,\mathsf{f}(y,\mathsf{i}(z))))$.

orient	$\mathcal{G} \uplus \{(s \approx t, \mathsf{O}, \sigma)\} \Rightarrow \mathcal{G} \cup \{(s \approx t, \mathsf{O} \cup \{x > y\}, \sigma), (s \approx t, \mathsf{O} \cup \{y > x\}, \sigma), (s\rho \approx t\rho, \mathsf{O}\rho, \sigma\rho)\}$
	if $x, y \in \mathcal{V}\mathrm{ar}(s \approx t)$ and where $\rho = \{x \mapsto y\}$
unsat	$\mathcal{G} \uplus \{(s \approx t, \mathsf{O}, \sigma)\} \Rightarrow \mathcal{G}$
	if $\succeq_{\mathsf{O}}$ does not cover any $\mathcal{F}$-grounding substitution for $s \approx t$
instantiate	$\mathcal{G} \uplus \{(s \approx t, \mathsf{O}, \sigma)\} \Rightarrow \mathcal{G} \cup \{(s\sigma_f \approx t\sigma_f, \mathsf{O}\sigma_f, \sigma\sigma_f\} \mid f \in \mathcal{F}\}$
	if $\mathcal{F}$ is finite, $\sigma_f = \{x \mapsto f(z_0, \dots, z_n)\}$ for $x \in \mathcal{V}\mathrm{ar}(s \approx t)$ and $z_0, \dots, z_n$ fresh
subreducible	$\mathcal{G} \uplus \{(s \approx t, \mathsf{O}, \sigma)\} \Rightarrow \mathcal{G}$
	if σ is $\mathcal{E}^{\succ_{\mathsf{O}}}$-reducible

The inference system GJ extends the transformation relation used by Martin and Nipkow [8] by the last three rules. A sequence of transformation steps $\gamma\colon \mathcal{G}_0 \Rightarrow \mathcal{G}_1 \Rightarrow \mathcal{G}_2 \Rightarrow \dots$ is called a *derivation*. The aim of this section is the following correctness result:

Lemma 4. *If* $\{(s \approx t, \varnothing, \varnothing)\} \Rightarrow^+ \varnothing$ *then* $s \Downarrow_{\mathcal{F}} t$ *holds.*

Before proving Lemma 4 we illustrate our approach by an example, which also shows that GJ is strictly more powerful than the transformation relation by Martin and Nipkow [8].

Example 5. Consider the following system $\mathcal{E}$ for commutative groups, which is ground complete on both $\mathcal{F} = \{\mathsf{f}, \mathsf{i}, \mathsf{0}\}$ and $\mathcal{F}_{\mathsf{a}} = \mathcal{F} \cup \{\mathsf{a}\}$ [8].

$$
\begin{array}{cccc}
\mathsf{f}(\mathsf{i}(x),x) \to \mathsf{0} & \mathsf{f}(y,\mathsf{f}(\mathsf{i}(y),x)) \to x & \mathsf{i}(\mathsf{0}) \to \mathsf{0} & \mathsf{f}(\mathsf{f}(x,y),z) \to \mathsf{f}(x,\mathsf{f}(y,z)) \\
\mathsf{f}(x,\mathsf{i}(x)) \to \mathsf{0} & \mathsf{f}(\mathsf{i}(y),\mathsf{f}(y,x)) \to x & \mathsf{i}(\mathsf{i}(x)) \to x & \mathsf{i}(\mathsf{f}(x,y)) \to \mathsf{f}(\mathsf{i}(y),\mathsf{i}(x)) \\
\mathsf{f}(x,\mathsf{0}) \to x & \mathsf{f}(\mathsf{i}(y),\mathsf{f}(x,y)) \to x & \mathsf{f}(x,y) \approx \mathsf{f}(y,x) & \mathsf{f}(x,\mathsf{f}(y,z)) \approx \mathsf{f}(y,\mathsf{f}(x,z)) \\
\mathsf{f}(\mathsf{0},x) \to x & \mathsf{f}(y,\mathsf{f}(x,\mathsf{i}(y))) \to x & &
\end{array}
$$

