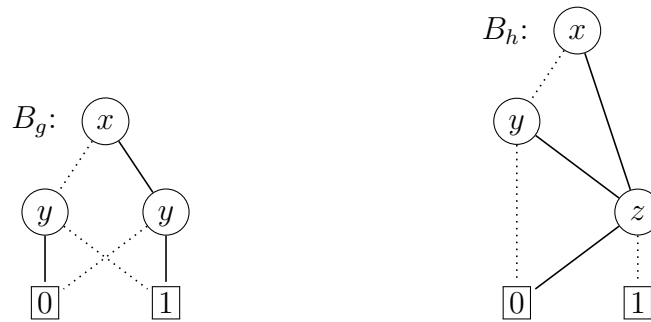


This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass. ***Explain your answers to the first four exercises!***

- [1] Consider the boolean function f defined by $f(x, y, z) = x(y \oplus z) \oplus (x + \bar{z})$ and the following two reduced OBDDs:



- [6] (a) Construct a reduced OBDD for f with variable ordering $[x, y, z]$.
 [7] (b) Compute $\text{apply}(\oplus, B_g, B_h)$.
 [7] (c) Determine which of the five properties from Post's adequacy theorem hold for f . Is the set $\{0, f\}$ adequate?

- [7] [2] (a) Use DPLL to determine satisfiability of

$$\varphi = (s \wedge r \rightarrow q) \wedge (\neg p \rightarrow \neg s) \wedge (\neg p \vee r) \wedge (\neg p \vee \neg q) \wedge (s \vee p) \wedge (\neg q \rightarrow s \vee \neg r)$$

- [7] (b) Determine satisfiability of the Horn formula

$$\psi = (r \wedge s \rightarrow q) \wedge (q \wedge u \rightarrow t) \wedge (\top \rightarrow p) \wedge (p \wedge r \rightarrow s) \wedge (t \wedge r \rightarrow \perp) \wedge (p \rightarrow r)$$

- [6] (c) Transform the following formula χ into an equisatisfiable Skolem normal form:

$$\chi = \exists x (\exists y P(x, y) \rightarrow \exists z H(f(z))) \rightarrow \forall z Q(z)$$

- [3] For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

- [6] (a) $\vdash \forall x (x = a \rightarrow x = b) \rightarrow a = b$
 [7] (b) $\forall x (P(x) \rightarrow Q(x)) \vdash \exists x P(x) \rightarrow \forall x Q(x)$
 [7] (c) $\forall x (P(x) \vee Q(x)) \vdash \forall x P(x) \vee \exists x Q(x)$

[4] This exercise is about the construction of the labeled Büchi automaton $A_{\neg\varphi}$ in the context of the LTL model checking algorithm. Let φ be an arbitrary LTL formula.

[4] (a) The closure $\mathcal{C}(\varphi)$ of φ consists of all subformulas φ and their negations, where $\neg\neg\psi$ and ψ are identified. Complete the following definition:

The states of the automaton $A_{\neg\varphi}$ are the maximal (with respect to $\subseteq$) subsets S of $\mathcal{C}(\varphi)$ such that

1. for all non-negated $\psi \in \mathcal{C}(\varphi)$ either $\psi \in S$ or $\boxed{}$ ¹
2. $\psi_1 \vee \psi_2 \in \mathcal{C}(\varphi) \implies (\psi_1 \vee \psi_2 \in S \iff \psi_1 \in S \text{ or } \psi_2 \in S)$
3. $\psi_1 \wedge \psi_2 \in \mathcal{C}(\varphi) \implies (\psi_1 \wedge \psi_2 \in S \iff \psi_1 \in S \text{ and } \psi_2 \in S)$
4. $\psi_1 \rightarrow \psi_2 \in \mathcal{C}(\varphi) \implies (\psi_1 \rightarrow \psi_2 \in S \iff \boxed{}^{\mathbf{2}})$
5. $\psi_1 \mathbf{U} \psi_2 \in S \implies \boxed{\phantom{\psi_1 \mathbf{U} \psi_2 \in S}}^{\mathbf{3}}$ or $\psi_2 \in S$
6. $\boxed{}^{\mathbf{4}} \implies \neg\psi_2 \in S$

[4] (b) Complete the following definition of the transition relation δ of $A_{\neg\varphi}$:

$(S, T) \in \delta$ if and only if the following conditions are satisfied:

1. $\mathbf{X}\psi \in S \implies \psi \in T$
2. $\neg\mathbf{X}\psi \in S \implies \boxed{}^{\mathbf{1}}$
3. $\psi_1 \mathbf{U} \psi_2 \in S$ and $\boxed{\phantom{\psi_1 \mathbf{U} \psi_2 \in T}}^{\mathbf{2}} \implies \psi_1 \mathbf{U} \psi_2 \in T$
4. $\neg(\psi_1 \mathbf{U} \psi_2) \in S$ and $\boxed{\phantom{\neg\psi_1 \mathbf{U} \psi_2 \in T}}^{\mathbf{3}} \implies \boxed{\phantom{\neg\psi_1 \mathbf{U} \psi_2 \in T}}^{\mathbf{4}}$

[4] (c) What are the initial states of $A_{\neg\varphi}$? A trace t is an infinite sequence of valuations of propositional atoms. When is t accepted by $A_{\neg\varphi}$?

[8] (d) Construct $A_{\neg\chi}$ for the LTL formula $\chi = p \mathbf{U} (\neg\mathbf{X}p)$ and determine which of the following traces are accepted by $A_{\neg\chi}$:

1. $\{p\}^\omega$
2. $\{p\} \emptyset \{p\}^\omega$
3. $\emptyset \{p\} \emptyset \{p\}^\omega$

- [20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement

Propositional resolution is sound and complete.

The CTL formula $\text{AG EF } q$ can be expressed in LTL.

The formulas $(p \vee q) \wedge \neg p$ and $q \wedge \neg q$ are equisatisfiable.

The term $f(y)$ is free for x in $\forall x \exists y (P(x) \wedge P(y) \wedge P(z))$.

Satisfaction of LTL formulas in finite models is undecidable.

Every propositional formula ϕ has a unique conjunctive normal form.

The LTL formulas $\text{G}(\phi \wedge \neg\psi)$ and $\text{G}\phi \wedge \neg\text{F}\psi$ are semantically equivalent.

The set $\{\text{EX}, \text{EG}, \text{EU}\}$ is an adequate set of temporal connectives for CTL.

Every boolean function $f: \{0, 1\}^n \rightarrow \{0, 1\}$ has a unique algebraic normal form.

If two terms s and t are not unifiable, then the unification algorithm on $s \approx t$ does not terminate.