universität innsbruck

Logic | 26S | LVA 703026 + 703027

Lecture 4 | April 16, 2026

Selected Solutions

1 (a) The sequent $p,\ r \to \neg p,\ \neg r \wedge s \to t,\ \neg t \vdash \neg s$ is valid:

1	p	premise
2	$r \to \neg p$	premise
3	$\neg r \wedge s \to t$	premise
4	$\neg t$	premise
5	s	assumption
6	$\neg\neg p$	$\neg\neg$i 1
7	$\neg r$	MT 2, 6
8	$\neg r \wedge s$	$\wedge$i 7, 5
9	t	$\to$e 3, 8
10	$\bot$	$\neg$e 9, 4
11	$\neg s$	$\neg$i 5–10

(b) For the valuation $v(p) = \mathsf{F}$ and $v(q) = v(r) = \mathsf{T}$, the formula $p \to q \to r$ evaluates to T but $r \to q \to p$ evaluates to F. Hence, $p \to q \to r \not\models r \to q \to p$. Using soundness of natural deduction it follows that the sequent $p \to q \to r \vdash r \to q \to p$ is not valid.

(c) The sequent $p \vdash (p \wedge q) \vee ((q \to \bot) \wedge (r \to p))$ is valid:

1	p	premise
2	$q \vee \neg q$	LEM
3	q	assumption
4	$p \wedge q$	$\wedge$i 1, 3
5	$(p \wedge q) \vee ((q \to \bot) \wedge (r \to p))$	$\vee$i$_1$ 4
6	$\neg q$	assumption
7	q	assumption
8	$\bot$	$\neg$e 7, 6
9	$q \to \bot$	$\to$i 7–8
10	r	assumption
11	p	copy 1
12	$r \to p$	$\to$i 10–11
13	$(q \to \bot) \wedge (r \to p)$	$\wedge$i 9, 12
14	$(p \wedge q) \vee ((q \to \bot) \wedge (r \to p))$	$\vee$i$_2$ 13
15	$(p \wedge q) \vee ((q \to \bot) \wedge (r \to p))$	$\vee$e 2, 3–5, 6–14

3 (a) The BDD B is reduced as the transformations C1, C2, C3 are not applicable. The BDD B is not ordered: The root node demands that y precedes x, but the right node labeled x demands that x precedes y.

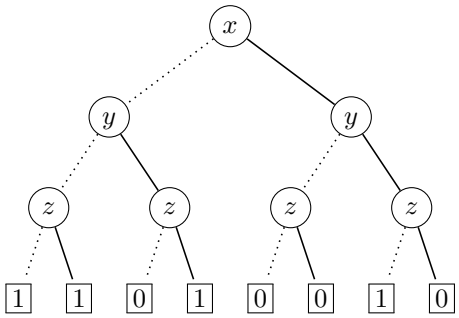
(b) We obtain the boolean function $f(x,y,z) = \overline{y}\,\overline{x} + y(\overline{x}\,\overline{y} + x(\overline{z}\,\overline{x} + z\overline{y}))$ by decomposing B. This can be simplified as follows:

$$f(x,y,z) \;=\; \overline{y}\,\overline{x} + y(\overline{x}\,\overline{y} + x(\overline{z}\,\overline{x} + z\overline{y})) \;=\; \overline{y}\,\overline{x} + yx(\overline{z}\,\overline{x} + z\overline{y})) \;=\; \overline{y}\,\overline{x}$$

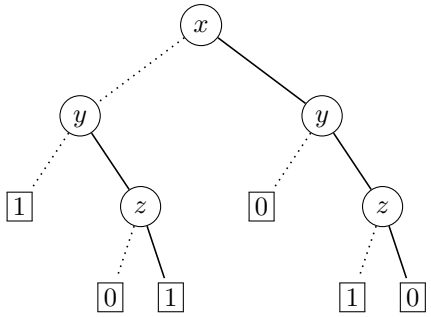
4 From the table

x	y	z	$f(x,y,z)$
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

we obtain the binary decision tree



This tree is an ordered BBD (with variable order $[x, y, z]$) but is not reduced because transformations C1 and C2 are applicable. Using transformation C2 twice produces



An application of transformation C1 produces the reduced OBDD