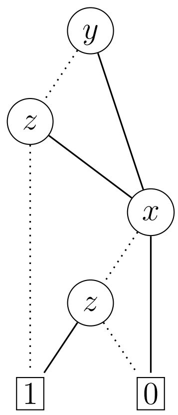
universität innsbruck

Logik	SS 2024	LVA 703026
EXAM 3		February 26, 2025

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass. ***Explain your answers to the first four exercises!***

1 Consider the boolean function $f(x,y,z) = (x + y \cdot z) \cdot \overline{y} \oplus \overline{z}$ and the BDD B_g



[7] (a) Transform B_g into an equivalent reduced OBDD with variable ordering $[x, y, z]$.

[6] (b) Compute the algebraic normal forms of f and g.

[5] (c) Determine which of the five properties from Post's adequacy theorem hold for f.

[2] (d) Can $\overline{x}$ be expressed only using f (and x)?

[7] 2 (a) Transform the following formula into an equisatisfiable Skolem normal form:

$$\neg(\forall x\, \exists y\, (P(x) \vee Q(y)) \rightarrow (\forall y\, (P(y) \wedge Q(y)) \wedge \exists x\, \neg Q(x)))$$

[5] (b) Compute a most general unifier of the terms $g(x, f(a, h(y)), x)$ and $g(b, z, y)$ or argue why this is not possible. Here, a and b are constants, while x, y and z are variables.

[8] (c) Use resolution to determine satisfiability of the clausal form

$$\{\, \{P(f(x), y),\ Q(w),\ P(z, y)\},\ \{\neg P(x, y),\ \neg P(f(z), y)\},\ \{\neg Q(a)\}\,\}$$

where a is a constant.

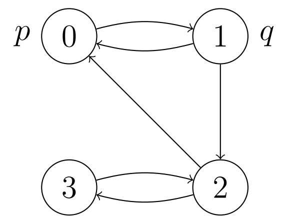
3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[6] (a) $\vdash\ (p \rightarrow q) \vee (q \rightarrow p)$

[7] (b) $\forall x\,(x = a \vee b = x),\ P(a),\ \exists x\,(x = b \wedge P(x))\ \vdash\ \forall x\, P(x)$

[7] (c) $\forall x\,(P(x) \vee Q(x))\ \vdash\ \forall x\, P(x) \vee \forall x\, Q(x)$

4 Consider the model $\mathcal{M}$:



[7] (a) Determine in which states of $\mathcal{M}$ the CTL formula $\varphi = \mathsf{AX}\,\mathsf{E}[\mathsf{EG}\,\neg q \,\mathsf{U}\, \mathsf{AF}\,p]$ holds.

[7] (b) Determine in which states of $\mathcal{M}$ the LTL formula $\psi = \neg q \,\mathsf{U}\, (p \vee \mathsf{G}\,\neg p)$ holds.

[6] (c) For each $0 \leqslant i \leqslant 3$ construct a CTL formula χ_i which holds only in state i.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
Every unary boolean function is affine.
The function $f(x, y) = xy \oplus x\overline{y}$ is self-dual.
Every valid sequent has a proof without ¬¬i.
The sequent $\forall x\, \exists y\, P(x, y) \vdash \exists y\, \exists x\, P(x, y)$ is valid.
Every Skolem normal form is in prenex normal form.
Every adequate set of temporal connectives for CTL contains AU.
$\mathcal{M}, s \vDash \mathsf{A}_{\{p\}}\mathsf{F}(q \rightarrow p)$ holds for every model $\mathcal{M}$ and every state s in $\mathcal{M}$.
$\{Q(h(x)), P(y, z)\}$ is a resolvent of $\{Q(x), Q(h(x))\}$ and $\{\neg Q(y), P(y, z)\}$.
The CTL* state formulas $\mathsf{E}[\mathsf{G}\,\mathsf{E}[\mathsf{F}\,p]]$ and $\mathsf{E}[\mathsf{G}\,\mathsf{F}\,p]$ are semantically equivalent.
The Horn formula $(p \wedge q \rightarrow \bot) \wedge (r \rightarrow \bot) \wedge (\top \rightarrow q) \wedge (q \wedge r \rightarrow \top)$ is satisfiable.