Logic 26S LVA 703026 + 703027

Lecture 3 March 26, 2026

Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. Solutions for bonus exercises must be submitted separately. The (strict) deadline is 7 am on March 26.

Exercises

⟨2⟩ 1. Prove the validity of the following sequents using natural deduction:
   - (a) $(q \wedge p) \vee p \vdash p$
   - (b) $p \rightarrow q \vdash (r \rightarrow p) \rightarrow (r \rightarrow q)$

⟨3⟩ 2. Prove the validity of the following sequents using natural deduction:
   - (a) $\vdash p \rightarrow (\neg p \rightarrow (p \rightarrow q))$
   - (b) $p \rightarrow (q \vee \neg r), \neg q, r \vdash \neg p$
   - (c) $p \wedge q \rightarrow r \vdash (p \rightarrow r) \vee (q \rightarrow r)$

⟨2⟩ 3. In this exercise we ban the negation operator $\neg$ from propositional logic and use $\varphi \rightarrow \bot$ to represent $\neg\varphi$. Naturally, such a logic cannot rely on the natural deduction rules for negation. Which of the rules $\neg$i, $\neg$e, $\neg\neg$i and $\neg\neg$e can you simulate with the remaining proof rules?

⟨3⟩ 4. In this exercise we consider the question whether there exists a blue/red coloring of the set of numbers $\{1, \ldots, n\}$ such that there is no monochromatic solution of the equation $a + b = c$ for $1 \leqslant a < b < c \leqslant n$. For instance, for $n = 5$ the answer is yes because the only solutions of the equation are

$$1 + 2 = 3 \qquad 1 + 3 = 4 \qquad 1 + 4 = 5 \qquad 2 + 3 = 5$$

and so we can (e.g.) use the coloring 1 2 3 4 5:

$$1 + 2 = 3 \qquad 1 + 3 = 4 \qquad 1 + 4 = 5 \qquad 2 + 3 = 5$$

   - (a) Show that the answer is yes when $n \leqslant 8$.
   - (b) Construct a CNF formula φ such that satisfiability of φ answers the question for $n = 9$, and encode φ into DIMACS format. Use a SAT solver to obtain the answer.

Bonus Exercise

The deadline for this exercise is April 16.

⟨5⟩ 5. Write a program that solves picture logic puzzles using a SAT solver. Use your program to solve the following puzzle:

	9 1 2 1 1	1 2 3 4 2 1 1	1 2 1 3 8 1	1 2 5 4 2 2 1	11 2 2 2 1 1	2 1 3 1 3 1 6 1	1 1 4 4 2 2	1 2 2 1 1 4 2	1 2 4 3 2 1	11 1 1 5	1 3 5 1 2 2	1 2 1 1 2 2 1	1 3 1 1 3 1	1 1 3 2 1 2 4 3	1 3 5 4 3 2	3 6 1 1 1 3 1	3 5 2 1 1 2 1	2 3 2 3 2 3 2	1 4 2 3 1 4	2 6 2 1	3 3 2 2 1	1 2 5 2 1	2 4 5 1 1 1	3 1 3 4 1 2 1 1	1 1 6 1 2 1 3 1	1 1 5 9 1 1	2 1 4 1 2 1 1 1	1 3 7 1 3 3 2	3 5 6 2 1 1	1 5 3 1 2 1 1 1
1 1 1 3 2 1																														
1 2 1 1 1 2 5																														
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