A well-founded involutive monoid for confluence

Vincent van Oostrom

Universiteit Utrecht

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Involutive monoids (0.4)

A well-founded order on French strings (0.4)

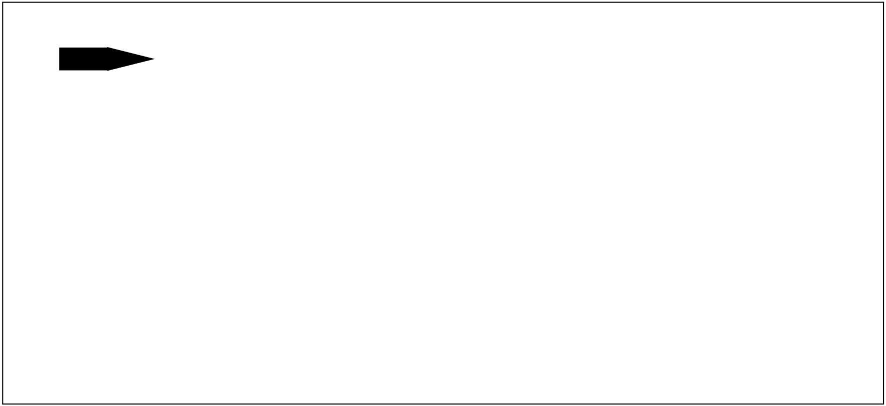
An application to proving confluence (0.2)

Boustrophedon

Gortyn code, Crete, 5th century B.C. (wikipedia)

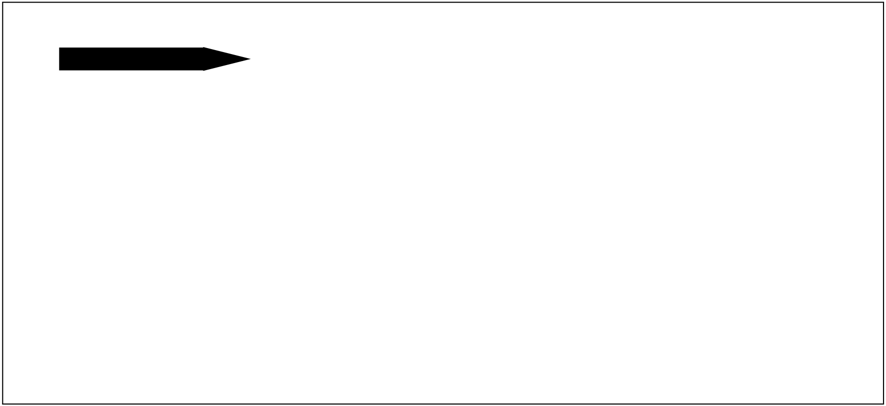
Boustrophedon

Boustrophedon



how the cow ploughs

Boustrophedon



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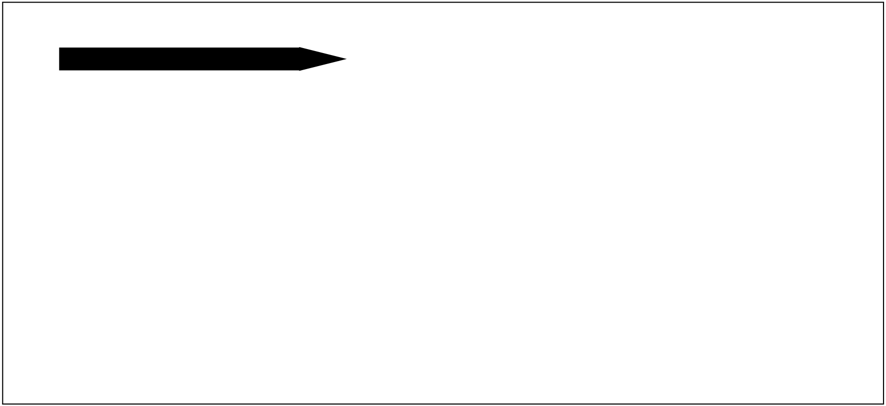

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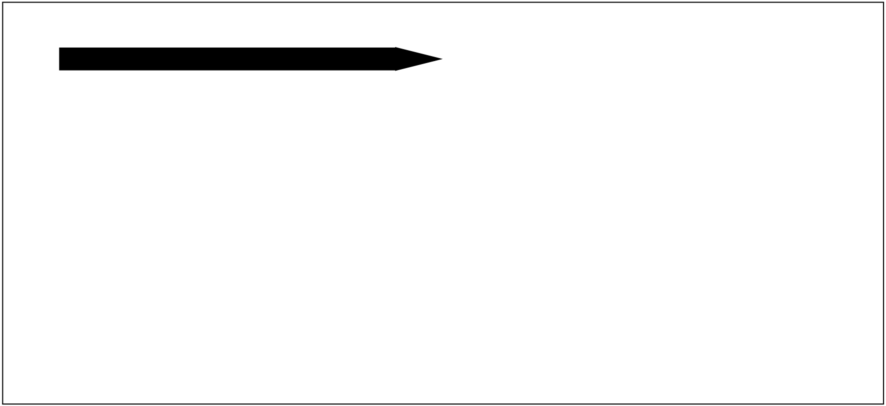

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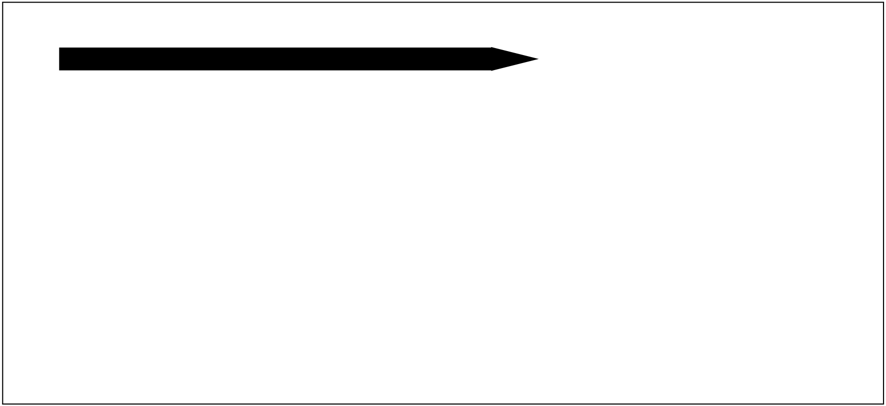

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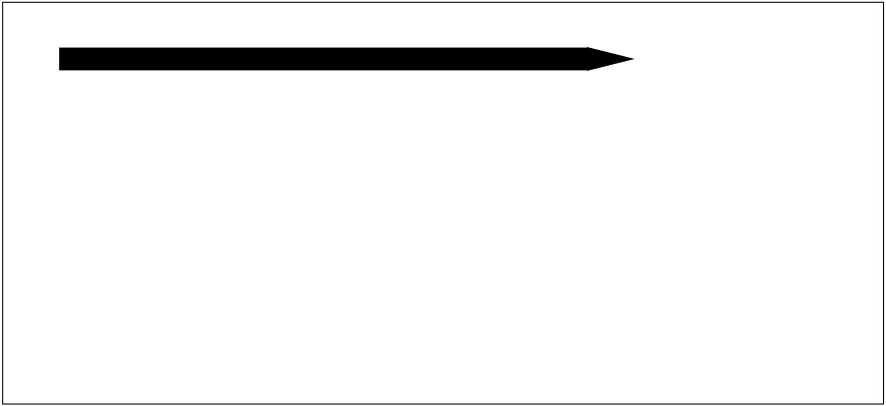

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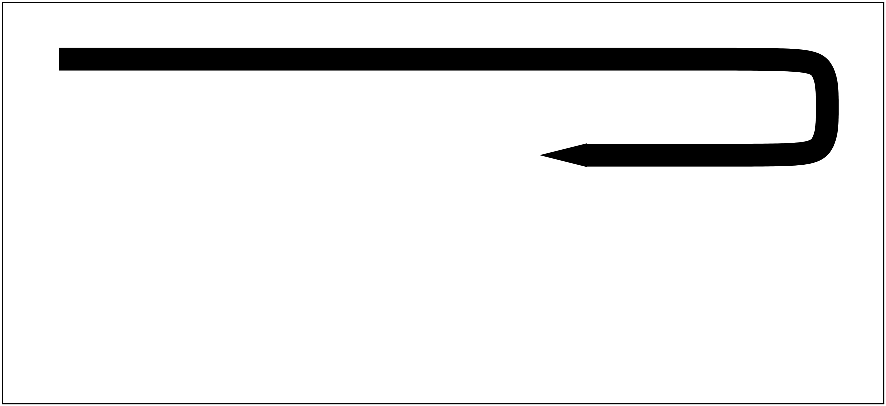

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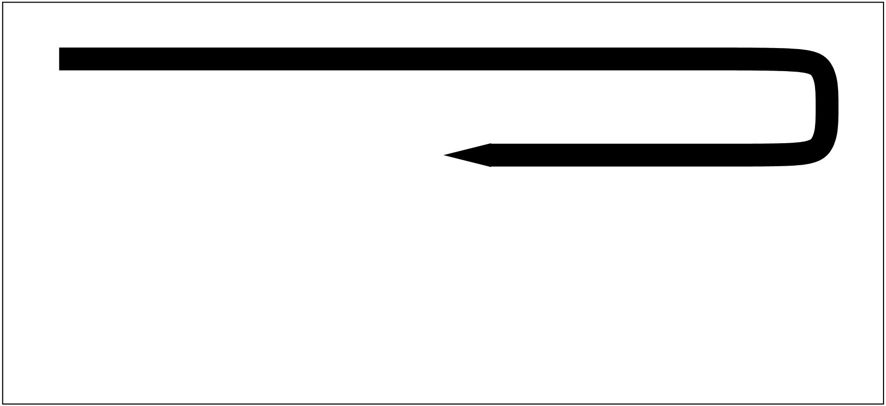

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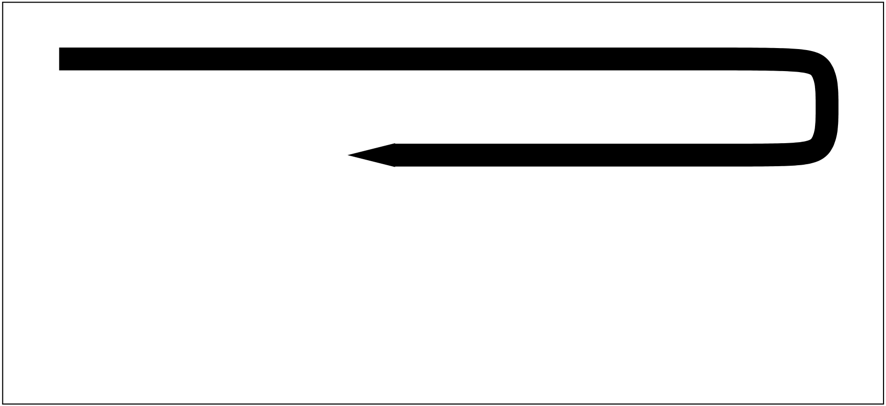

