universität innsbruck

26S lecture 1

Logic

Luca Campa Philipp Dablander Aaron Groß Aart Middeldorp
Alexander Montag Johannes Niederhauser Vera Schmitt

Outline

VO is streamed and recorded

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Particify with session ID **6893 6178** for anonymous questions

ars.uibk.ac.at

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Particify ars.uibk.ac.at with session ID **6893 6178** for anonymous questions

Important Information

- LVA 703026 (VO 3) + 703027 (PS 2)

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- OLAT links for VO and PS

Time and Place

VO	Monday	8:30 – 11:00	HSB 1	Aart

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TU	Tuesday	15:30 – 16:15	HS G	Aaron

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PS	Thursday	8:15 – 9:45	HS 10	Alexander	group 1
	Thursday	12:00 – 13:30	HS 10	Vera	group 2
	Thursday	8:30 – 10:00	HSB 4	Luca	group 3
	Thursday	12:00 – 13:30	HSB 8	Philipp	group 4
	Thursday	8:30 – 10:00	SR 12	Vera	group 5
	Thursday	12:00 – 13:30	HS C	Johannes	group 6
	Thursday	12:00 – 13:30	HS E	Alexander	group 7

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Consultation Hours

Lucas Campa	3M03	Wednesday	10:00 – 11:30
Philipp Dablander	3M03	Wednesday	10:00 – 11:30
Aart Middeldorp	3M07	Monday	12:00 – 13:30
Alexander Montag	ÖH Technik	Monday	11:30 – 12:30
Johannes Niederhauser	3M03	Wednesday	10:00 – 11:30
Vera Schmitt			by arrangement

Schedule

lecture 1	02.03 & 05.03 & 12.03	lecture 8	04.05 & 21.05
lecture 2	09.03 & 19.03	lecture 9	11.05 & 21.05
lecture 3	16.03 & 26.03	lecture 10	18.05 & 28.05
lecture 4	23.03 & 16.04	lecture 11	01.06 & 11.06
lecture 5	13.04 & 23.04	lecture 12	08.06 & 18.06
lecture 6	20.04 & 30.04	lecture 13	15.06 & 25.06
lecture 7	27.04 & 07.05	lecture 14	22.06

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lecture 7	27.04 & 07.05	lecture 14	22.06

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- PS in presence

Grading — VO

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- second exam on September 15
- third exam on February 4, 2027

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grade : $[0,50) \to \mathbf{5}$ $[50,63) \to \mathbf{4}$ $[63,75) \to \mathbf{3}$ $[75,88) \to \mathbf{2}$ $[88,100] \to \mathbf{1}$

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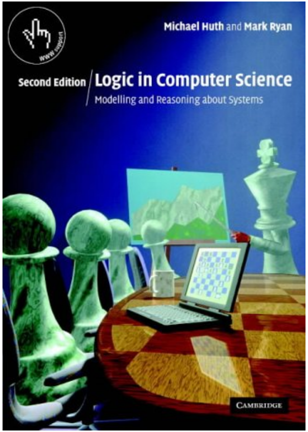

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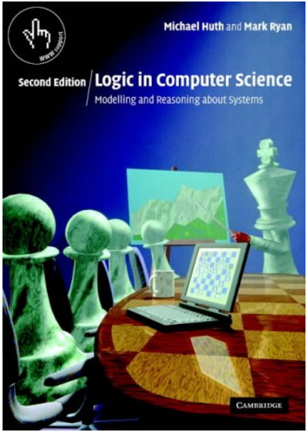

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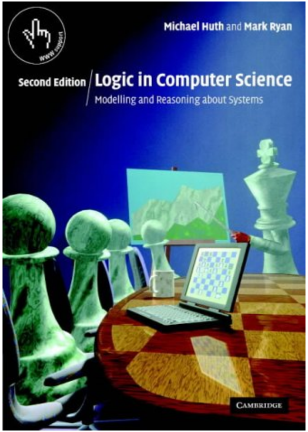

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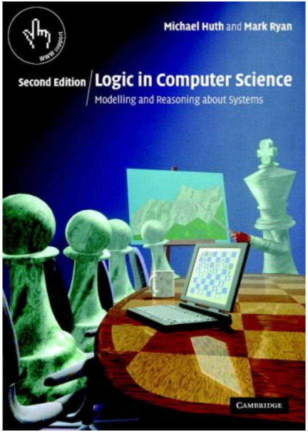

