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1. Non-modularity in higher-order

Vincent van Oostrom

UU

2. Modularity

Properties preserved by taking union?

3. First-Order Modularity

Many modular properties, e.g.

- Confluence
- Normalisation
- Left-linear completeness

3.1. Proof Technique: Induction on Rank

Rank = number of alternating layers

3.2. Connecting $\Rightarrow$ Destructive Collapsing

$$f(a) \rightarrow b$$

$$g(x) \rightarrow x$$

3.2. Connecting $\Rightarrow$ Destructive Collapsing

$$f(a) \rightarrow b$$

$$g(x) \rightarrow x$$

$$f(g(a)) \rightarrow f(a)$$

destructive collapse!

3.3. Orthogonal Acyclicity

3.4. Least common collapsing reduction

4. Higher-Order Modularity

Almost no property modular!

4.1. Connecting $\not\Rightarrow$ Destructive Collapsing

$$\mu(x.Z(x)) \longrightarrow Z(\mu(x.Z(x)))$$

μ-recursion rule

4.1. Connecting $\not\Rightarrow$ Destructive Collapsing

$$\mu(x.Z(x)) \to Z(\mu(x.Z(x)))$$

μ-recursion rule
connects context with subterm without collapse!

4.2. Termination

TRSs: fails (Toyama)

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$$f(a,b,x) \to f(x,x,x)$$

$$\begin{aligned} g(x,y) &\to x \\ g(x,y) &\to y \end{aligned}$$

4.2. Termination

TRSs: fails (Toyama)

$$f(a,b,x) \to f(x,x,x)$$

$$\begin{aligned} g(x,y) &\to x \\ g(x,y) &\to y \end{aligned}$$

$$\begin{aligned} &\underline{f(a,b,g(a,b))} \\ &\to \; f(\underline{g(a,b)},g(a,b),g(a,b)) \\ &\to \; f(a,\underline{g(a,b)},g(a,b)) \\ &\to \; f(a,b,g(a,b)) \end{aligned}$$

4.3. Confluence

TRSs: holds (Toyama)

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TRSs: holds (Toyama)
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$$\begin{aligned} f(x,x) &\rightarrow a \\ f(x,s(x)) &\rightarrow b \end{aligned}$$

$$\mu(x.Z(x)) \rightarrow Z(\mu(x.Z(x)))$$

4.3. Confluence

TRSs: holds (Toyama)
PRSs: fails (Klop)

$$\begin{aligned} f(x,x) &\rightarrow a \\ f(x,s(x)) &\rightarrow b \end{aligned}$$

$$\mu(x.Z(x)) \rightarrow Z(\mu(x.Z(x)))$$

$$f(\mu(x.s(x)),\mu(x.s(x))) \rightarrow a$$

$$\begin{aligned} &f(\mu(x.s(x)),\mu(x.s(x))) \\ &\rightarrow\ f(\mu(x.s(x)),s(\mu(x.s(x)))) \\ &\rightarrow\ b \end{aligned}$$

μ-rule increases rank!

4.4. Left-linear Confluence

TRSs: holds (Rosen)

4.4. Left-linear Confluence

TRSs: holds (Rosen)

PRSs: holds (vO)

Proof. Hindley–Rosen Lemma + commutation of steps from distinct TRSs $\square$

4.5. Left-linear Completeness

TRSs: holds (Marchiori)

4.5. Left-linear Completeness

TRSs: holds (Marchiori)
PRSs: fails

$$f(x.x, xy.Z(x, y)) \rightarrow g(Z(a, f(x.Z(x, a), xy.Z(x, y))))$$

$$h(x, y) \rightarrow x$$

4.5. Left-linear Completeness

TRSs: holds (Marchiori)
PRSs: fails

$$f(x.x, xy.Z(x,y)) \to g(Z(a, f(x.Z(x,a), xy.Z(x,y))))$$

$$h(x,y) \to x$$

$$\begin{aligned} &\underline{f(x.x, xy.h(x,y))} \\ &\quad \to \ g(h(a, f(x.\underline{h(x,a)}, xy.h(x,y)))) \\ &\quad \to \ g(h(a, f(x.x, xy.h(x,y)))) \end{aligned}$$

$$t \twoheadrightarrow g(h(a,t))$$

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4.6. Normalisation

TRSs: easy induction on terms

4.6. Normalisation

TRSs: easy induction on terms
PRSs: fails

$$\begin{aligned} f(x.Z(x), y.y) &\rightarrow f(x.Z(x), y.Z(Z(y))) \\ f(x.x, y.Z(y)) &\rightarrow a \end{aligned}$$

$$g(g(x)) \rightarrow x$$

4.6. Normalisation

TRSs: easy induction on terms
PRSs: fails

$$
\begin{aligned}
f(x.Z(x), y.y) &\rightarrow f(x.Z(x), y.Z(Z(y))) \\
f(x.x, y.Z(y)) &\rightarrow a
\end{aligned}
$$

$$g(g(x)) \rightarrow x$$

$$f(x.g(x), y.y) \leftrightarrow f(x.g(x), y.g(g(y)))$$

4.7. Orthogonal Acyclicity

TRSs: holds (vO)

4.7. Orthogonal Acyclicity

TRSs: holds (vO)
PRSs: fails

$$
\begin{aligned}
& f(xyz.Z(x,y,z),W,V) \\
& \ \rightarrow \ Z(W, Z(V, W, f(xyz.Z(x,y,z),W,V)),V)
\end{aligned}
$$

$$
\begin{aligned}
g(a,x,y) &\rightarrow x \\
g(b,x,y) &\rightarrow y
\end{aligned}
$$

4.7. Orthogonal Acyclicity

TRSs: holds (vO)
PRSs: fails

$$\begin{array}{l} f(xyz.Z(x,y,z),W,V) \\ \to \; Z(W,Z(V,W,f(xyz.Z(x,y,z),W,V)),V) \end{array}$$

$$\begin{array}{rcl} g(a,x,y) & \to & x \\ g(b,x,y) & \to & y \end{array}$$

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$$
\begin{aligned}
&\underline{f(xyz.g(x,y,z),a,b)}\\
&\quad\longrightarrow\ \underline{g(a,g(b,a,f(xyz.g(x,y,z),a,b)),b)}\\
&\quad\longrightarrow\ \underline{g(b,a,f(xyz.g(x,y,z),a,b))}\\
&\quad\longrightarrow\ f(xyz.g(x,y,z),a,b)
\end{aligned}
$$

5. Concluding Remarks

- PRSs have new form of connexions
- Modularity fails for 2nd order PRS (CRS) + TRS
- Modularity fails for λ-calculus + TRS
- Modularity fails for 'closed' fragments
- Search for interesting modular subclasses