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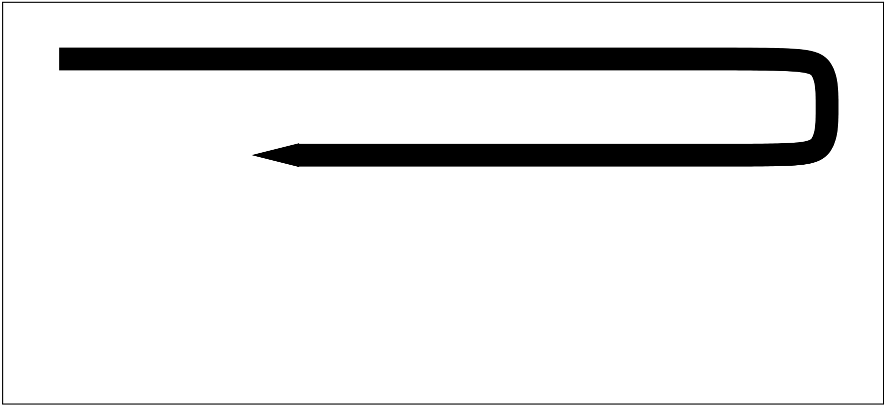

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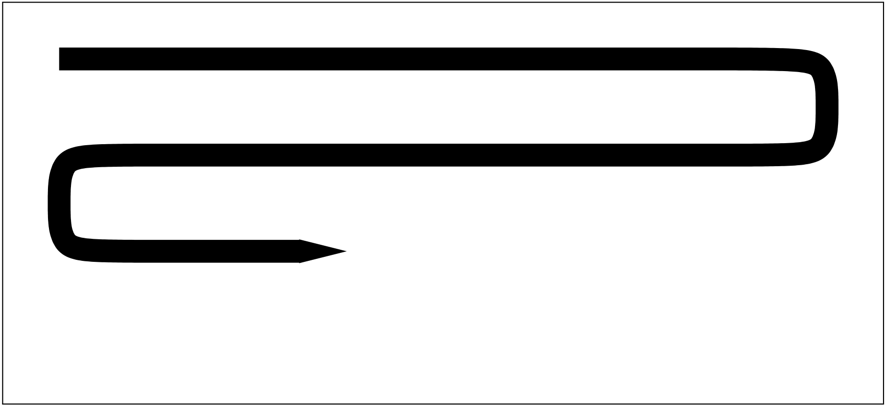

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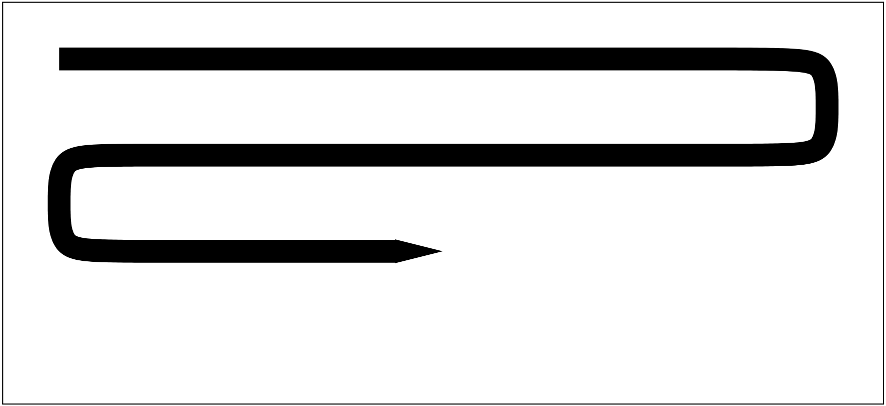

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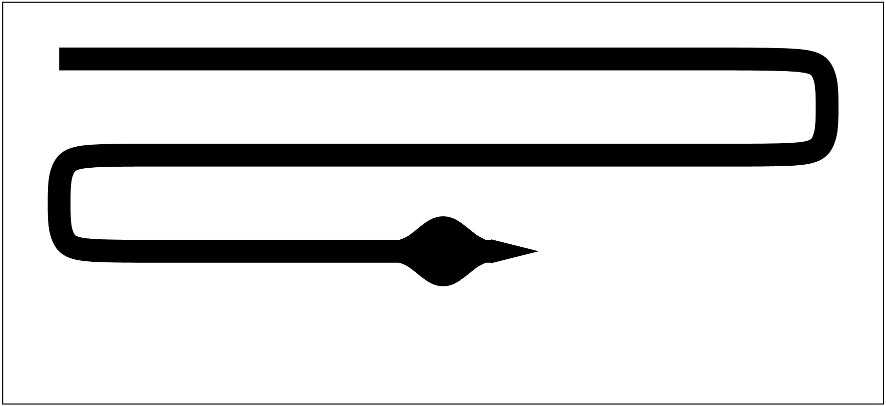
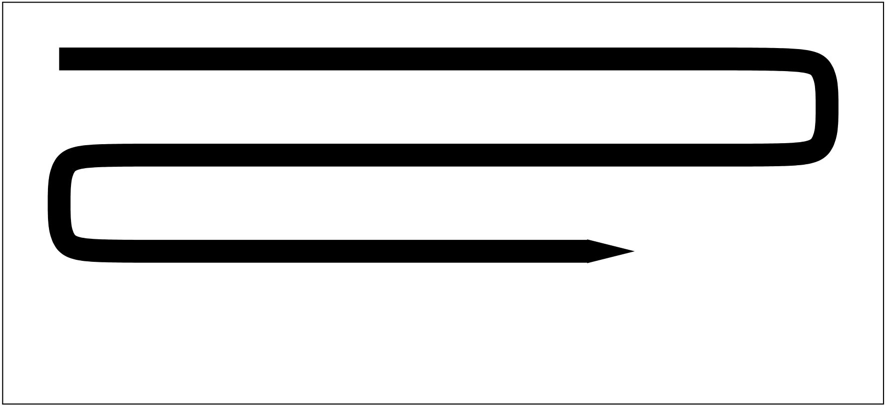


how the plow coughs

Boustrophedon



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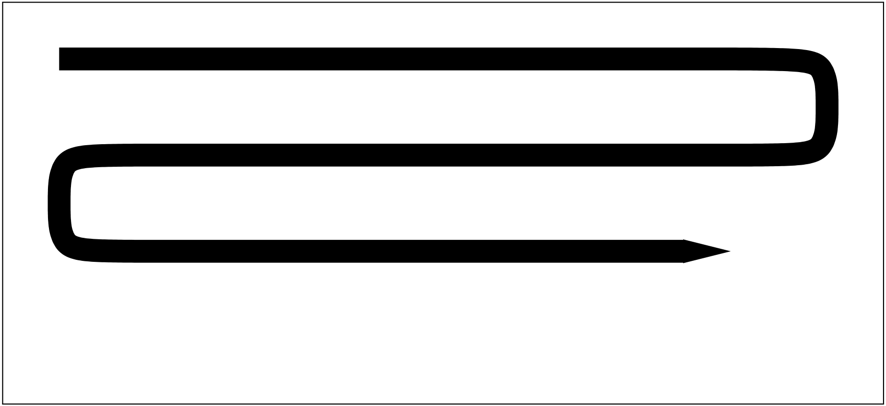

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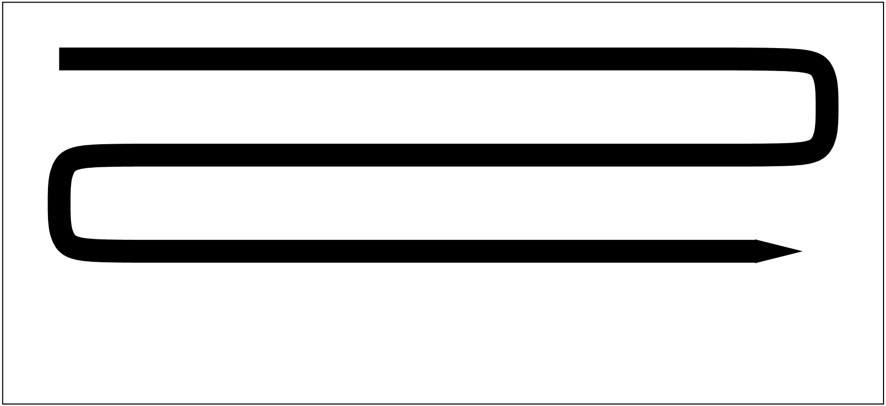

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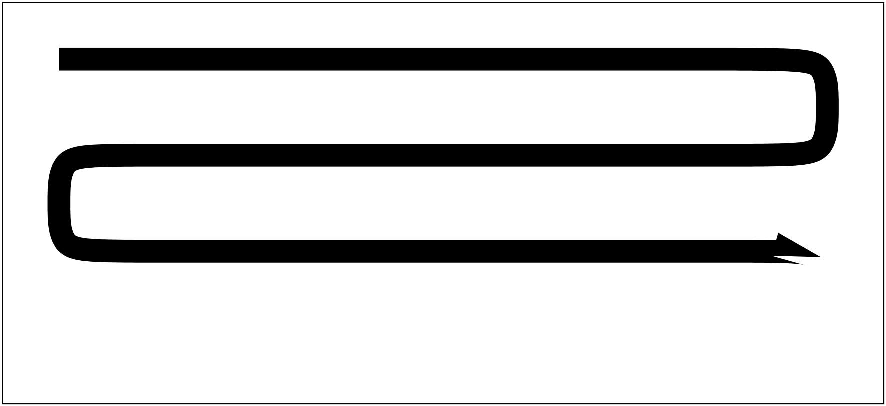

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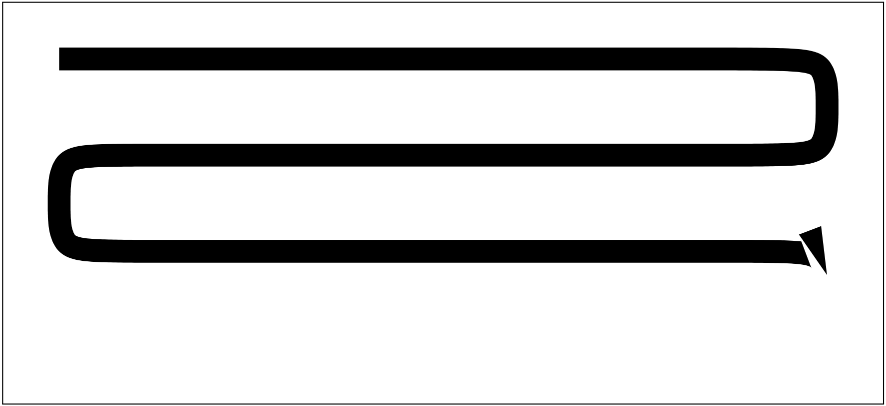