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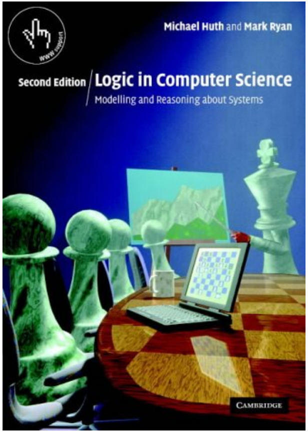

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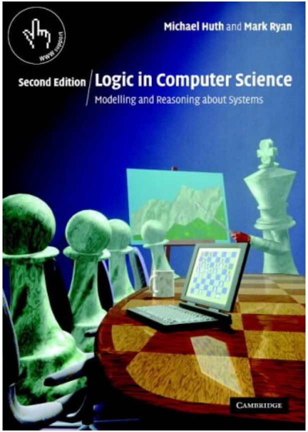

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- There were some times where I became lost on what was going on in the lecture, but that was more so a skill issue on my part being distracted and missing important parts.

Evaluation 25S (selected comments, cont'd)

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- Maybe make it more easier.

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Evaluation 25S (selected comments, cont'd)

- Zu viel Stoff vielleicht ein bisschen weniger da der abgedeckte Bereich riesig ist, dürfen wir jetzt einen cheatsheet nutzen bei der Klausur?
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- how well organized it is, especially when taking together with the PS. If you do homework a little bit early, there are parts that are kind of difficult, but the new material in the next lecture helps understanding the previous content way better. The topics either build on each other or start something else, which feels very like it's one continuous lesson. The lecturer is sometimes funny and that makes a lot of the content easier to memorize. Quizes during the break are also a pretty good way to help comprehend the content.

Evaluation 25S (selected comments, cont'd)

- I like that during the semester we revisit past topics to refresh our knowledge on past topics and towards the end of each lecture we sometimes do a task to prepare for the exam, which is very effective. Also questions are answered with engagement. I like that for mist theorems a proof was provided (and if not there was a good reasoning not to do so) Topics (other than ANF) where explained very intuitively

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- Best prepared VO I've visited sofar.

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- One of the best VOs I'νe visited
- Please make it easier maybe
- Thank you very much to teach us and i learned lots of things !

Outline

Logic in Innsbruck

Donnerstag, 11. Mai 2006 **WISSEN HEUTE** *LFU INNSBRUCK* **5**

ZEITTAFEL der LFU Innsbruck

1669	Gründung der Universität Innsbruck aus dem seit 100 Jahren bestehenden Jesuitengymnasiums durch Leopold I.
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1810	Aufhebung durch die Bayern

WISSENSWERT

Mit Maria-Theresia kam die Bibliothek an die Universität: Am 22. Mai 1745 genehmigte Maria Theresia die Errichtung einer Innsbrucker Bibliothek. Grundstein bildete die Büchersammlung der Tiroler Habsburger. Die Bibliothek war öffentlich zugänglich und die Benützerordnung war streng: Es durfte immer nur ein Buch vor Ort gelesen werden und auf Bücherentwendungen stand die Exmatrikulation.

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Formal Logic at Department of Christian Philosophy

Logic in Computer Science

- During the past 30 years, there has been an extensive and growing interaction between logic and computer science.
- Concepts and methods of logic occupy a central place in computer science, insomuch that logic has been called the calculus of computer science.
- Logic has been much more effective in computer science than it has been in mathematics.

Phokion G. Kolaitis, Moshe Y. Vardi (2001)

Example （数独 Sudoku）

	6		1		4		5	
		8	3		5	6		
2								1
8			4		7			6
		6				3		
7			9		1			4
5								2
		7	2		6	9		
	4		5		8		7	

propositional logic is very useful to quickly develop efficient solver for Sudoku

Example（数独 Sudoku）

8								
		3	6					
	7			9		2		
	5				7			
				4	5	7		
			1				3	
		1					6	8
		8	5			4	1	
	9							

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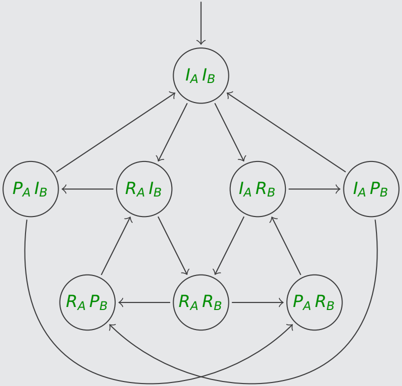
5								2
		7	2		6	9		
	4		5		8		7	

propositional logic is very useful to quickly develop efficient solver for Sudoku and all kinds of other tasks