Equation $\mathsf{f}(x,y) \approx \mathsf{f}(z,\mathsf{f}(x,\mathsf{f}(y,\mathsf{i}(z))))$ $(\star)$ occurs in the set $\mathrm{CP}_>(\mathcal{E})$ for all reduction orders $>$ that orient the rules as indicated. Figure 1 depicts a possible GJ-derivation for this problem. We cannot rewrite Equation $(\star)$, so we may start by applying the orient rule to x and z. If $z >_{\mathsf{O}} x$ holds then also $\mathsf{i}(z) >_{\mathsf{O}} x$ does because $>$ and hence $>_{\mathsf{O}}$ has the subterm property. It is then easy to check that repeated applications of rewrite turn the problem into a trivial one. But for instance for the variable order $y >_{\mathsf{O}} x >_{\mathsf{O}} z$ no rewrite step is possible. We instead continue by applying instantiate to z, writing t for $\mathsf{f}(z,\mathsf{f}(x,\mathsf{f}(y,\mathsf{i}(z))))$:

- If $\sigma_{\mathsf{0}}(z) = \mathsf{0}$ then repeated applications of rewrite turn the problem into a trivial one because of the rewrite steps $t\sigma_{\mathsf{0}} = \mathsf{f}(\mathsf{0},\mathsf{f}(x,\mathsf{f}(y,\mathsf{i}(\mathsf{0})))) \to \mathsf{f}(x,\mathsf{f}(y,\mathsf{i}(\mathsf{0}))) \to^+ \mathsf{f}(x,y)$.
- If $\sigma_{\mathsf{i}}(z) = \mathsf{i}(z_1)$ we have $y >_{\mathsf{O}} x >_{\mathsf{O}} \mathsf{i}(z_1)$. Since $>$ is a simplification order, $x >_{\mathsf{O}} \mathsf{i}(z_1)$ implies $x >_{\mathsf{O}} z_1$ and similar for y. The following rewrite steps make the problem trivial:

$$t\sigma_{\mathsf{i}} \to \mathsf{f}(\mathsf{i}(z_1),\mathsf{f}(x,\mathsf{f}(y,z_1))) \to \mathsf{f}(\mathsf{i}(z_1),\mathsf{f}(x,\mathsf{f}(z_1,y))) \to \mathsf{f}(\mathsf{i}(z_1),\mathsf{f}(z_1,\mathsf{f}(x,y))) \to \mathsf{f}(x,y)$$

- Finally, if $z = \mathsf{f}(z_1,z_2)$, so $y >_{\mathsf{O}} x >_{\mathsf{O}} \mathsf{f}(z_1,z_2)$ then we can first apply rewrite to obtain $\mathsf{f}(\mathsf{f}(z_1,z_2),\mathsf{f}(x,\mathsf{f}(y,\mathsf{i}(\mathsf{f}(z_1,z_2))))) \to^+ \mathsf{f}(z_1,\mathsf{f}(z_2,\mathsf{f}(x,\mathsf{f}(y,\mathsf{f}(\mathsf{i}(z_2),\mathsf{i}(z_1)))))) =: u$. Now suppose we can ensure that the reduction order satisfies $\mathsf{f}(z_1,z_2) > \mathsf{i}(z_1)$ and $\mathsf{f}(z_1,z_2) > \mathsf{i}(z_1)$, and thus by transitivity $x, y > \mathsf{i}(z_1), \mathsf{i}(z_2)$. This admits the ordered rewrite steps

$$u \to^+ \mathsf{f}(z_1,\mathsf{f}(z_2,\mathsf{f}(\mathsf{i}(z_2),\mathsf{f}(\mathsf{i}(z_1),\mathsf{f}(x,y))))) \to \mathsf{f}(z_1,\mathsf{f}(\mathsf{i}(z_1),\mathsf{f}(x,y))) \to \mathsf{f}(x,y)$$

So $(\star)$ is $\mathcal{F}$-ground-joinable if the constraints $\mathsf{f}(z_1,z_2) > \mathsf{i}(z_1), \mathsf{i}(z_2)$ are satisfied. Indeed the KBO with $\mathsf{i} > \mathsf{f} > \mathsf{0}$, $w_0 = w(\mathsf{0}) = 1$ and $w(\mathsf{i}) = w(\mathsf{f}) = 0$ orients $\mathcal{E}$ and satisfies these constraints. (For the signature $\mathcal{F}_{\mathsf{a}}$ some further case analysis is required, as indicated in Figure 1.)