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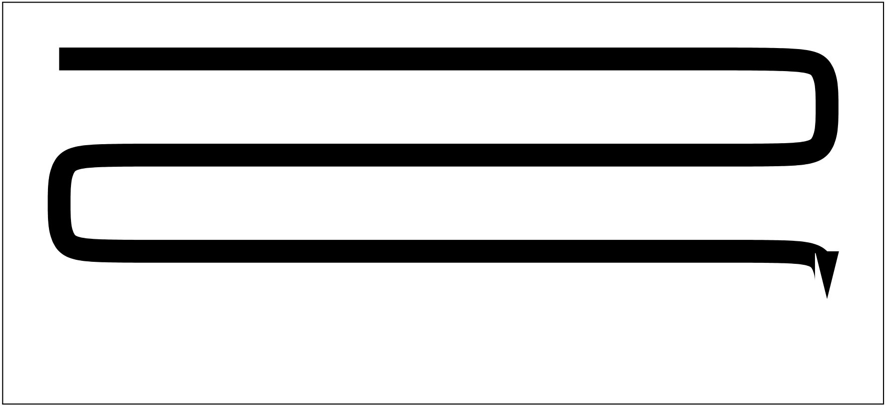

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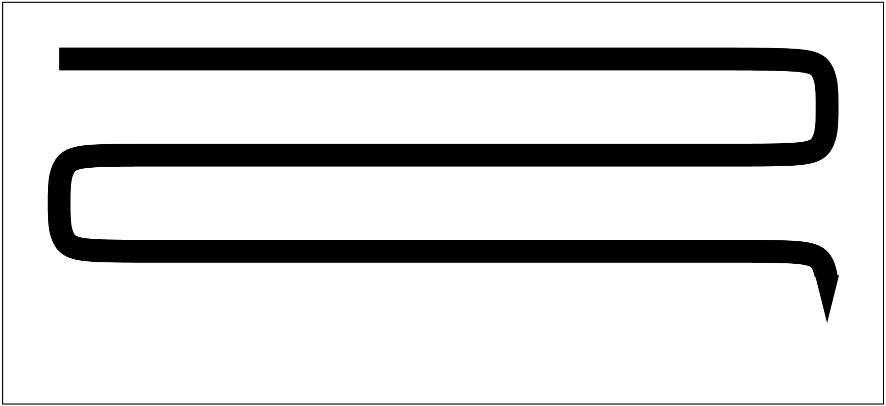

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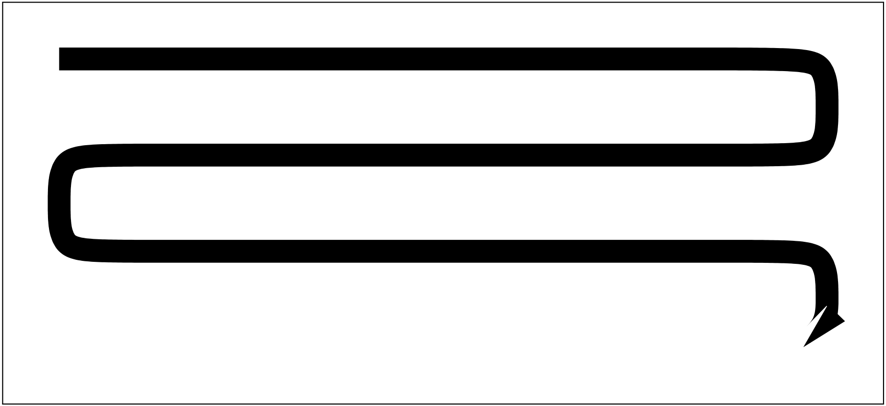

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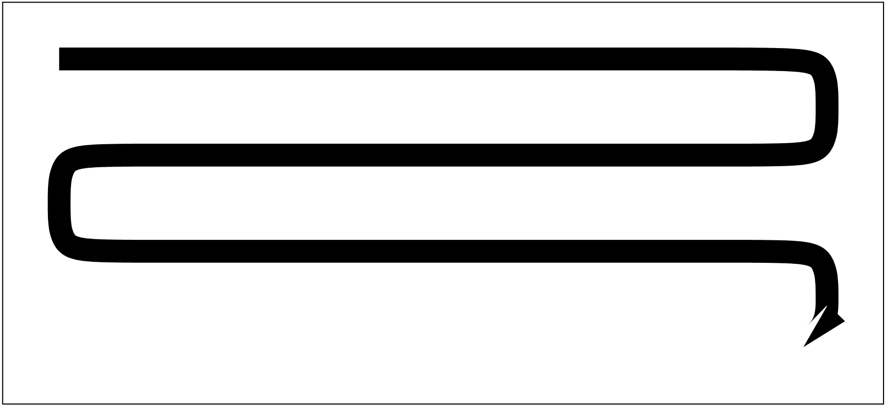

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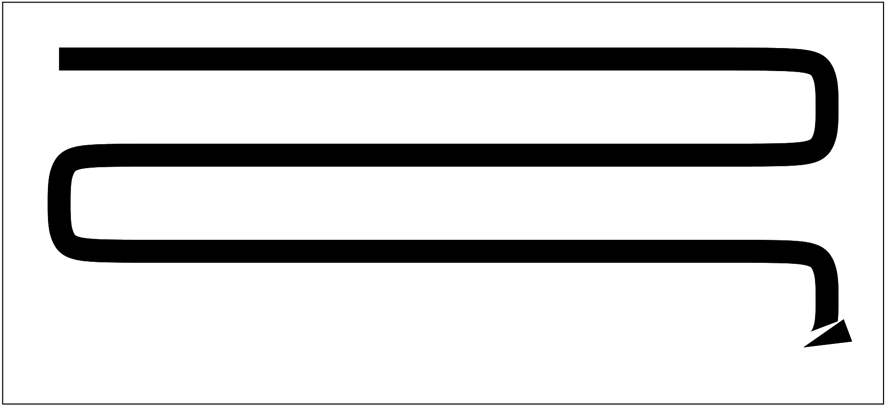

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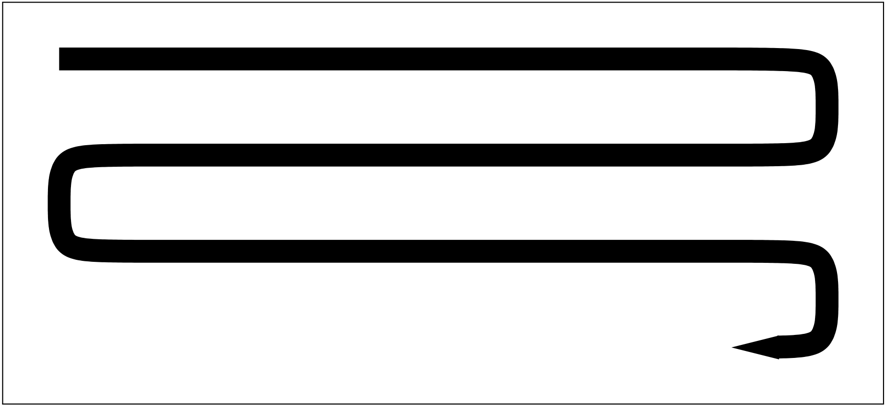

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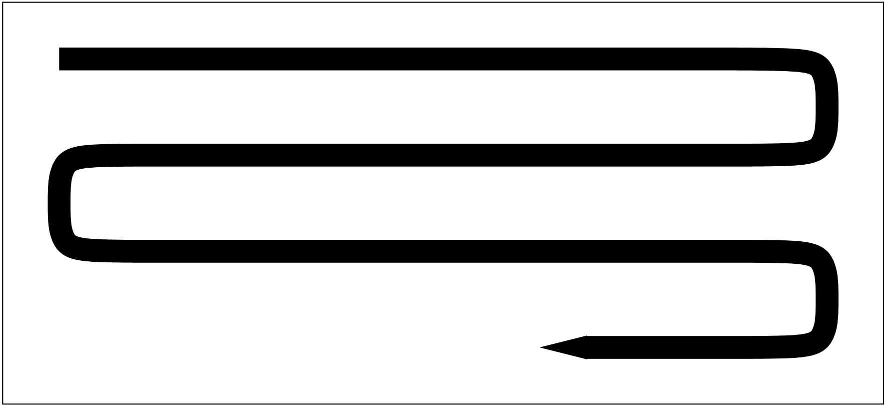

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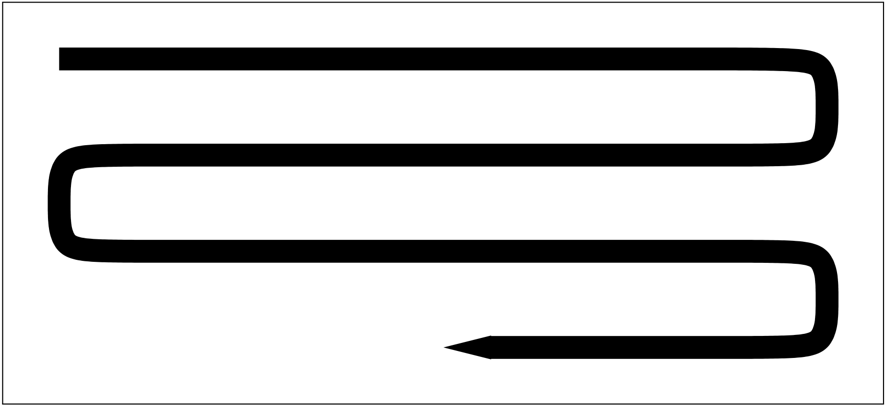

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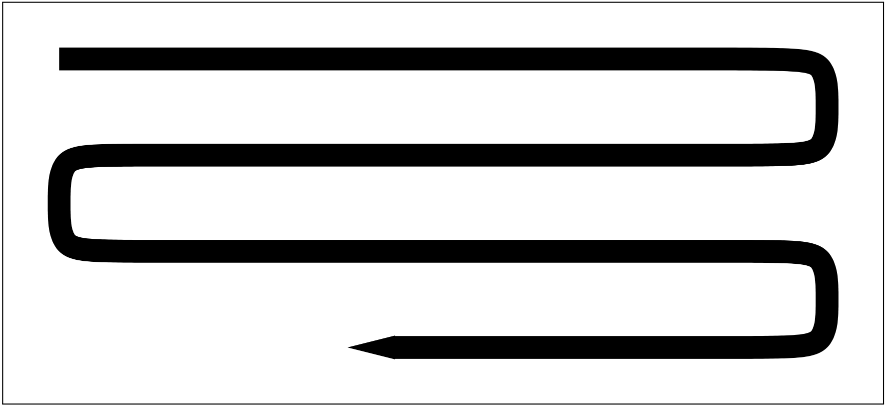

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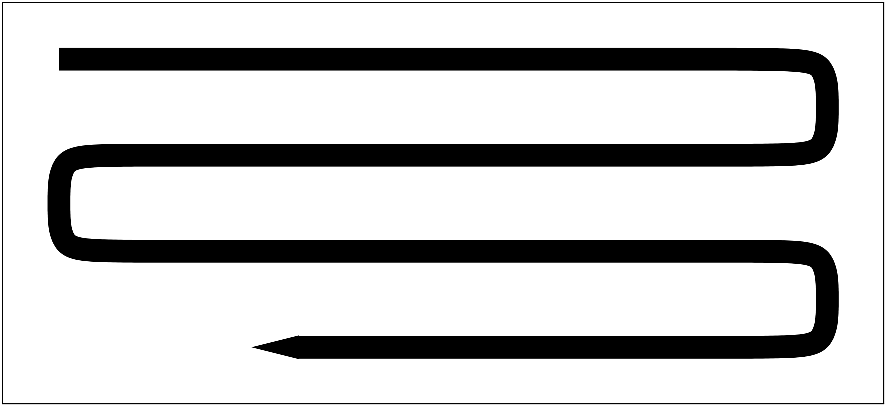