Example (Printer Manager)



two users A and B

- I_i user i is idle
- R_i print request by user i
- P_i printing document for user i

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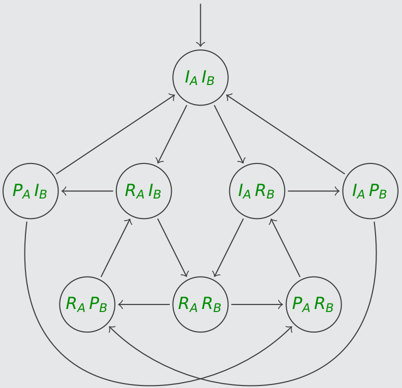

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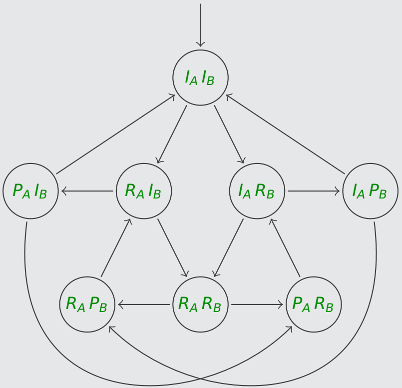

two users A and B

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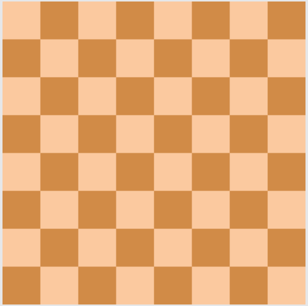
some questions

- is every P_i preceded by R_i ?
- is every R_i eventually followed by P_i ?

Example (Eight Queens Puzzle)



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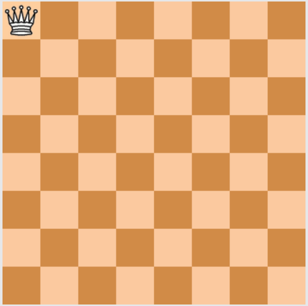

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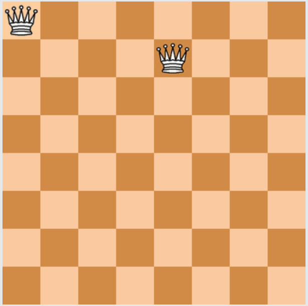

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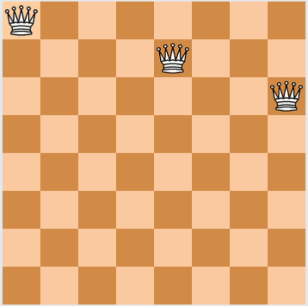

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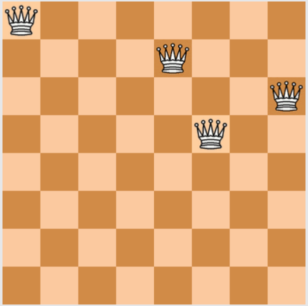

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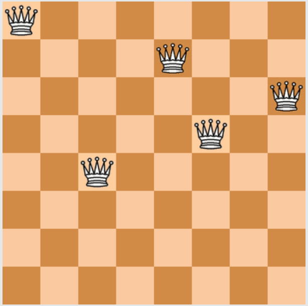

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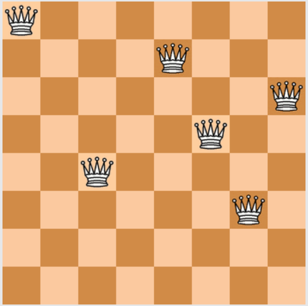

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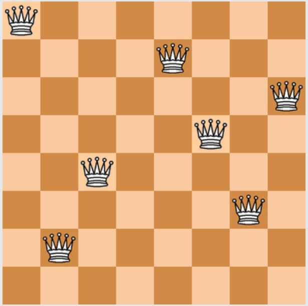

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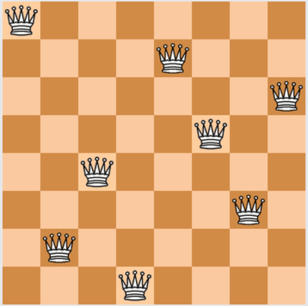