Ground confluence of $\mathcal{E}^>$ is established by checking that the union of the joinability constraints that arise from equations in $\mathrm{CP}_>(\mathcal{E})$ can be satisfied by some reduction order that is compatible with $\mathcal{E}$. MædMax employs an SMT solver for this task and can indeed show ground confluence. The tools Waldmeister and E fail on this problem.

For a derivation tree as in Figure 1 it is intuitively clear that any grounding substitution corresponds to exactly one branch. This fact needed for the proof of Lemma 4 is made formal in Lemma 6. We write $G \Rightarrow_1 G'$ if there is some $\mathcal{G}$ such that $\{G\} \Rightarrow \mathcal{G}$ and $G' \in \mathcal{G}$.

Lemma 6. *Let $s\sigma, t\sigma$ be ground and $\gamma\colon \mathcal{G}_0 = \{(s \approx t, \varnothing, \varnothing)\} \Rightarrow^+ \mathcal{G}_n$. Then there is some $m \leqslant n$ such that for all $i < m$ there are $G_i \in \mathcal{G}_i$ such that $G_i = (s_i \approx t_i, \mathsf{O}_i, \tau_i)$ and (1) $s\tau_i = s_i$, $t\tau_i = t_i$ and there is some δ_i such that $\sigma = \tau_i\delta_i$, (2) $\succeq_{\mathsf{O}_i}$ covers δ_i, and (3) $G_i \Rightarrow_1 G_{i+1}$ for all $i < m$, and if $m < n$ then $\{G_m\} \Rightarrow \varnothing$. The sequence $G_0, \dots, G_m$ is called the* projection *of γ to σ.*

Proof of Lemma 4. Suppose σ is a substitution that is $\mathcal{F}$-grounding for s and t. By assumption $\mathcal{G}_0 \Rightarrow^+ \mathcal{G}_n$ holds for $\mathcal{G}_0 = \{(s \approx t, \varnothing, \varnothing)\}$ and $\mathcal{G}_n = \varnothing$. By Lemma 6 there is a projection $G_0, \dots, G_m$ to σ for some $m \leqslant n$, so for all $G_i = (s_i \approx t_i, O_i, \tau_i)$ with $i < m$ there is some δ_i satisfying $\sigma = \tau_i\delta_i$ and for all $x >_{\mathsf{O}} y$ in O_i it holds that $\delta_i(x) > \delta_i(y)$.

We show by induction on $\{s\sigma, t\sigma\}$ with respect to $>_{\mathsf{mul}}$ that $s\sigma \downarrow_{\mathcal{E}^>} t\sigma$ holds. In a nested induction on $m - i$ we establish $s_i\delta_i \downarrow_{\mathcal{E}^>} t_i\delta_i$. We first consider the base cases where $i = m$ and do a case distinction on the last step. If delete is applied then $s_i\delta_i \downarrow_{\mathcal{E}^>} s_i\delta_i$ trivially holds. In case of an equation step, if $s_i\delta = t_i\delta$ then joinability trivially holds. Otherwise $s_i \leftrightarrow_{\mathcal{E}} t_i$ implies $s_i\delta_i \to_{\mathcal{E}^>} t_i\delta_i$ or $t_i\delta_i \to_{\mathcal{E}^>} s_i\delta_i$ because of ground totality. Note that there cannot be an unsat step as $\succeq_{\mathsf{O}_i}$ covers σ. For the final base case, suppose subreducible was applied. As τ_i is reducible for some $x \in \mathcal{D}\mathrm{om}(\tau_i)$ also $\sigma = \tau_i\delta_i$ is reducible to, say, σ'. Then $s\sigma > s\sigma'$ or $t\sigma > t\sigma'$

must hold since ordered rewriting is compatible with $>$, so we have $\{s\sigma, t\sigma\} >_{\mathsf{mul}} \{s\sigma', t\sigma'\}$. Therefore the (outer) induction hypothesis yields $s\sigma' \downarrow_{\mathcal{E}^>} t\sigma'$, which implies $s\sigma \downarrow_{\mathcal{E}^>} t\sigma$.