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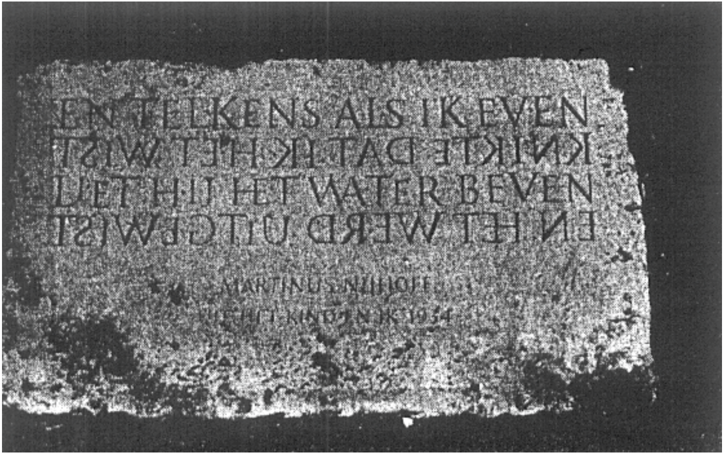

Martinus Nijhoff, Het kind en ik, Nieuwe Gedichten, 1934
(Hortus Botanicus, Universiteitsmuseum Utrecht, next to pond)

Boustrophedon

EN TELKENS ALS IK EVEN
KNIKTE DAT IK HET WIST
LIET HIJ HET WATER BEVEN
EN HET WERD UITGEWIST

Boustrophedon

EN TELKENS ALS IK EVEN
TSIW TEH KI TAD ETKINK
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How to represent linearly?

French strings (chaînes)

Definition

- French letter is an accented (acute or grave) letter

French strings (chaînes)

Definition

- French letter is an accented (acute or grave) letter
- juxtaposition ␣ èv̀èǹ juxtaposed to ḱńíḱt́é gives èv̀èǹḱńíḱt́é

French strings (chaînes)

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French strings (chaînes)

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- mirroring $\,^{-1}$
- $\widehat{L}$ set of French Strings on L ($\hat{\mathrm{a}}$ for either à or á)

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letter markup (representation preserves length,prefix,suffix)

Monoid of strings

$$
\begin{aligned}
(sr)q &= s(rq) && \text{(associativity)}\\
s\varepsilon &= \varepsilon && \text{(right identity)}\\
\varepsilon s &= s && \text{(left identity)}
\end{aligned}
$$

Involutive monoid of French strings

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(sr)q & = & s(rq) & \text{(associativity)} \\
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Proof.

$$
\varepsilon^{-1} = \varepsilon\varepsilon^{-1} = (\varepsilon^{-1})^{-1}\varepsilon^{-1} = (\varepsilon\varepsilon^{-1})^{-1} = (\varepsilon^{-1})^{-1} = \varepsilon
$$

□

Involutive monoid

Definition

set with

- associative binary operation $\cdot$
- identity element e
- involutive anti-automorphism $^{-1}$

$$
\begin{aligned}
(a \cdot b) \cdot c &= a \cdot (b \cdot c) && \text{(associative)} \\
a \cdot e &= a && \text{(right identity)} \\
e \cdot a &= a && \text{(left identity)} \\
(a^{-1})^{-1} &= a && \text{(involutive)} \\
(a \cdot b)^{-1} &= b^{-1} \cdot a^{-1} && \text{(anti-automorphic)}
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Involutive monoid examples

- $\{*\}$ with binary, nullary, unary constant-$*$ map

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- integers with addition, zero, unary minus

Involutive monoid examples

- $\{*\}$ with binary, nullary, unary constant-$*$ map
- positive rationals with multiplication, one, inverse

Involutive monoid examples

- $\{*\}$ with binary, nullary, unary constant-$*$ map
- group

Involutive monoid examples

- $\{*\}$ with binary, nullary, unary constant-$*$ map
- group (examples $(\mathbb{Z}, +, 0, -)$, $(\mathbb{Q}^+, \cdot, 1, \ ^{-1})$)
- natural numbers with addition, zero, identity map

Involutive monoid examples

- $\{*\}$ with binary, nullary, unary constant-$*$ map
- group (examples $(\mathbb{Z}, +, 0, -)$, $(\mathbb{Q}^+, \cdot, 1, \ ^{-1})$)
- multisets with multiset sum, empty multiset, identity map

Involutive monoid examples

- $\{*\}$ with binary, nullary, unary constant-$*$ map
- group (examples $(\mathbb{Z}, +, 0, -)$, $(\mathbb{Q}^+, \cdot, 1, \ ^{-1})$)
- commutative monoid with identity map

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- diagrams of $\diagdown$ with gluing, point, mirroring in vertical axis

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- diagrams of $\diagdown$ with gluing, point, mirroring in vertical axis
- number pairs with pointwise addition, $(0, 0)$, swapping

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- $\{*\}$ with binary, nullary, unary constant-$*$ map
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- number triples with composition given by $(n_1, m_1, k_1) \cdot (n_2, m_2, k_2) = (n_1 + n_2, m_1 + k_1 \cdot n_2 + m_2, k_1 + k_2)$, zero $(0, 0, 0)$, involution $(n, m, k)^{-1} = (k, m, n)$

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$$
\begin{aligned}
&((n_1, m_1, k_1) \cdot (n_2, m_2, k_2)) \cdot (n_3, m_3, k_3) \\
&\quad = \quad (n_1 + n_2, m_1 + k_1 \cdot n_2 + m_2, k_1 + k_2) \cdot (n_3, m_3, k_3) \\
&\quad = \quad (n_1 + n_2 + n_3, m_1 + k_1 \cdot n_2 + m_2 + (k_1 + k_2) \cdot n_3 + m_3, k_1 + k_2 + k_3) \\
&\quad = \quad (n_1 + n_2 + n_3, m_1 + k_1 \cdot (n_2 + n_3) + m_2 + k_2 \cdot n_3 + m_3, k_1 + k_2 + k_3) \\
&\quad = \quad (n_1, m_1, k_1) \cdot (n_2 + n_3, m_2 + k_2 \cdot n_3 + m_3, k_2 + k_3) \\
&\quad = \quad (n_1, m_1, k_1) \cdot ((n_2, m_2, k_2) \cdot (n_3, m_3, k_3))
\end{aligned}
$$

Involutive monoid homomorphisms

Definition

maps preserving operations

Examples

- involutive monoid to itself (identity)

Involutive monoid homomorphisms

Definition

maps preserving operations

Examples

- involutive monoid to itself (identity)
- French strings → number pairs (grave,acute)
  ćèńàř ↦ $(3, 2)$

Involutive monoid homomorphisms

Definition
maps preserving operations

Examples

- involutive monoid to itself (identity)
- number pairs → natural numbers (sum)
  $(3, 2) \mapsto 5$

Involutive monoid homomorphisms

Definition

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Examples

- involutive monoid to itself (identity)
- French strings → natural numbers (length)
  composition of previous two

Involutive monoid homomorphisms

Definition

maps preserving operations

Examples

- involutive monoid to itself (identity)
- French strings → natural numbers (length)
- French strings → multisets (letters)
  b́áŕb̀àŕó $\mapsto [a, a, b, b, o, r, r]$

Involutive monoid homomorphisms

Definition

maps preserving operations

Examples

- involutive monoid to itself (identity)
- French strings → natural numbers (length)
- French strings → multisets (letters)
- French strings → diagrams

  ćèńàr̀ ↦ 

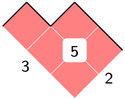
Involutive monoid homomorphisms

Definition

maps preserving operations

Examples

- involutive monoid to itself (identity)
- French strings → natural numbers (length)
- French strings → multisets (letters)
- diagrams → triples

 $\mapsto (3, 5, 2)$ cf. 

Involutive monoid homomorphisms

Definition

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Examples

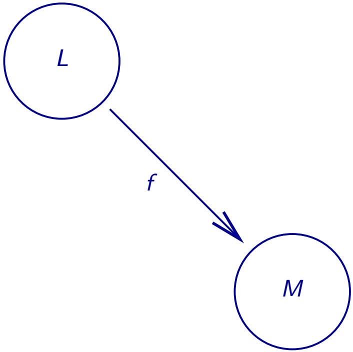
- involutive monoid to itself (identity)
- French strings → natural numbers (length)
- French strings → multisets (letters)
- French strings → triples (area)
  composition of previous two

Free involutive monoid on generators

Theorem

French strings on L give free involutive monoid on L

Freeness of involutive monoid of French Strings



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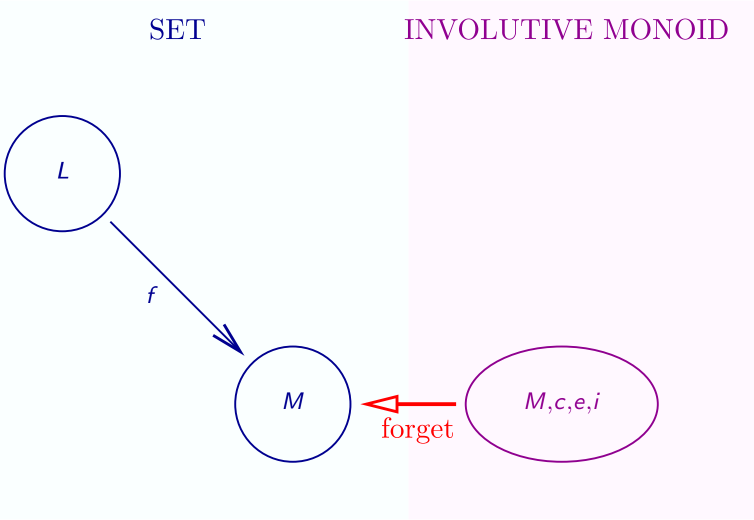

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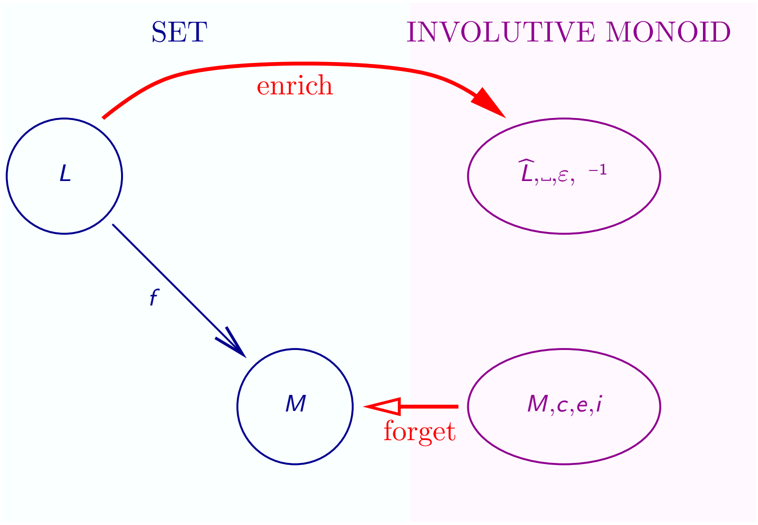

