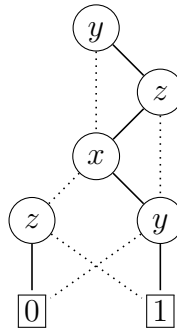


This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass. **Explain your answers to the first four exercises!**

- [1] Consider the boolean function $f(x, y, z) = \bar{x}y + x\bar{y}\bar{z}$ and the BDD B_g

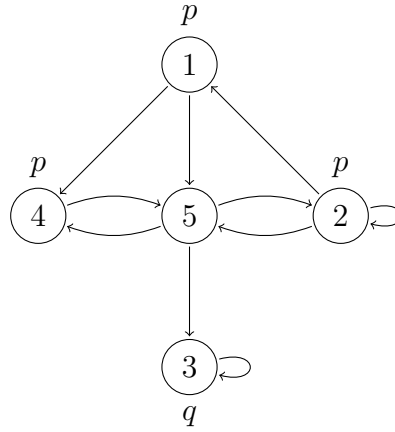


- [6] (a) Is f monotone? Is f self-dual?
- [8] (b) Compute the algebraic normal forms of f and g .
- [6] (c) Can $\bar{x}$ be expressed using f , g and the variable x ?
- [6] [2] (a) Determine whether the terms $f(g(x, y), h(z, z))$ and $f(g(g(w, z), a), w)$ are unifiable and compute a most general unifier if possible. Here, w, x, y, z are variables and a is a constant.
- [7] (b) Transform the following formula into an equisatisfiable Skolem normal form:
- $$\psi = \forall x \exists y (P(x, y) \wedge (\forall z (P(x, z) \wedge P(z, y))) \rightarrow Q(x, y))$$
- [7] (c) Use resolution to determine satisfiability of the clausal form
- $$\{\{P(x), P(f(x))\}, \{\neg P(x), \neg P(f(f(x)))\}\}$$

- [3] For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

- [6] (a) $\neg x \wedge \neg y \vdash \neg(x \vee y)$
- [7] (b) $\forall x (P(x) \vee Q(x)), \exists x \neg P(x) \vdash \forall x (R(x) \rightarrow \neg Q(x)) \rightarrow \exists x \neg R(x)$
- [7] (c) $\forall x (P(x) \vee Q(x)), \exists x \neg P(x) \vdash \forall x (R(x) \rightarrow \neg P(x)) \rightarrow \exists x \neg R(x)$

4 Consider the model $\mathcal{M}$:



- [6] (a) Use the CTL model checking algorithm to determine in which states of $\mathcal{M}$ the CTL formula $\varphi = \text{EXA}[\text{EF } p \cup \text{AG } q]$ holds.
- [7] (b) Construct an LTL formula ψ that distinguishes states 1 and 2.
- [7] (c) Transform φ into an equivalent CTL formula that uses only temporal connectives from $\{\text{AX}, \text{EG}, \text{EU}\}$.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement

Propositional resolution always terminates.

The LTL formula α is equivalent to the CTL* formula $\text{A}[\alpha]$.

Every predicate logic formula has an equivalent prenex normal form.

A predicate logic formula ϕ is valid if and only if $\neg\phi$ is *not* satisfiable.

For every boolean function f and variable x , $f = \bar{x} \cdot f[0/x] \oplus x \cdot f[1/x]$.

The set $\{\leftrightarrow, \neg\}$ is an adequate set of connectives for propositional logic.

There exists an algorithm for deciding validity of predicate logic formulas.

The set $\{\text{EX}, \text{EG}, \text{AU}\}$ is an adequate set of temporal connectives for CTL.

For every propositional formula an equivalent CNF can be computed in linear time.

The substitution $\{x \mapsto f(a, a), y \mapsto a\}$ is a most general unifier of the terms $f(a, y)$ and x .