universität innsbruck

Logic	26S	LVA 703026 + 703027
Lecture 8 + 9		May 21, 2026

Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. The (strict) deadline is 7 am on May 21.

Exercises

⟨1⟩ 1. (a) Use resolution to determine satisfiability of the following clausal form:

$$\{\{P(x), Q(x,y)\}, \{\neg Q(f(x),y)\}, \{\neg P(f(g(x)))\}\}$$

For each resolvent compute the mgu of the clashing literals.

⟨2⟩ (b) Use resolution to determine satisfiability of the following clausal form:

$$\{\{\neg P(x), Q(x)\}, \{\neg Q(a)\}, \{P(b), R(x,y)\}, \{S(x), \neg R(a,b)\}, \{\neg S(a)\}\}$$

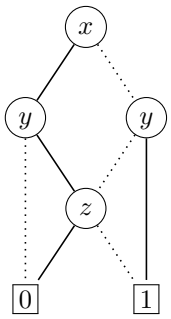
Here a and b are constants.

⟨2⟩ 2. Consider the PCP instance $C = ((0,00),(100,1))$.

(a) Construct the formula φ according to the proof on slide 30 of lecture 8.

(b) Is φ valid? If yes, provide a natural deduction proof of $\vdash \varphi$. If no, construct a model in which φ is not satisfied.

⟨1⟩ 3. Compute the algebraic normal form of the function f represented by the BDD

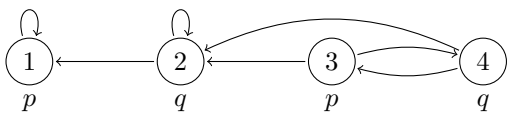


⟨2⟩ 4. Consider the boolean functions $f(x,y) = y \oplus (x + y)$ and $g(x,y) = \overline{x}y$.

(a) Is f monotone? Is f self-dual? Is f affine?

(b) Can $g(x,y)$ be expressed (only) using f and the variables x and y?

⟨2⟩ 5. Consider the following model $\mathcal{M}$:



and the formula $\varphi = \mathsf{EF}\,\mathsf{A}[\mathsf{EF}\,p\,\mathsf{U}\,\mathsf{AX}\,q]$.

(a) Draw the parse tree of φ and list all its subformulas.

(b) Use the CTL model checking algorithm to determine in which states of $\mathcal{M}$ φ holds.