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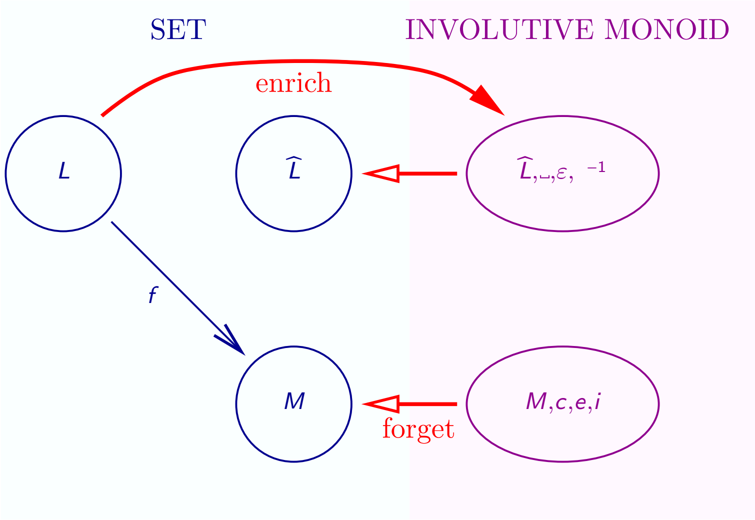

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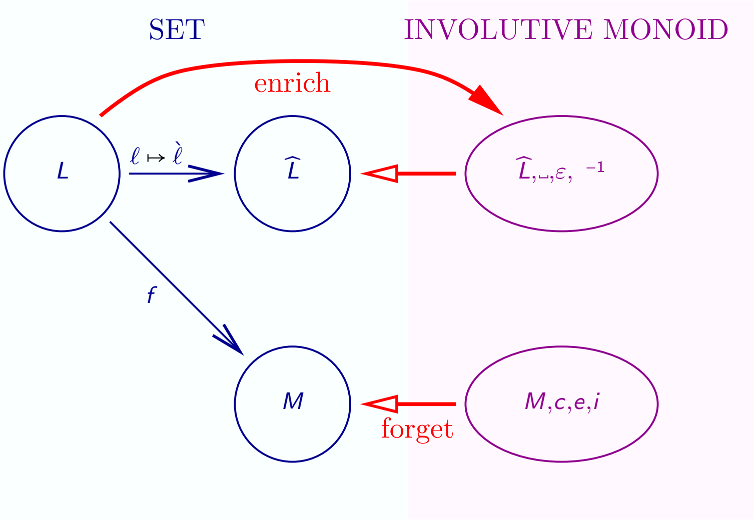

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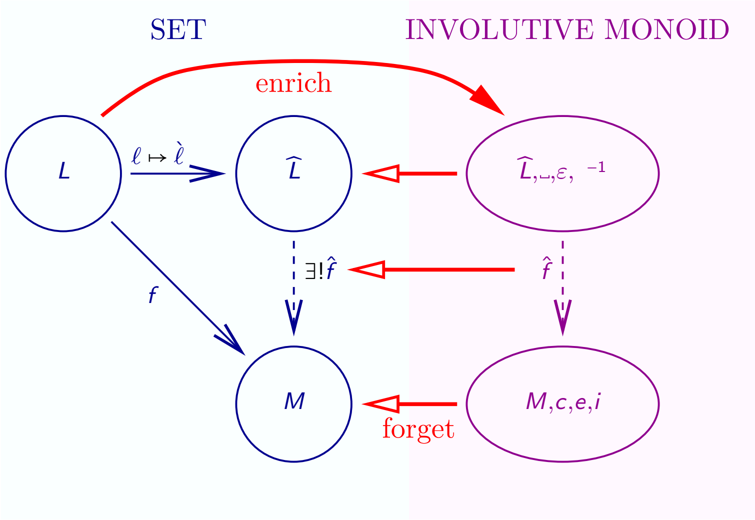

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Proof.

$\widehat{L}$ in bijection via $\grave{\ell} \mapsto \ell$ with

$$N ::= e \mid \ell \mid i(\ell) \mid c(\ell, N) \mid c(i(\ell), N)$$

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Proof.

$\widehat{L}$ in bijection via $\grave{\ell} \mapsto \ell$ with

$$N ::= e \mid \ell \mid i(\ell) \mid c(\ell, N) \mid c(i(\ell), N)$$

N set of normal forms on L for TRS completing axioms

$$\begin{array}{rcl} c(c(x,y),z) & \to & c(x,c(y,z)) \\ c(x,e) & \to & x \\ c(e,x) & \to & x \\ i(i(x)) & \to & x \\ i(c(x,y)) & \to & c(i(y),i(x)) \\ i(e) & \to & e \end{array}$$

Involutive monoid on French terms $L^\sharp$

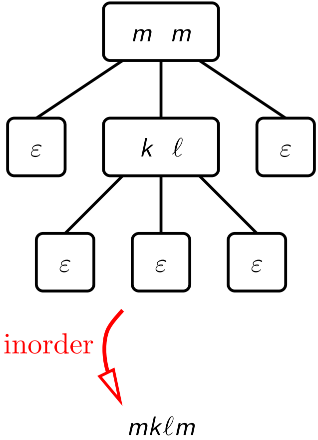
Definition

certain terms on certain French strings

Involutive monoid on French terms $L^\sharp$

Definition

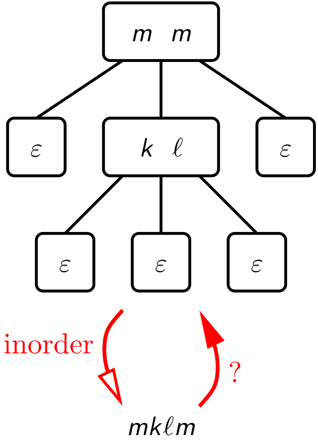
terms on strings



Involutive monoid on French terms $L^\sharp$

Definition

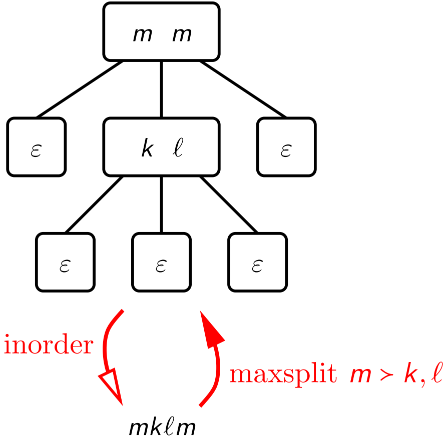
terms on strings



Involutive monoid on French terms $L^\sharp$

Definition

terms on strings on $\succ$-ordered letters



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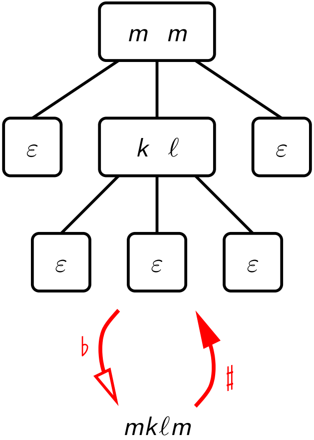
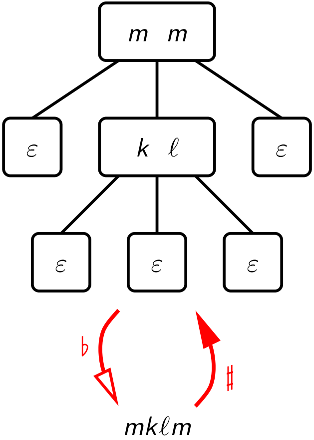
terms on strings on $\succ$-ordered letters



Involutive monoid on French terms $L^\sharp$

Definition

terms on strings on $\succ$-ordered letters where $\flat \circ \sharp$ identity



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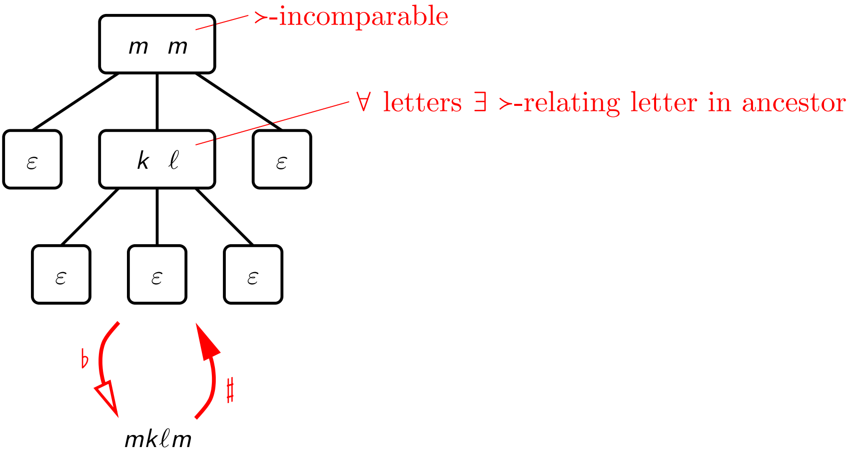
terms on strings on $\succ$-ordered letters where $\flat \circ \sharp$ identity



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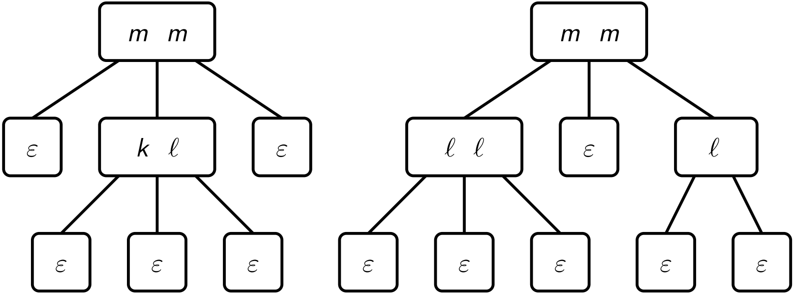
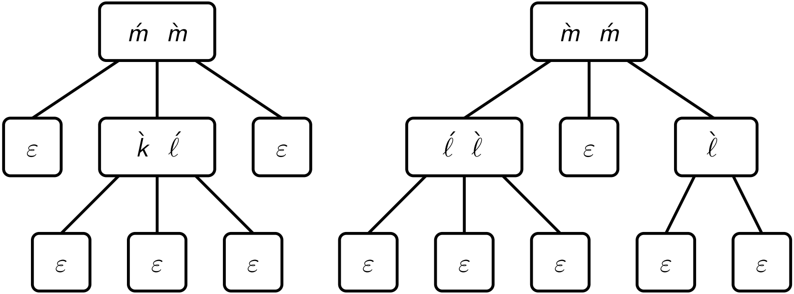


