universität innsbruck

Logik	25S	LVA 703026
EXAM 1		June 23, 2025

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass. ***Explain your answers to the first four exercises!***

1 Consider the boolean functions $f(x,y) = (x+y) \oplus y$ and $g(x,y) = \overline{x}y$.

[6] (a) Is f monotone? Is f self-dual? Is f affine?

[7] (b) Compute a reduced OBDD for $\overline{f(x,y)} + g(x,y)$ with variable ordering $[x,y]$.

[7] (c) Can $g(x,y)$ be expressed (only) using f and the variables x and y?

2 In parts (b) and (c) of this exercise P is a unary predicate symbol, Q is a binary predicate symbol, and f and g are unary function symbols.

[6] (a) Use resolution to determine satisfiability of the following clausal form:

$$\{\{p\}, \{\neg p, q\}, \{\neg q, r\}, \{\neg p, \neg q, \neg r, \neg s\}\}$$

[7] (b) Transform the following formula into an equisatisfiable Skolem normal form:

$$\varphi = \forall x\,(P(x) \to \exists y\,(Q(x,y) \wedge Q(f(x), g(y))))$$

[7] (c) Use resolution to determine satisfiability of the following clausal form:

$$\{\{P(x), Q(x,y)\}, \{\neg Q(f(x),y)\}, \{\neg P(f(g(x)))\}\}$$

For each resolvent compute the mgu of the clashing literals.

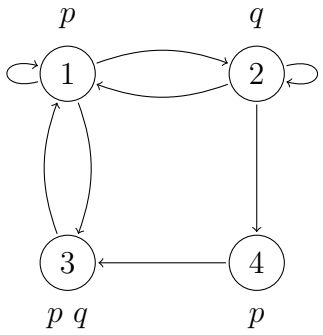
3 For each of the following sequents, either give a natural deduction proof or explain why such a proof does not exist.

[6] (a) $p \to q \to r \vdash r \to q \to p$

[7] (b) $\exists x\,(\neg P(x) \wedge \forall y\,(x = y \to \neg Q(y))) \vdash \neg\forall x\,(P(x) \vee Q(x))$

[7] (c) $\exists x\,(P(x) \wedge \exists y\,(\neg(x = y) \wedge \neg Q(y))) \vdash \neg\forall x\,(P(x) \wedge Q(x))$

4 Consider the following model $\mathcal{M}$:



[6] (a) Use the CTL model checking algorithm to determine in which states of $\mathcal{M}$ the CTL formula $\varphi = \mathsf{A}[\mathsf{EX}\,\neg p \vee \mathsf{AF}\,q\,\mathsf{U}\,\mathsf{EG}\,p]$ holds.

[7] (b) Determine in which states of $\mathcal{M}$ the LTL formulas $\psi_1 = p \to \mathsf{X}\,q$, $\psi_2 = \mathsf{F}(p \wedge q)$, and $\psi_3 = p\,\mathsf{U}\,q$ hold.

[7] (c) Find a CTL formula χ that is satisfied only in states 2 and 4 of $\mathcal{M}$.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
The sequent $\neg p \vdash p \vee \neg\neg p$ is valid.
Every finite LTL model satisfies either $\mathsf{F}\,p$ or $\mathsf{G}\,\neg p$.
The set of boolean functions $\{0, \overline{\ }, \oplus\}$ is adequate.
The term $g(f(x), y)$ is free for x in $\exists x\, P(x, g(f(x), y))$.
The boolean function $f(x, y, z) = x\overline{y} + zy$ is monotone.
$\mathcal{M}, s \vDash \mathsf{A}[p\,\mathsf{U}\,q] \to (\mathsf{AG}\,p \vee \mathsf{AF}\,q)$ for every model $\mathcal{M}$ and state s.
Validity of formulas in first-order logic is decidable.
The Skolem normal form of any valid formula in predicate logic is valid.
The clause $\{\neg P(a), R(b)\}$ is a resolvent of $\{\neg P(a), Q(x)\}$ and $\{\neg Q(z), R(b)\}$.
The number of nodes of a reduced OBDD for a boolean function depends on the variable order.