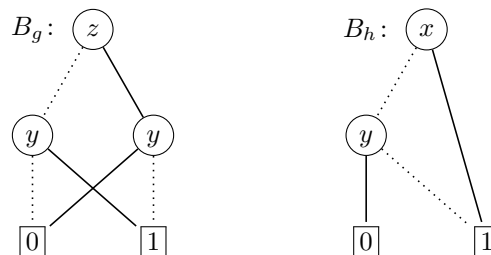


Solved exercises must be marked and solutions (as a single PDF file) uploaded in **OLAT**. The (strict) deadline is 7 am on April 23.

### Exercises

- (3) 1. Consider the function  $f(x, y, z) = \text{HWB}_4(x, x \oplus y, \bar{x}, z) \oplus x\bar{z}$  and the following two reduced OBDDs:



- (a) Construct a reduced OBDD for  $f$  with variable order  $[x, y, z]$ .
- (b) Compute  $\text{apply}(\cdot, B_g, B_h)$ .
- (c) Starting from  $B_g$ , compute a reduced OBDD that is equivalent to  $\exists y.g$ .
- (2) 2. let  $\varphi$  be a boolean formula.
- (a) Show that  $\varphi$  is satisfiable if and only if  $\exists x.\varphi$  is satisfiable.
- (b) Show that  $\varphi$  is valid if and only if  $\forall x.\varphi$  is valid.
- (3) 3. For each of the following sequents, either give a natural deduction proof or explain why such a proof does not exist:
- (a)  $\vdash (\top \rightarrow \perp) \vee (\perp \rightarrow \top)$
- (b)  $p, \neg q, p \wedge r \rightarrow q \vdash (q \rightarrow p) \rightarrow r$
- (c)  $\neg(p \rightarrow \neg q) \vdash p \wedge q$
- (2) 4. Consider the predicate logic formula  $\varphi = \forall x (P(y, f(z)) \vee \exists y (P(x, y) \rightarrow Q(x, g(y, z))))$ . Here  $f$  is a unary function symbol,  $g$  is a binary function symbol, and  $P$  and  $Q$  are binary predicate symbols.
- (a) Draw the parse tree of  $\varphi$  and list all its subformulas.
- (b) Which variable occurrences are bound? Which are free?
- (c) For each of the following terms  $t$ , compute  $\varphi[t/x]$ ,  $\varphi[t/y]$  and  $\varphi[t/z]$ . Is  $t$  free for  $x/y/z$  in  $\varphi$ ?
- $f(y)$
  - $g(x, z)$
  - $g(f(z), z)$