$mk\ell m$ $\qquad$ $\ell\ell mm\ell$

Involutive monoid on French terms $L^\sharp$

Definition

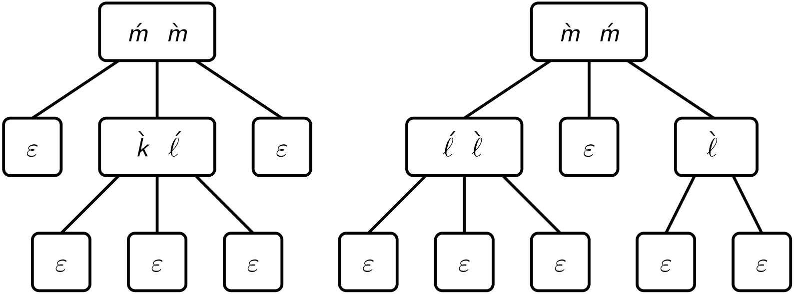
terms on French strings on $\succ$-ordered letters where $\flat \circ \sharp$ identity
operations on $L^\sharp$ defined via $\widehat{L}$, e.g. $t \cdot u = (t^\flat u^\flat)^\sharp$



$\acute{m}\grave{k}\acute{\ell}\grave{m}$ $\quad$ $\acute{\ell}\grave{\ell}\,\grave{m}\acute{m}\grave{\ell}$

A well-founded order on French terms

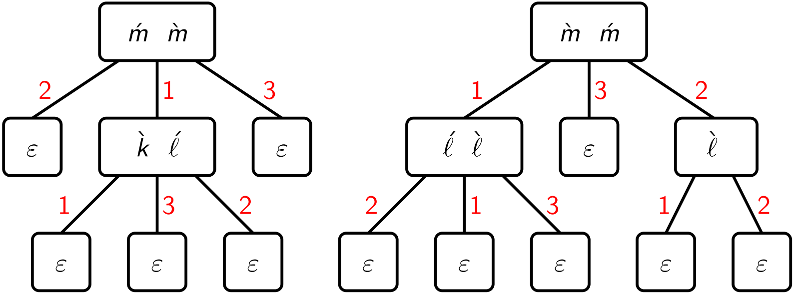
- (iterative) lexicographic path order based on $>$



$\acute{m}\grave{k}\acute{\ell}\grave{m}$ $\qquad$ $\acute{\ell}\grave{\ell}\,\grave{m}\acute{m}\grave{\ell}$

A well-founded order on French terms

- (iterative) lexicographic path order based on $\succ$
- lexicographic order on argument places compatible with marks



$\acute{m}\grave{k}\acute{\ell}\grave{m}$ $\qquad$ $\acute{\ell}\grave{\ell}\,\grave{m}\acute{m}\grave{\ell}$

A well-founded order on French terms

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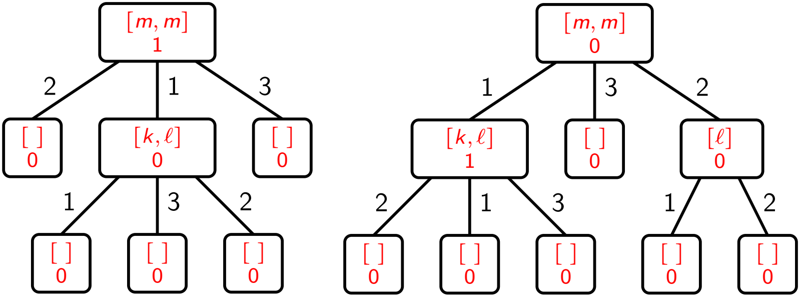
- lexicographic order on argument places compatible with marks
- signature ordered by $\succ = \binom{\succ_{mul}}{>}$ via $\binom{\text{multiset}}{\text{area}}$



$\acute{m}\grave{k}\acute{\ell}\grave{m}$ $\acute{\ell}\grave{\ell}\,\grave{m}\acute{m}\grave{\ell}$

A well-founded order on French strings/terms

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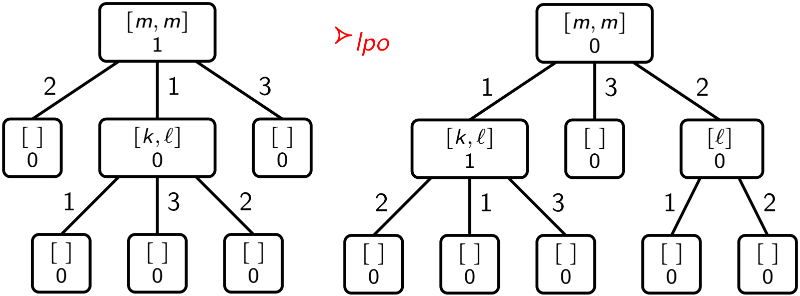
- lexicographic order on argument places compatible with marks
- signature ordered by $\succ = \binom{\succ_{mul}}{>}$ via $\binom{\text{multiset}}{\text{area}}$



$\acute{m}\grave{k}\acute{\ell}\grave{m} \quad \succ_{lpo} \quad \acute{\ell}\grave{\ell}\ \grave{m}\acute{m}\grave{\ell}$

Properties of $\succ_{lpo}$

- head of term $\succ$-related to heads of all subterms

Properties of $\succ_{lpo}$

- head of term $\succ$-related to heads of all subterms
- $\succ_{lpo}$ not an ordered monoid: $\grave{k}\grave{\ell} \succ_{lpo} \acute{\ell}$ but $\grave{k}\grave{\ell}\grave{\ell} \not\succ_{lpo} \acute{\ell}\grave{\ell}$

Properties of $\succ_{lpo}$

- head of term $\succ$-related to heads of all subterms
- $\succ_{lpo}$ not an ordered monoid
- $s\hat{\ell}r \succ_{lpo} s\{\ell\succ\}r$ (in EBNF { } is arbitrary repetition)

Properties of $\succ_{lpo}$

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Proof.

induction on length sr, cases whether ℓ is $\succ$-maximal in $s\hat{\ell}r$

yes decrease in multiset of head

no induction on substring/term $\hat{\ell}$ is in □

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- $s\acute{\ell}\grave{m}r \succ_{lpo} s\{\ell\succ\}[\grave{m}]\{\ell, m\succ\}[\acute{\ell}]\{m\succ\}r$ ([ ] is option)

Properties of $\succ_{lpo}$

- head of term $\succ$-related to heads of all subterms
- $\succ_{lpo}$ not an ordered monoid
- $s\hat{\ell}r \succ_{lpo} s\{\ell{>}\}r$

Proof.
induction on length sr, cases whether ℓ is $>$-maximal in $s\hat{\ell}r$

yes decrease in multiset of head
no induction on substring/term $\hat{\ell}$ is in ☐

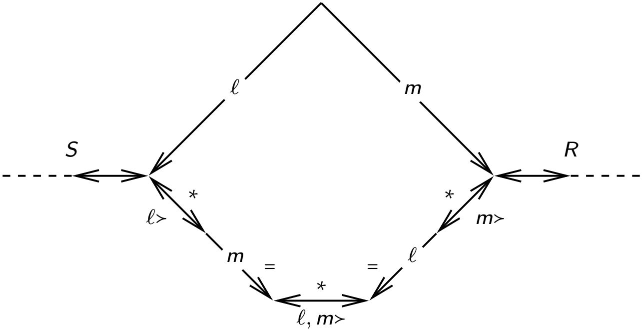
- $s\acute{\ell}\grave{m}r \succ_{lpo} s\{\ell{>}\}[\grave{m}]\{\ell, m{>}\}[\acute{\ell}]\{m{>}\}r$

Proof.
induction on length sr, cases whether ℓ, m are $>$-maximal in $s\acute{\ell}\grave{m}r$

both decrease in area of head
$\acute{\ell}$ decrease in the substring/term to the right of $\acute{\ell}$
$\grave{m}$ decrease in the substring/term to the left of $\grave{m}$
neither induction on substring/term $\acute{\ell}\grave{m}$ is in ☐

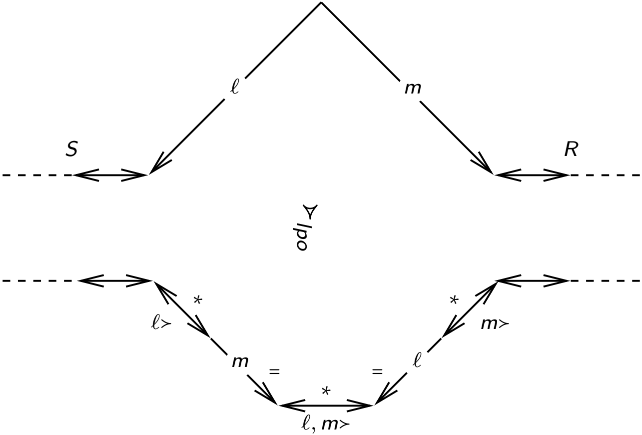
Filling in locally decreasing diagram decreases

Theorem



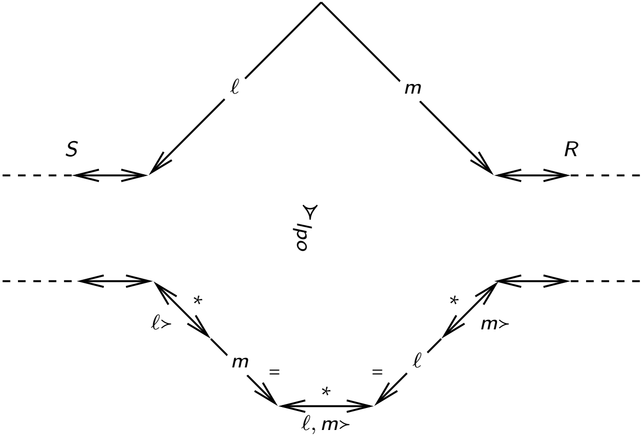
Filling in locally decreasing diagram decreases

Theorem



Filling in locally decreasing diagram decreases

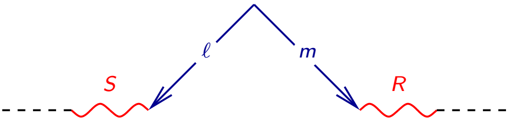
Theorem

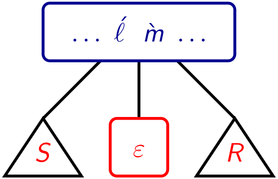


Proof.

