

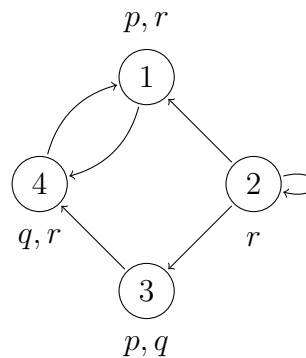
This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

- [1] Consider the boolean function f defined by $f(x, y, z) = x\bar{y} \oplus y\bar{z}$ and the following two reduced OBDDs:



- [6] (a) Compute a reduced OBDD for $f(x, y, z)$ with the variable ordering $[x, y, z]$.
 [7] (b) Compute $\text{apply}(+, B_g, B_h)$.
 [7] (c) Show that the set $\{f, \rightarrow\}$ is adequate.

- [2] Consider the model $\mathcal{M}$:



- [7] (a) Use the CTL model checking algorithm to determine in which states of $\mathcal{M}$ the CTL formula $\phi = \text{EG}(r \wedge \text{AFA}[q \text{ U } p])$ holds.
 [6] (b) Consider the LTL formula $\psi = \text{GF} p \rightarrow \text{F}(q \wedge r)$. For each state $1 \leq i \leq 4$ determine whether $\mathcal{M}, i \models \psi$ holds or not.
 [7] (c) Consider the CTL formula $\chi = \text{EF} p$. Either give an LTL formula χ' which is equivalent to χ or explain why χ is not expressible in LTL.

- [3] For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

- [6] (a) $\vdash \neg p \vee q \rightarrow (p \rightarrow q)$
 [6] (b) $\vdash ((p \rightarrow q) \rightarrow q) \rightarrow ((q \rightarrow p) \rightarrow p)$
 [8] (c) $\forall x (P(x) \vee Q(x)), \exists x \neg Q(x), \forall x (R(x) \rightarrow \neg P(x)) \vdash \exists x \neg R(x)$

- [6] [4] (a) Use resolution to determine satisfiability of the CNF

$$\phi = (\neg p \vee q \vee r) \wedge (p \vee \neg q \vee r) \wedge (p \vee q)$$

- [7] (b) Determine satisfiability of the Horn formula

$$\psi = (p \wedge q \rightarrow r) \wedge (\top \rightarrow p) \wedge (p \rightarrow q) \wedge (r \rightarrow \perp)$$

- [7] (c) Use DPLL to determine satisfiability of the CNF

$$\chi = (\neg 1 \vee 2 \vee 3) \wedge (\neg 1 \vee \neg 2) \wedge (\neg 1 \vee \neg 3) \wedge (1 \vee 4)$$

starting with the decision 1.

- [20] [5] Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement

$\neg \forall x P(x) \not\models \neg \exists y \neg P(y)$

Every unary affine boolean function is monotone.

The boolean function $f(x, y) = x \oplus y \oplus \bar{x}$ is affine.

Every Skolem normal form is a prenex normal form.

The set of propositional connectives $\{\top, \neg, \oplus\}$ is adequate.

There exists a sorting network for 4 wires with 4 comparators.

There exists an LTL formula without equivalent CTL formula.

The CTL formulas $A[\phi \text{ U } \psi]$ and $\neg(E[\neg\psi \text{ U } (\neg\phi \wedge \neg\psi)] \vee EG \psi)$ are equivalent.

The disjunction of all decision literals when a conflict occurs is a backjump clause in basic DPLL.

The set $\{\text{U}, \text{R}\}$ is an adequate set of connectives for the LTL fragment consisting of negation-normal forms without X .