universität innsbruck

Logic	26S	LVA 703026 + 703027
Lecture 12		June 18, 2026

Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. The (strict) deadline is 7 am on June 18.

Exercises

⟨2⟩ 1. Which of the following pairs of CTL* formulas are equivalent?

(a) $\mathsf{A}[\mathsf{G}\,\mathsf{F}\,p]$ and $\mathsf{A}[\mathsf{G}\,\mathsf{E}[\mathsf{F}\,p]]$

(b) $\mathsf{A}[\mathsf{X}\,p \vee \mathsf{X}\,\mathsf{X}\,p]$ and $\mathsf{A}[\mathsf{X}\,p] \vee \mathsf{A}[\mathsf{X}\,\mathsf{A}[\mathsf{X}\,p]]$

(c) $\mathsf{E}[\mathsf{G}\,\mathsf{F}\,p]$ and $\mathsf{E}[\mathsf{G}\,\mathsf{E}[\mathsf{F}\,p]]$

(d) $\mathsf{A}[(p\,\mathsf{U}\,r) \vee (q\,\mathsf{U}\,r)]$ and $\mathsf{A}[(p \vee q)\,\mathsf{U}\,r]$

⟨2⟩ 2. Use DPLL to determine satisfiability of the formula (which answers exercise 4(b) of lecture 3)

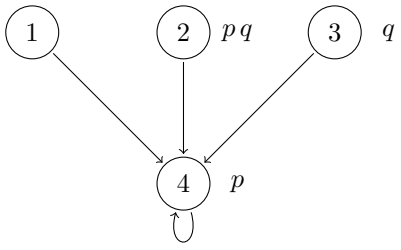
$$\begin{aligned}\varphi = {} & (x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2 \vee \neg x_3) \wedge (x_2 \vee x_4 \vee x_6) \wedge (\neg x_2 \vee \neg x_4 \vee \neg x_6) \wedge \\ & (x_1 \vee x_3 \vee x_4) \wedge (\neg x_1 \vee \neg x_3 \vee \neg x_4) \wedge (x_2 \vee x_5 \vee x_7) \wedge (\neg x_2 \vee \neg x_5 \vee \neg x_7) \wedge \\ & (x_1 \vee x_4 \vee x_5) \wedge (\neg x_1 \vee \neg x_4 \vee \neg x_5) \wedge (x_2 \vee x_6 \vee x_8) \wedge (\neg x_2 \vee \neg x_6 \vee \neg x_8) \wedge \\ & (x_1 \vee x_5 \vee x_6) \wedge (\neg x_1 \vee \neg x_5 \vee \neg x_6) \wedge (x_2 \vee x_7 \vee x_9) \wedge (\neg x_2 \vee \neg x_7 \vee \neg x_9) \wedge \\ & (x_1 \vee x_6 \vee x_7) \wedge (\neg x_1 \vee \neg x_6 \vee \neg x_7) \wedge (x_3 \vee x_4 \vee x_7) \wedge (\neg x_3 \vee \neg x_4 \vee \neg x_7) \wedge \\ & (x_1 \vee x_7 \vee x_8) \wedge (\neg x_1 \vee \neg x_7 \vee \neg x_8) \wedge (x_3 \vee x_5 \vee x_8) \wedge (\neg x_3 \vee \neg x_5 \vee \neg x_8) \wedge \\ & (x_1 \vee x_8 \vee x_9) \wedge (\neg x_1 \vee \neg x_8 \vee \neg x_9) \wedge (x_3 \vee x_6 \vee x_9) \wedge (\neg x_3 \vee \neg x_6 \vee \neg x_9) \wedge \\ & (x_2 \vee x_3 \vee x_5) \wedge (\neg x_2 \vee \neg x_3 \vee \neg x_5) \wedge (x_4 \vee x_5 \vee x_9) \wedge (\neg x_4 \vee \neg x_5 \vee \neg x_9)\end{aligned}$$

⟨3⟩ 3. Consider the propositional formulas $\varphi_n = (p_1 \leftrightarrow p_2 \leftrightarrow \cdots \leftrightarrow p_n)$ for $n > 0$. Let ψ_n be a CNF of φ_n.

(a) Prove that every clause in ψ_n contains exactly n literals.

(b) How many clauses does ψ_n have?

⟨3⟩ 4. Consider the following model $\mathcal{M}$



We want to use symbolic model checking to determine the states in which the CTL formula $\varphi = q \wedge \mathsf{EX}\,p$ holds. Using the constructions on slide 14 of lecture 10, perform the following tasks.

(a) Construct a reduced OBDD $B_{\to}$ that represents the transition relation of $\mathcal{M}$.

(b) Construct reduced OBDDs which represent the sets $[\![p]\!]$ and $[\![q]\!]$.

(c) Construct the reduced OBDD for the set $[\![q \wedge \mathsf{EX}\,p]\!]$. Use this BDD to determine in which states φ holds.