Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. Solutions for bonus exercises must be submitted separately. The (strict) deadline is 7 am on April 30.

Exercises

⟨2⟩ 1. Consider the sentence

$$\varphi \;=\; \forall x\, \exists y\, (P(x,y) \wedge \forall z\, (P(y,z) \to \neg P(z,x)))$$

Which of the following models satisfies φ?

(a) The model $\mathcal{M}_1$ consisting of $\mathbb{N}$ and the interpretation $P^{\mathcal{M}_1} = \{(a,b) \mid b = a+1\}$.

(b) The model $\mathcal{M}_2$ consisting of $\mathbb{N}$ and the interpretation $P^{\mathcal{M}_2} = \{(a,b) \mid 2a \neq b\}$.

(c) The model $\mathcal{M}_3$ consisting of $\mathbb{N}$ and the interpretation $P^{\mathcal{M}_3} = \{(a,b) \mid a < b\}$.

⟨2⟩ 2. Prove the validity of the following sequents using natural deduction:

(a) $Q(a) \vdash \forall x\, (x = a \to Q(x))$

(b) $\exists x\, \exists y\, (P(x,y) \vee P(y,x)),\ \neg \exists x\, P(x,x) \vdash \exists x\, \exists y\, \neg(x = y)$

For each application of $=$e, make sure to specify φ, t_1 and t_2.

⟨3⟩ 3. For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

(a) $\exists x\, (\neg P(x) \wedge \forall y\, (x = y \to \neg Q(y))) \vdash \neg \forall x\, (P(x) \vee Q(x))$

(b) $\exists x\, (\neg P(x) \wedge \forall y\, (\neg(x = y) \vee \neg Q(y))) \vdash \exists x\, (P(x) \vee Q(x))$

(c) $\exists x\, (P(x) \wedge \exists y\, (\neg(x = y) \wedge \neg Q(y))) \vdash \neg \forall x\, (P(x) \wedge Q(x))$

⟨3⟩ 4. Prove the validity of the following sequents using natural deduction:

(a) $\exists x\, (P \to Q(x)) \vdash P \to \exists x\, Q(x)$

(b) $\forall x\, Q(x) \to P \vdash \exists x\, (Q(x) \to P)$

(c) $P \to \exists x\, Q(x) \vdash \exists x\, (P \to Q(x))$

Bonus Exercises

⟨2⟩ 5. Prove the soundness of propositional resolution: A set S of clauses that admits a refutation is unsatisfiable.

⟨3⟩ 6. Prove the completeness of propositional resolution: Every unsatisfiable set S of clauses admits a refutation.