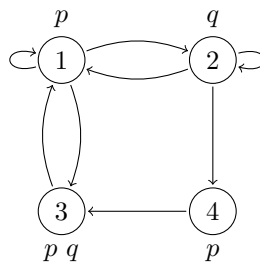


Solved exercises must be marked and solutions (as a single PDF file) uploaded in [OLAT](#). Solutions for bonus exercises must be submitted separately. The (strict) deadline is 7 am on May 28.

### Exercises

- (3) 1. Consider the boolean function  $f(x, y, z) = (((x \oplus y) \rightarrow \bar{z}) \rightarrow \bar{y}) \rightarrow \bar{x}$ . Here  $u \rightarrow v$  stands for  $\bar{u} + v$ .
  - (a) Compute the algebraic normal form of  $f$ .
  - (b) Is  $f$  monotone? Is  $f$  self-dual?
  - (c) Determine all minimal adequate subsets of  $\{\oplus, f, +, \bar{f}\}$ .
- (2) 2. Consider the LTL formula  $\varphi = \neg p \text{ U } (F q \vee G \neg p \rightarrow q \text{ U } \neg p)$ .
  - (a) Draw the parse tree of  $\varphi$  and list all its subformulas.
  - (b) Construct a model  $\mathcal{M}_1$  with two states such that  $\mathcal{M}_1, s \models \varphi$  for both states  $s$  of  $\mathcal{M}_1$ .
  - (c) Construct a model  $\mathcal{M}_2$  with two states such that  $\mathcal{M}_2, s \models \varphi$  for no state  $s$  of  $\mathcal{M}_2$ .
  - (d) Construct a model  $\mathcal{M}_3$  with two states such that  $\mathcal{M}_3, s \models \varphi$  for exactly one state  $s$  of  $\mathcal{M}_3$ .
- (2) 3. Prove the following equivalences.
  - (a)  $\varphi \text{ U } \psi \equiv \varphi \text{ W } \psi \wedge F \psi$
  - (b)  $\varphi \text{ W } \psi \equiv \psi \text{ R } (\varphi \vee \psi)$
- (3) 4. Consider the model  $\mathcal{M}$ :



- (a) Use the CTL model checking algorithm to determine in which states of  $\mathcal{M}$  the CTL formula  $\varphi = A[EX \neg p \vee AF q \text{ U } EG p]$  holds.
- (b) Determine in which states of  $\mathcal{M}$  the LTL formulas  $\psi_1 = p \rightarrow X q$ ,  $\psi_2 = F(p \wedge q)$  and  $\psi_3 = p \text{ U } q$  hold.
- (c) Find a CTL formula  $\chi$  that is satisfied only in states 2 and 4 of  $\mathcal{M}$ .

### Bonus Exercise

- (5) 5. Download the [Haskell file](#) for this exercise and complete the function `check` that implements the CTL model checking algorithm.