$s\acute{\ell}\grave{m}r \succ_{lpo} s\{\ell\succ\}[\grave{m}]\{\ell, m\succ\}[\acute{\ell}]\{m\succ\}r$ □

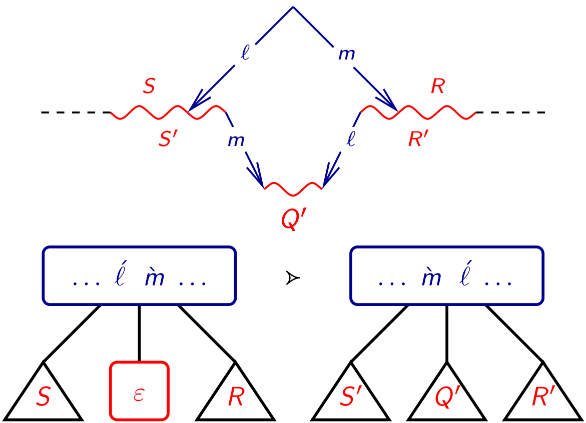
Idea: ≻-maximal steps modulo non-≻-maximal steps





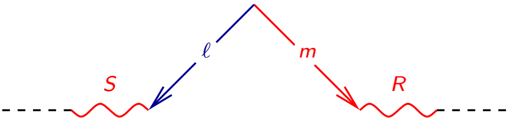
case 1: local confluence peak of ≻-maximal steps

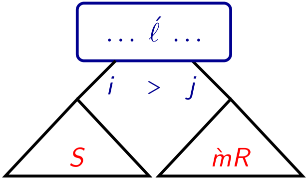
Idea: ≻-maximal steps modulo non-≻-maximal steps



area decrease

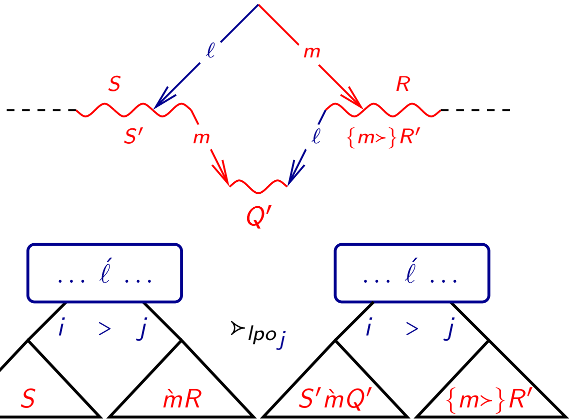
Idea: ≻-maximal steps modulo non-≻-maximal steps





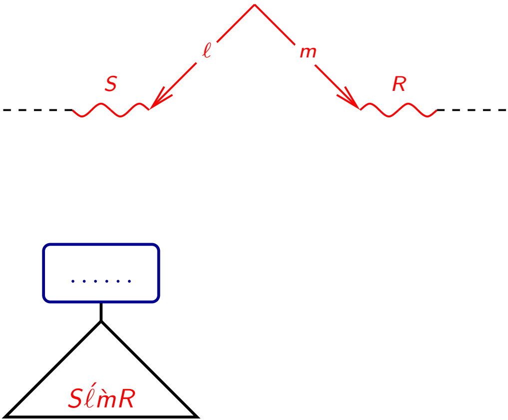
case 2: local coherence peak of ≻-maximal and non-≻-maximal step

Idea: $\succ$-maximal steps modulo non-$\succ$-maximal steps



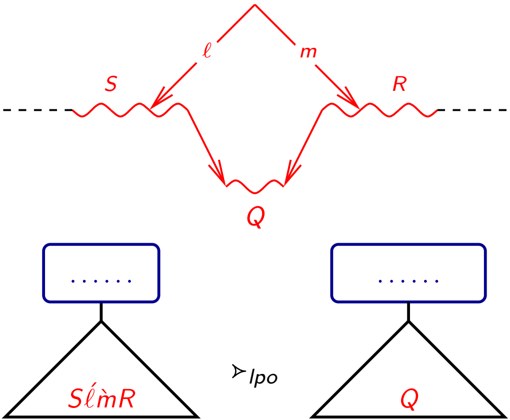
decrease in jth argument, lexicographically before ith

Idea: ≻-maximal steps modulo non-≻-maximal steps



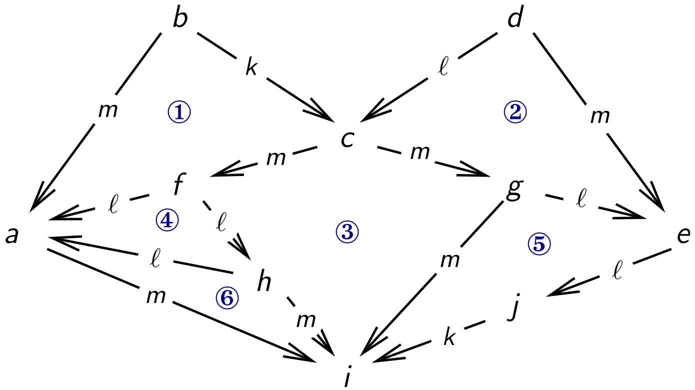
case 3: local modulo peak of non-≻-maximal steps

Idea: ≻-maximal steps modulo non-≻-maximal steps

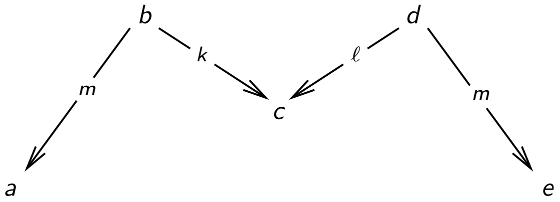


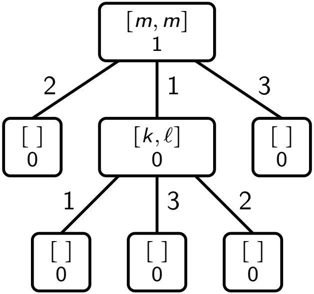
decrease in argument both steps are in

$\succ_{lpo}$ at work



Filling in local diagrams ①





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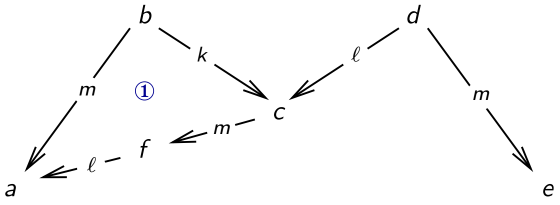
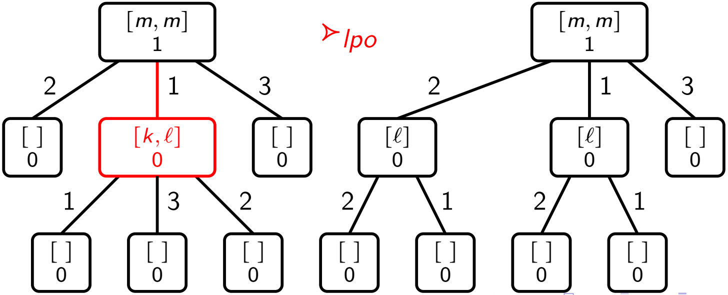
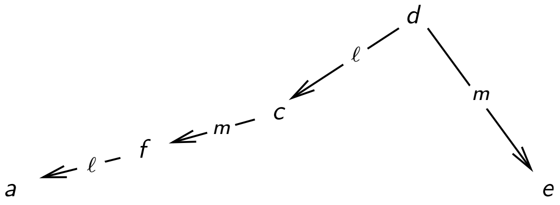
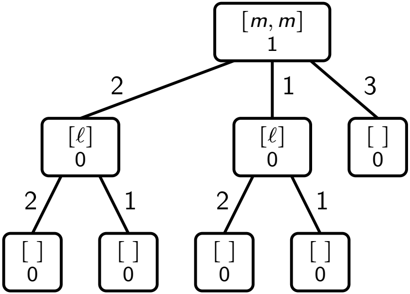




Filling in local diagrams ②





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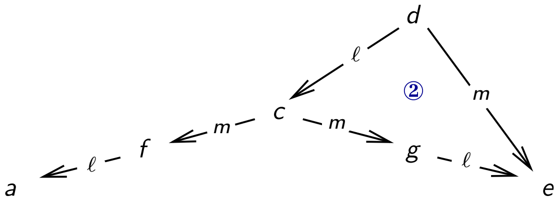
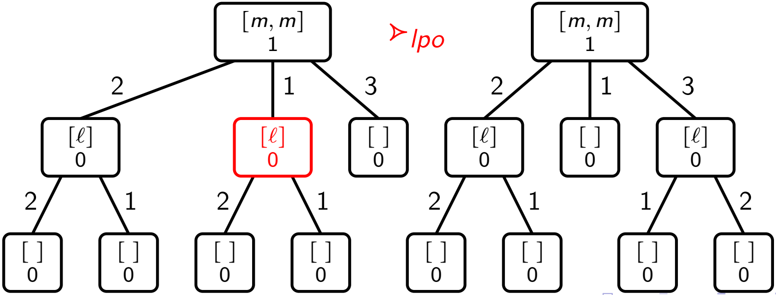
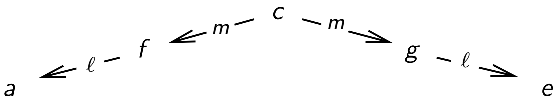
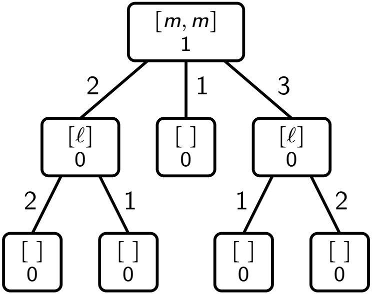




Filling in local diagrams ③





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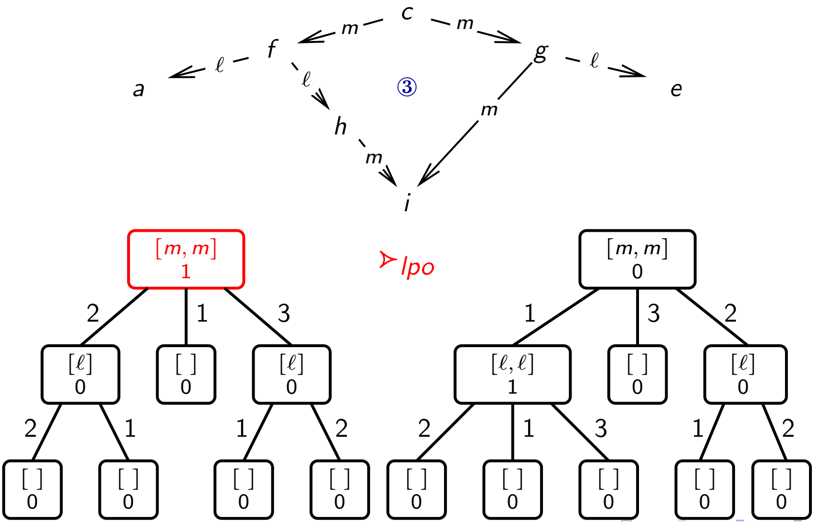
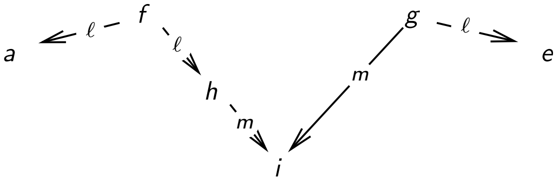
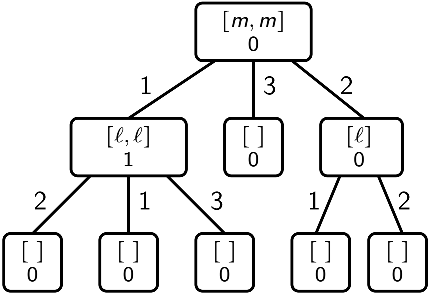