Prolog code

```
:- use_module(library(clpfd)).
nqueens(N,Qs) :-
  length(Qs,N), Qs ins 1 .. N,
  all_different(Qs),
  constraint_queens(Qs), label(Qs).
constraint_queens([]).
constraint_queens([Q|Qs]) :-
  noattack(Q,Qs,1),
  constraint_queens(Qs).
noattack(_,[],_).
noattack(X,[Q|Qs],N) :-
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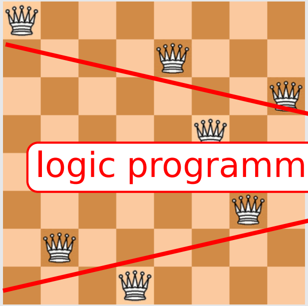

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```
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```

logic programming is sometimes taught in elective module

Example (Coloring Austria)

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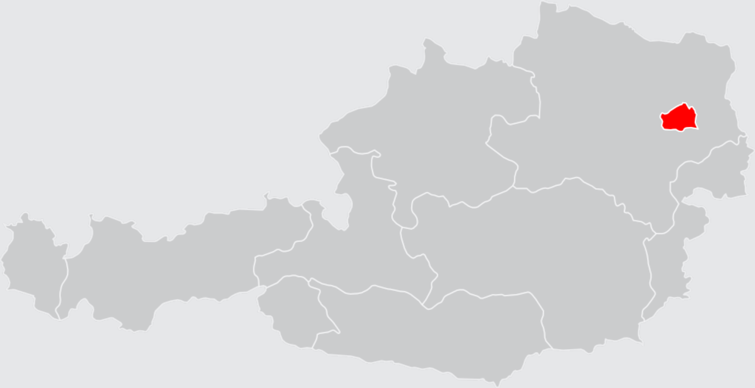

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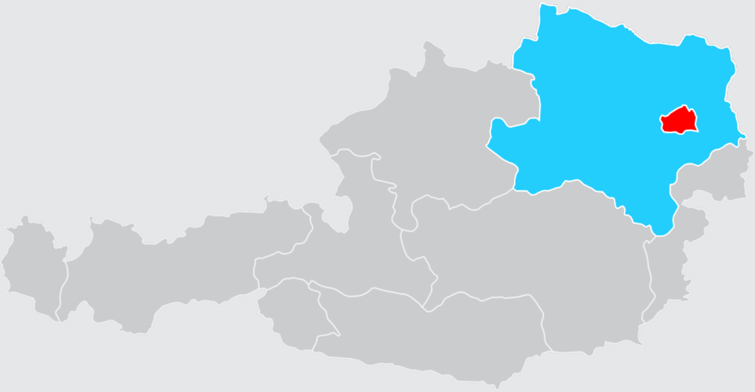

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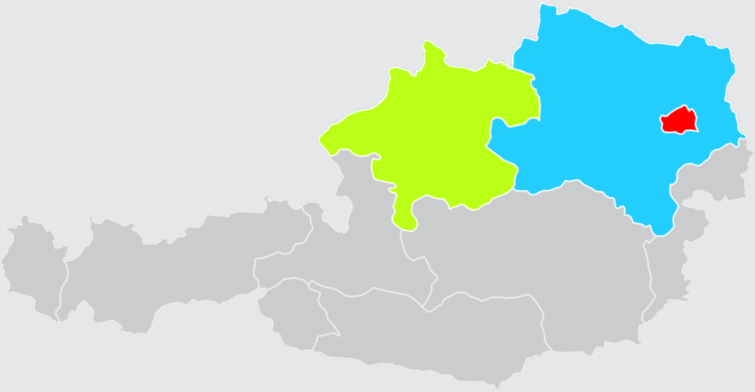

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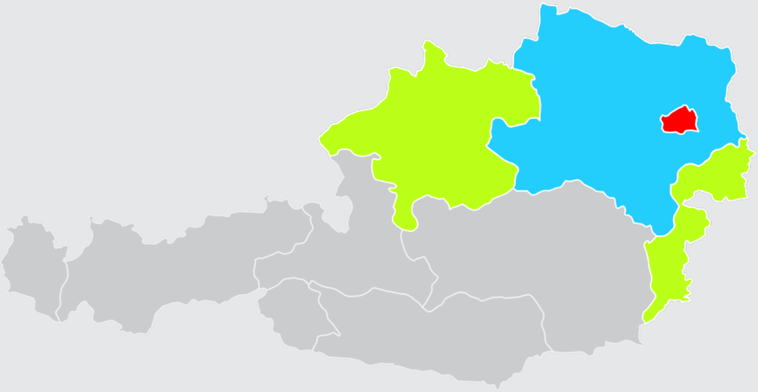

