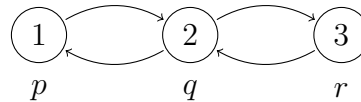


This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.
Explain your answers!!

- [1] Consider the boolean functions $f(x, y, z) = (x + y) \oplus (y + z)$ and $g(x, y) = \bar{x} \oplus y$.
- [6] (a) Is f monotone? Is f self-dual? Is f affine?
- [7] (b) Compute a reduced OBDD for $\overline{f(x, y, z)} + g(x, y)$ with variable ordering $[x, y, z]$.
- [7] (c) Can $g(x, y)$ be expressed (only) using f and the variables x and y ?

- [2] Consider the model $\mathcal{M}$:



- [7] (a) Use the CTL model checking algorithm to determine in which states of $\mathcal{M}$ the CTL formula $\varphi = \text{AG}(\text{E}[\text{EF } p \text{ U } r])$ holds.
- [6] (b) Consider the LTL formulas $\psi_1 = \text{G } p$, $\psi_2 = \text{F } q$, and $\psi_3 = \text{X } r$. For all $1 \leq i, j \leq 3$ determine whether $\mathcal{M}, i \models \psi_j$ holds or not.
- [7] (c) Show that the CTL formulas $\chi_1 = p \rightarrow \text{AG } q$ and $\chi_2 = \text{AG}(p \rightarrow q)$ are not equivalent.

- [3] For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.
- [6] (a) $p \vee \neg q, \neg p \rightarrow q, p \rightarrow r \vdash p \wedge r$
- [7] (b) $\forall x \forall y (Q(x, y) \rightarrow x = y), \exists x \exists y Q(x, y) \vdash \exists x \forall y Q(x, y)$
- [7] (c) $\forall x \forall y (x = y \rightarrow \neg P(x, y)), \forall x (Q(x) \rightarrow P(x, x)) \vdash \forall x \neg Q(x)$

- [7] 4 (a) Use resolution to determine whether the formula

$$\varphi = (r \rightarrow p \vee q) \wedge (p \vee q \vee r) \wedge \neg((q \rightarrow p) \rightarrow p)$$

is satisfiable or not.

- [6] (b) Use DPLL to determine satisfiability of the CNF

$$\psi = (p \vee q \vee r) \wedge (p \vee \neg q \vee r) \wedge (q \vee \neg r) \wedge (\neg q \vee \neg r) \wedge (\neg p \vee s) \wedge (\neg p \vee r \vee \neg s)$$

- [7] (c) Determine whether the terms $f(x, h(x), z, g(a))$ and $f(g(z), h(g(y)), y, x)$ are unifiable and compute a most general unifier if possible. Here x, y, z are variables and a is a constant.

- [20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement

The LTL formulas $\text{FGF}p$ and $\text{GF}p$ are equivalent.

Every prenex normal form is a Skolem normal form.

The boolean function $f(x, y) = x \oplus y \oplus x$ is monotone.

The proof rule $\perp\text{e}$ is a derived rule in natural deduction.

The boolean function $f(x, y, z) = x \oplus (y + \bar{z})$ is self-dual.

The terms $f(x, x, y, g(y))$ and $f(y, g(z), g(z), x)$ are unifiable.

The clause $\{p, \neg p\}$ is a resolvent of the clauses $\{\neg p, q\}$ and $\{p, \neg q\}$.

It is decidable whether a set of predicate logic formulas is consistent.

The sequent $\vdash \forall x \exists y (P(x) \rightarrow Q(y)) \rightarrow \forall x \neg(P(x) \wedge \exists y \neg Q(y))$ is valid.

If $f(x, y)$ is an adequate boolean function then also $g(x, y, z) = f(x, y) + z$ is adequate.