Filling in local diagrams ④





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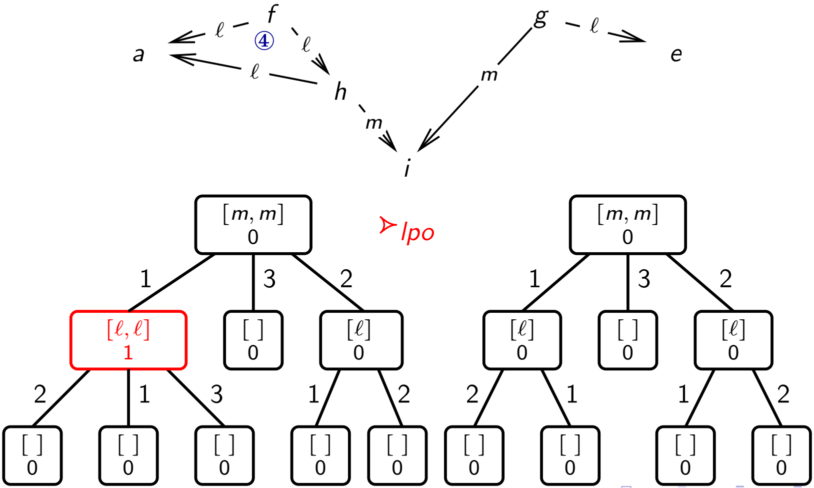
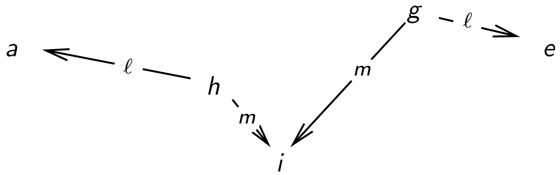
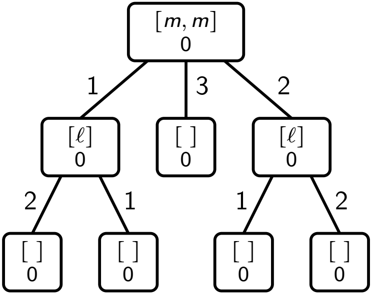


Filling in local diagrams ⑤





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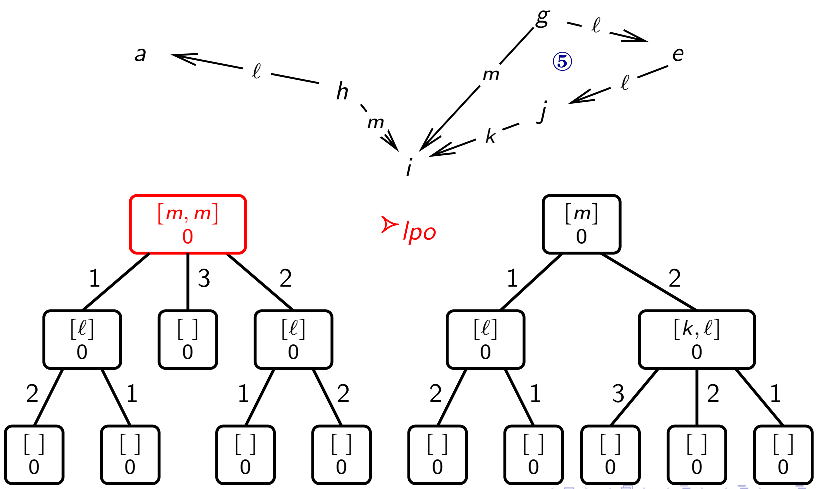

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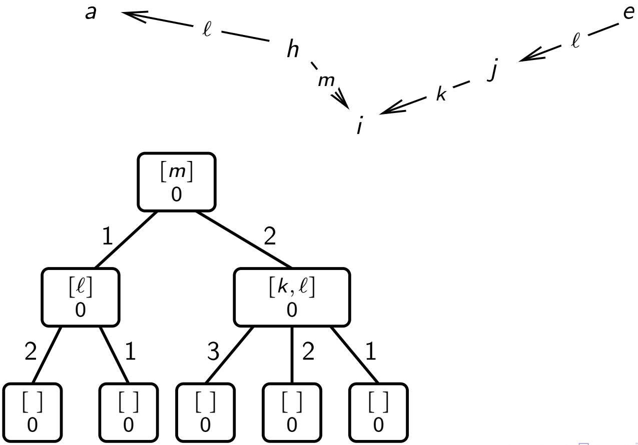

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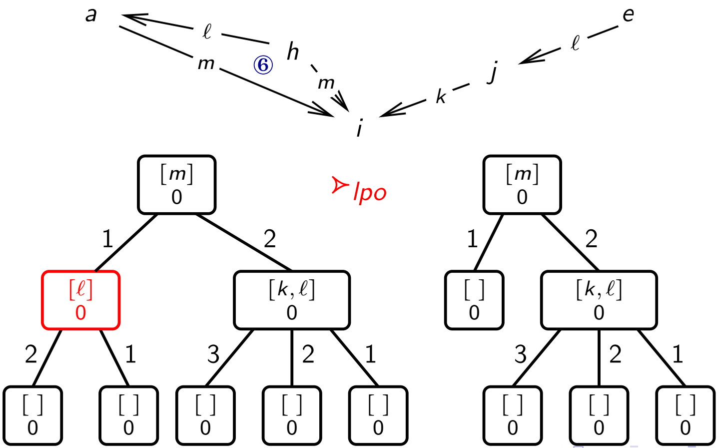

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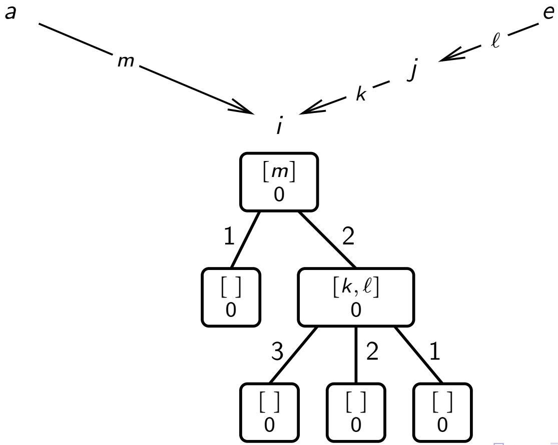

Conclusion

- alternative correctness proof of decreasing diagrams
  (De Bruijn,vO,Klop,de Vrijer,Bezem,Jouannaud)

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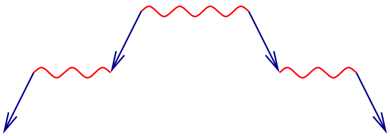

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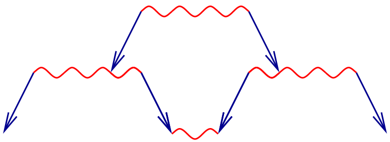

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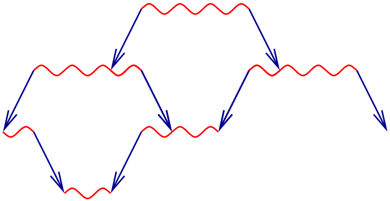

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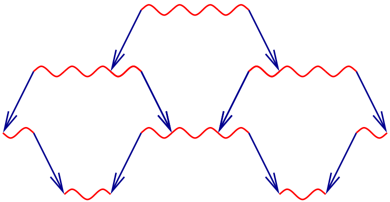

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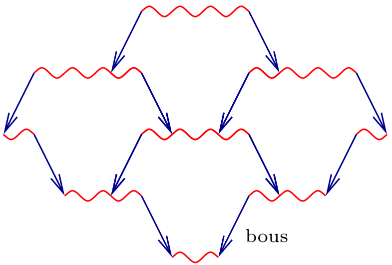

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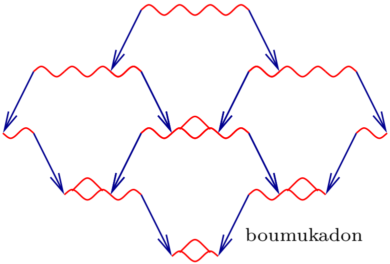

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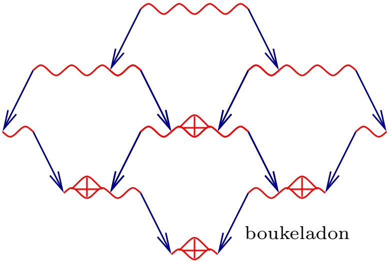

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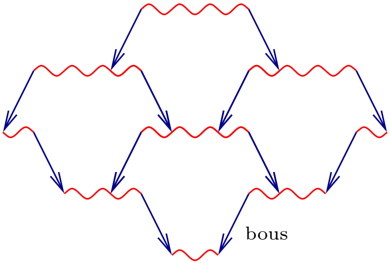
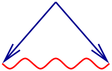
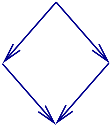


- Newman's Lemma (multiset)+Lemma of Hindley–Rosen (area)





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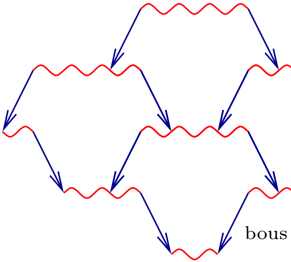
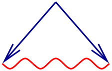
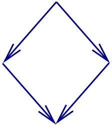


- Newman's Lemma+Lemma of Hindley–Rosen





Conclusion

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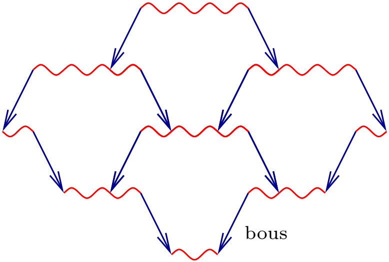

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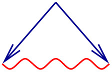

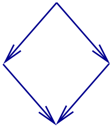

- decreasing diagrams modulo: involutive letters $\dot{\ell}$, i.e. $\dot{\ell}^{-1} = \dot{\ell}$

Conclusion

- alternative correctness proof of decreasing diagrams
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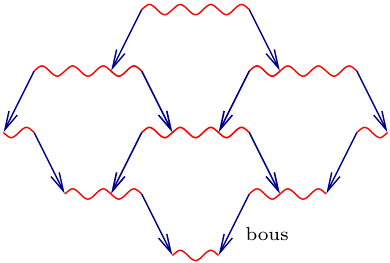

- Newman's Lemma+Lemma of Hindley–Rosen

- involutive rewriting ($\varrho : s \to r$ converse of $\varrho^{-1} : s^{-1} \to r^{-1}$)

Het kind en ik

Ik zou een dag uit vissen,
ik voelde mij moedeloos.
Ik maakte tussen de lissen
met de hand een wak in het kroos.

Er steeg licht op van beneden
uit de zwarte spiegelgrond.
Ik zag een tuin onbetreden
en een kind dat daar stond.

Het stond aan zijn schrijftafel
te schrijven op een lei.
Het woord onder de griffel
herkende ik, was van mij.

Maar toen heeft het geschreven,
zonder haast en zonder schroom,
al wat ik van mijn leven
nog ooit te schrijven droom.

En telkens als ik even
knikte dat ik het wist,
liet hij het water beven
en het werd uitgewist.

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TeReSe business meeting

din

ner