In the step case where $i \neq m$ we can assume $s_{i+1}\delta_{i+1} \downarrow_{\mathcal{E}^>} t_{i+1}\delta_{i+1}$ by the (inner) induction hypothesis. First, suppose rewrite is applied, so $s_i \to_{\mathcal{E}^{\succ_{o_i}}} s_{i+1}$ and $\delta_i = \delta_{i+1}$. By Lemma 2 this implies $s_i\delta_i \to_{\mathcal{E}^>} s_{i+1}\delta_i$. So $s_i\delta_i \downarrow_{\mathcal{E}^>} t_i\delta_i$ follows from $s_{i+1}\delta_{i+1} \downarrow_{\mathcal{E}^>} t_{i+1}\delta_{i+1}$. The second rewrite case is symmetric. Next, if orient is applied the statement is obvious unless $x\delta_i = y\delta_i$. But then $\rho\delta_{i+1} = \delta_i$, so $s_i\delta_i \downarrow_{\mathcal{E}^>} t_i\delta_i$ follows from $s_{i+1}\delta_{i+1} \downarrow_{\mathcal{E}^>} t_{i+1}\delta_{i+1}$. For instantiate the choice of G_i ensures $s = s_{i+1}\delta_{i+1} = s_i\delta_i$ and $t = t_{i+1}\delta_{i+1} = t_i\delta_i$.

We ultimately conclude that $s_0\sigma = s\sigma \downarrow_{\mathcal{E}^>} t\sigma = t_0\sigma$ holds. □

4 Conclusion

We presented a new criterion for ground joinability that can be exploited to check ground completeness. For the original signature, the resulting test is strictly more powerful than the approach by Martin and Nipkow.

The criterion is implemented in the tool MædMax[1], which implements an ordered-completion version of maximal completion [6]. A key characteristic of this approach is that a pool of equations is maintained, from which in every iteration a maximal terminating TRS $\mathcal{R}_i$ is extracted by exploiting maxSAT/maxSMT and a SAT/SMT-encoding of a reduction order (LPO, KBO, or a choice between the two). If $\mathcal{R}_i$ together with the remaining equations is ground confluent we are done, otherwise the new extended critical pairs are added and the procedure is reiterated.

As sketched in Example 5, the proposed criterion allows to express conditions on ground joinability as conditions on the reduction order. Moreover, LPO and KBO constraint solving is known to be NP-complete [9, 7], so satisfiability of order constraints can be SAT-encoded. This fact is exploited in MædMax to apply the unsat rule. Experiments show that our test allows MædMax to gain power over other tools, the results can be found on-line.

References

[1] J. Avenhaus, T. Hillenbrand, and B. Löchner. On using ground joinable equations in equational theorem proving. *JSC*, 36(1–2):217–233, 2003.

[2] F. Baader and T. Nipkow. *Term Rewriting and All That.* Cambridge University Press, 1998.

[3] L. Bachmair, N. Dershowitz, and D.A. Plaisted. Completion without failure. In H. Aït Kaci and M. Nivat, editors, *Resolution of Equations in Algebraic Structures*, volume 2: Rewriting Techniques of *Progress in Theoretical Computer Science*, pages 1–30. Academic Press, 1989.

[4] H. Comon, P. Narendran, R. Nieuwenhuis, and M. Rusinowitch. Deciding the confluence of ordered term rewrite systems. *TOCL*, 4(1):33–55, 2003.

[5] D. Kapur, P. Narendran, and F. Otto. On ground-confluence of term rewriting systems. *Inform. Comput.*, 86(1):14–31, 1990.

[6] D. Klein and N. Hirokawa. Maximal completion. In *Proc. RTA-22*, volume 10 of *LIPIcs*, pages 71–80, 2011.

[7] K. Korovin and A. Voronkov. Knuth–Bendix constraint solving is NP-complete. *TOCL*, 6(2):361–388, 2005.

[8] U. Martin and T. Nipkow. Ordered Rewriting and Confluence. In *Proc. CADE-10*, volume 449 of *LNCS*, pages 366–380, 1990.

[9] R. Nieuwenhuis. Simple LPO constraint solving methods. *Inform. Process. Lett.*, 47(2):65–69, 1993.

[1] http://cl-informatik.uibk.ac.at/software/maedmax