universität innsbruck

Logic	26S	LVA 703026 + 703027
Lecture 8 + 9		May 21, 2026

Selected Solutions

1 (a) The following refutation shows that the clausal form is unsatisfiable:

1.	$\{P(x), Q(x,y)\}$		
2.	$\{\neg Q(f(x),y)\}$		
3.	$\{\neg P(f(g(x)))\}$		
4.	$\{\neg Q(f(x'),y')\}$	rename 2	$\{x \mapsto x', y \mapsto y'\}$
5.	$\{P(f(x'))\}$	resolve 1, 4	$\{x \mapsto f(x'), y \mapsto y'\}$
6.	$\square$	resolve 3, 5	$\{x' \mapsto g(x)\}$

In the first resolution step we solve the unification problem

$$\underline{Q(x,y) \approx Q(f(x'),y')}$$
$$\mathsf{d} \Downarrow$$
$$\underline{x \approx f(x'), y \approx y'}$$
$$\mathsf{v} \Downarrow \{x \mapsto f(x')\}$$
$$\underline{y \approx y'}$$
$$\mathsf{v} \Downarrow \{y \mapsto y'\}$$
$$\square$$

and obtain the mgu $\{x \mapsto f(x'), y \mapsto y'\}$. In the second resolution step we solve the unification problem

$$\underline{P(f(g(x))) \approx P(f(x'))}$$
$$\mathsf{d} \Downarrow$$
$$\underline{f(g(x)) \approx f(x')}$$
$$\mathsf{d} \Downarrow$$
$$\underline{g(x) \approx x'}$$
$$\mathsf{v} \Downarrow \{x' \mapsto g(x)\}$$
$$\square$$

and obtain the mgu $\{x' \mapsto g(x)\}$.

(b) Resolution produces the following clauses:

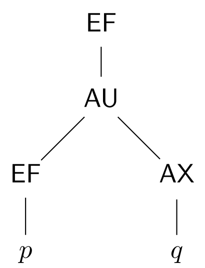
1.	$\{\neg P(x), Q(x)\}$		
2.	$\{\neg Q(a)\}$		
3.	$\{P(b), R(x,y)\}$		
4.	$\{S(x), \neg R(a,b)\}$		
5.	$\{\neg S(a)\}$		
6.	$\{\neg P(a)\}$	resolve 1, 2	$\{x \mapsto a\}$
7.	$\{\neg R(a,b)\}$	resolve 4, 5	$\{x \mapsto a\}$
8.	$\{P(b), R(u,v)\}$	rename 3	$\{x \mapsto u, y \mapsto v\}$
9.	$\{Q(b), R(u,v)\}$	resolve 1, 8	$\{z \mapsto b\}$
10.	$\{P(b), S(x)\}$	resolve 4, 8	$\{u \mapsto a, v \mapsto b\}$
11.	$\{P(b), S(z)\}$	rename 10	$\{x \mapsto z\}$
12.	$\{Q(b), S(z)\}$	resolve 1, 11	$\{x \mapsto b\}$
13.	$\{P(b)\}$	resolve 5, 10	$\{z \mapsto a\}$
14.	$\{Q(b)\}$	resolve 5, 12	$\{z \mapsto a\}$

As there are no further resolvents (modulo renaming), the formula is satisfiable.

3 Using Shannon's expansion we obtain

$$\begin{aligned}
f(x,y,z) &= \overline{x}(\overline{y}\,\overline{z} \oplus y) \oplus xy\overline{z} \\
&= (x \oplus 1)((y \oplus 1)(z \oplus 1) \oplus y) \oplus xy(z \oplus 1) \\
&= (x \oplus 1)(yz \oplus y \oplus z \oplus 1 \oplus y) \oplus xyz \oplus xy \\
&= (x \oplus 1)(yz \oplus z \oplus 1) \oplus xyz \oplus xy \\
&= xyz \oplus xz \oplus x \oplus yz \oplus z \oplus 1 \oplus xyz \oplus xy \\
&= xz \oplus x \oplus yz \oplus z \oplus 1 \oplus xy
\end{aligned}$$

5 (a) From the parse tree of φ



we obtain 6 subformulas:

$\mathsf{EF}\ \mathsf{A}[\mathsf{EF}\ p\ \mathsf{U}\ \mathsf{AX}\ q]$ $\qquad$ $\mathsf{A}[\mathsf{EF}\ p\ \mathsf{U}\ \mathsf{AX}\ q]$ $\qquad$ $\mathsf{EF}\ p$ $\qquad$ p $\qquad$ $\mathsf{AX}\ q$ $\qquad$ q

(b) From the table

	p	q	$\mathsf{EF}\ p$	$\mathsf{AX}\ q$	$\mathsf{A}[\mathsf{EF}\ p\ \mathsf{U}\ \mathsf{AX}\ q]$	φ
1	✓		✓			
2		✓	✓			
3	✓		✓	✓	✓	✓
4		✓	✓			✓

we conclude that the CTL formula $\varphi = \mathsf{EF}\ \mathsf{A}[\mathsf{EF}\ p\ \mathsf{U}\ \mathsf{AX}\ q]$ holds in states 3 and 4.