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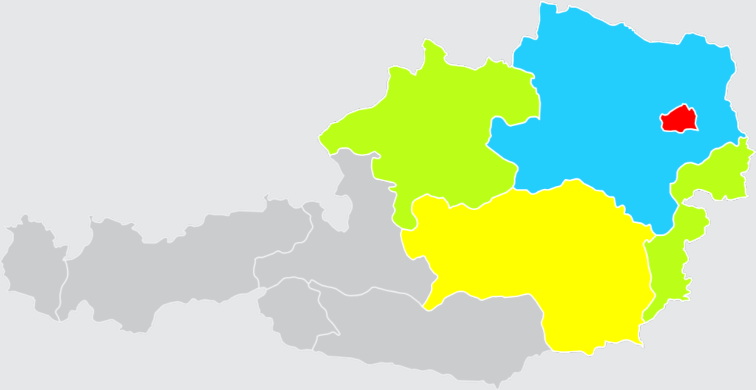

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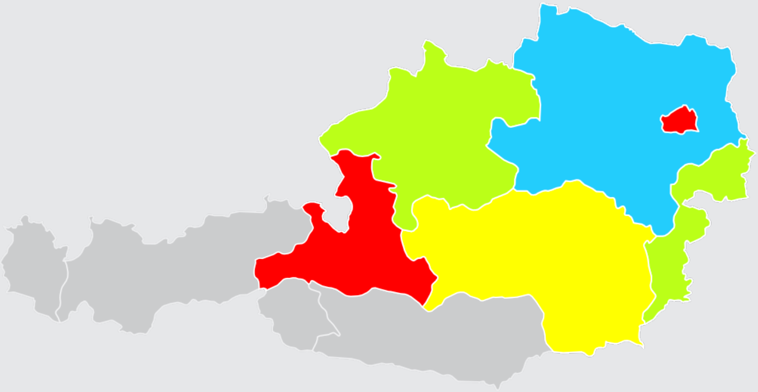

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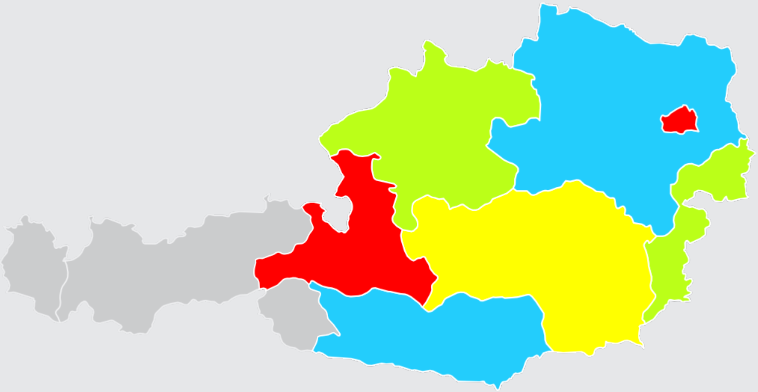

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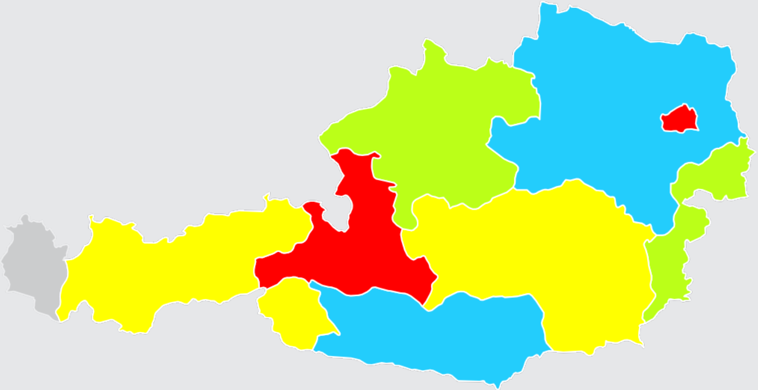

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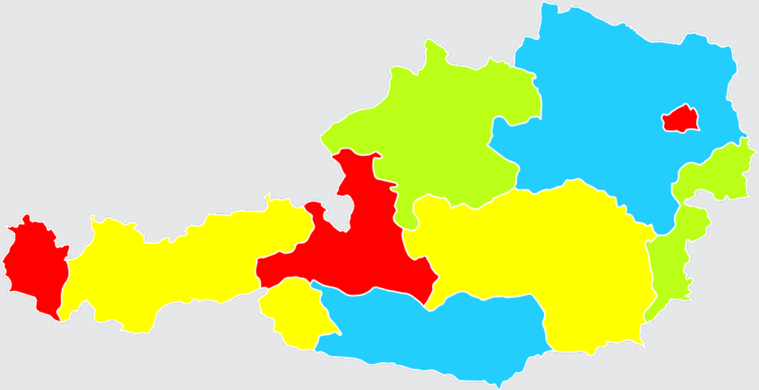

question: do three colors suffice to color Austria?

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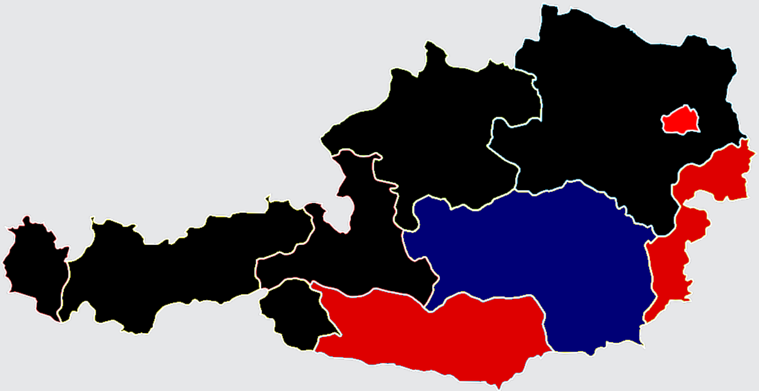

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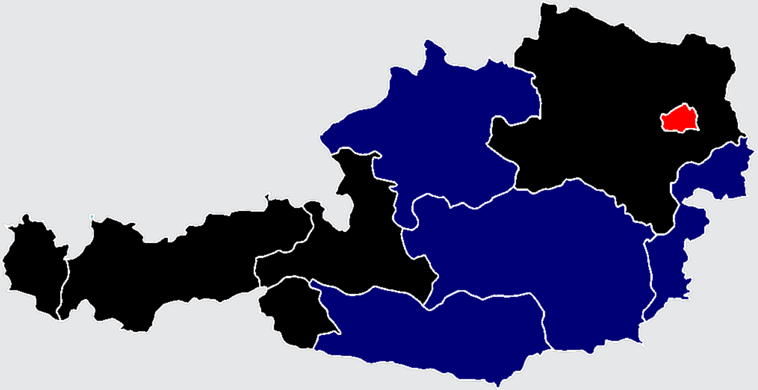

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Example (Programming Task)

function that computes $\sum_{i=1}^{n} i$ (exercise in C programming course at Nagoya University)

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some answers:

