Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. The (strict) deadline is 7 am on May 7.

Exercises

⟨3⟩ 1. For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

   (a) $\exists x\,(P(x) \to Q) \vdash \forall x\,P(x) \to Q$

   (b) $\forall x\,\forall y\,(P(x) \vee Q(y)) \vdash \forall x\,P(x) \vee \forall y\,Q(y)$

   (c) $\forall x\,\exists y\,(P(y) \vee Q(x)) \vdash \exists y\,\forall x\,((P(y) \vee Q(x))$

⟨3⟩ 2. Which of the following pairs of atomic formulas are unifiable? For those that are, compute a most general unifier. Here a and b are constants, and x, y and z are variables.

   (a) $P(x, f(y, x))$ and $P(f(a, y), f(z, y))$

   (b) $Q(f(x, f(b, a)), g(x, f(b, y)))$ and $Q(f(g(y, b), z), g(g(a, b), z))$

   (c) $R(h(f(z, b), a, f(b, y)), f(y, x))$ and $R(h(y, z, f(x, f(a, x))), z)$

⟨2⟩ 3. Transform the following formulas into equisatisfiable Skolem normal forms.

   (a) $(\exists x\,\forall y\,\neg P(x, y)) \to (\forall y\,\exists x\,\neg P(x, y))$

   (b) $\forall x\,\exists y\,\forall z\,(Q(x, y, z) \to \exists w\,R(y, z, w))$

⟨2⟩ 4. Determine the validity of the following formula:

$$\exists x\,(D(x) \to \forall y\,D(y))$$

   This formula models the Drinker Paradox: *There is someone in the pub such that, if he or she is drinking, then everyone in the pub is drinking.*