

Selected Solutions

2	(a)	1	p	assumption
		2	$\neg p$	assumption
		3	\perp	$\neg e$ 1, 2
		4	$p \rightarrow q$	$\perp e$ 3
		5	$\neg p \rightarrow (p \rightarrow q)$	$\rightarrow i$ 2-4
		6	$p \rightarrow (\neg p \rightarrow (p \rightarrow q))$	$\rightarrow i$ 1-5

(b)	1	$p \rightarrow (q \vee \neg r)$	premise
	2	$\neg q$	premise
	3	r	premise
	4	p	assumption
	5	$q \vee \neg r$	$\rightarrow e$ 1, 4
	6	q	assumption
	7	\perp	$\neg e$ 6, 2
	8	$\neg r$	assumption
	9	\perp	$\neg e$ 3, 8
	10	\perp	$\vee e$ 5, 6-7, 8-9
	11	$\neg p$	$\neg i$ 4-10

(c)	1	$p \wedge q \rightarrow r$	premise
	2	$\neg((p \rightarrow r) \vee (q \rightarrow r))$	assumption
	3	p	assumption
	4	q	assumption
	5	$p \wedge q$	$\wedge i$ 3, 4
	6	r	$\rightarrow e$ 1, 5
	7	$q \rightarrow r$	$\rightarrow i$ 4-6
	8	$(p \rightarrow r) \vee (q \rightarrow r)$	$\vee i_2$ 7
	9	\perp	$\neg e$ 8, 2
	10	r	$\perp e$ 9
	11	$p \rightarrow r$	$\rightarrow i$ 3-10
	12	$(p \rightarrow r) \vee (q \rightarrow r)$	$\vee i_1$ 11
	13	\perp	$\neg e$ 12, 2
	14	$(p \rightarrow r) \vee (q \rightarrow r)$	PBC 2-13

3 All of the rules involving negation can be simulated:

- $\neg i$:

1	φ	assumption
	\vdots	
n	\perp	
$n+1$	$\varphi \rightarrow \perp$	$\rightarrow i$ 1- n

- $\neg e$:

1	φ	premise
2	$\varphi \rightarrow \perp$	premise
3	\perp	$\rightarrow e$ 2, 1

• $\neg\neg i$:

1	φ	assumption
2	$\varphi \rightarrow \perp$	assumption
3	\perp	$\rightarrow e$ 2, 1
4	$(\varphi \rightarrow \perp) \rightarrow \perp$	$\rightarrow i$ 2-3

• $\neg\neg e$:

1	$(\varphi \rightarrow \perp) \rightarrow \perp$	premise
2	$\varphi \vee (\varphi \rightarrow \perp)$	LEM
3	φ	assumption
4	$\varphi \rightarrow \perp$	assumption
5	\perp	$\rightarrow e$ 1, 4
6	φ	$\perp e$ 5
7	φ	$\vee e$ 2, 3, 4-6

Note that we use LEM here. With some effort it can be shown that $\neg\neg e$ cannot be simulated by the other basic rules of natural deduction.