universität innsbruck

Logik	SS 2022	LVA 703027
EXAM 2		September 27, 2022

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass. ***Explain your answers to the first four exercises!***

[7] **1** (a) Compute a reduced OBDD for the following boolean function f with the variable ordering $[x, y, z]$:

$$f(x, y, z) = (x \to y) \oplus (y \to z) \oplus (z \to x)$$

Here $x \to y$ abbreviates $\overline{x} + y$.

[5] (b) Compute the algebraic normal form of f.

[8] (c) Which of the five properties from Post's adequacy theorem does f satisfy? Which of the three sets $\{f, 0\}$, $\{f, 1\}$, $\{f, \overline{f}\}$ are adequate?

[6] **2** (a) Are the terms $s = g(f(x), y, a)$ and $t = g(z, h(x, z), x)$ unifiable? Compute a most general unifier if possible. Here a is a constant while x, y and z are variables.

[7] (b) Transform the following formula φ into an equisatisfiable Skolem normal form:

$$(\forall x\, \exists y\, P(y, g(y, f(x))) \wedge \neg \forall z\, Q(z)) \vee \neg \forall x\, \exists y\, P(x, y)$$

[7] (c) Use resolution to determine satisfiability of the clausal form

$$\{\{\neg P(f(b)),\, R(a)\},\, \{P(f(x)),\, Q(x, y)\},\, \{\neg R(x),\, \neg R(a)\},\, \{\neg Q(y, f(z))\}\}$$

Here a and b are constants.

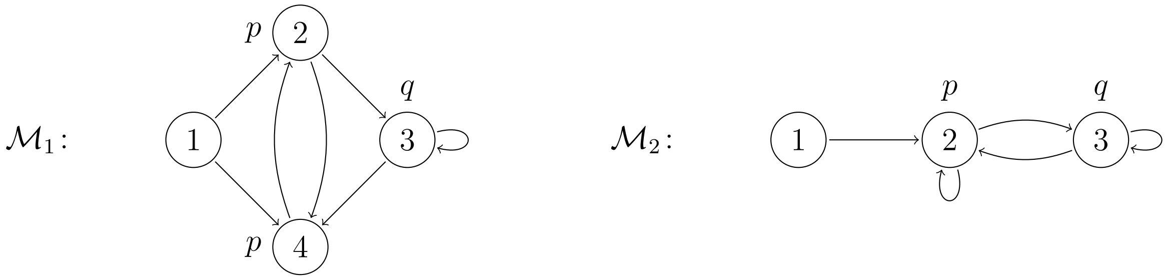
3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[7] (a) $\vdash\ ((p \to q) \to p) \to p$

[7] (b) $\forall x\, \forall y\, \exists z\, P(x, y, z)\ \vdash\ \forall x\, \exists z\, \forall y\, P(x, y, z)$

[6] (c) $\forall x\, \neg R(x, x),\ \forall x\, \exists y\, R(x, y)\ \vdash\ \neg \exists x\, \forall y\, (x = y)$

4 Consider the two models:



[7] (a) Use the CTL model checking algorithm to determine in which states of $\mathcal{M}_1$ the CTL formula $\varphi = \neg \mathsf{A}[\mathsf{AF}\, p \;\mathsf{U}\; \mathsf{EG}\, q]$ holds.

[6] (b) Determine in which states of $\mathcal{M}_2$ the LTL formulas $\psi_1 = \mathsf{F}\, p$, $\psi_2 = p \,\mathsf{U}\, \neg p$, and $\psi_3 = p \,\mathsf{W}\, q$ hold.

[7] (c) Find a CTL formula χ such that $\mathcal{M}_1, 1 \vDash \chi$ and $\mathcal{M}_2, 1 \nvDash \chi$.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
$\mathrm{HWB}_5(0,0,1,1,0) = 1$
Propositional tautologies are satisfiable.
Factoring is applicable to the clause $\{P(x),\ P(f(x))\}$.
The CTL* formula $\mathsf{A}[\mathsf{G}\,\mathsf{F}\, p \to \mathsf{F}\, q]$ is expressible in CTL.
The rule $\neg\neg$i is a derived proof rule in natural deduction.
The boolean function $f(x,y) = \overline{x}y \oplus xy \oplus y \oplus 1$ is self-dual.
In DPLL the backtrack rule can simulate the backjump rule.
The clauses $\{\neg p, q\}$ and $\{p, \neg q\}$ admit two different resolvents.
The terms $f(x, f(x,x))$ and $f(f(y,y), z)$ are unifiable. Here x, y and z are variables.
The automaton $A_{\neg\varphi}$ for the LTL formula $\varphi = (\mathsf{X}\, p)\,\mathsf{U}\, q$ contains the state $\{p, \neg q, \neg\varphi\}$.