```
int sum(int x) {
  int i, j, z;
  z = 0;
  for (i = 0; i <= x; i++)
    for (j = 0; j < i; j++)
      z++;
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}
```

```
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```

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question: are these programs correct ?

Greek Alphabet

alpha	α	A	eta	η	H	nu	ν	N	tau	τ	T
beta	β	B	theta	θ ϑ	Θ	xi	ξ	Ξ	upsilon	υ	Υ
gamma	γ	Γ	iota	ι	I	omicron	o	O	phi	ϕ φ	Φ
delta	δ	Δ	kappa	κ	K	pi	π	Π	chi	χ	X
epsilon	ϵ ε	E	lambda	λ	Λ	rho	ρ	P	psi	ψ	Ψ
zeta	ζ	Z	mu	μ	M	sigma	σ ς	Σ	omega	ω	Ω

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delta	δ	Δ	kappa	κ	K	pi	π	Π	chi	χ	X
epsilon	ϵ ε	E	lambda	λ	Λ	rho	ρ	P	psi	ψ	Ψ
zeta	ζ	Z	mu	μ	M	sigma	σ ς	Σ	omega	ω	Ω

Natural Numbers

$0 \in \mathbb{N}$

Outline

Part I: Propositional Logic

algebraic normal forms, binary decision diagrams, conjunctive normal forms, DPLL, Horn formulas, natural deduction, Post's adequacy theorem, resolution, SAT, semantics, sorting networks, soundness and completeness, syntax, Tseitin's transformation

Part II: Predicate Logic

natural deduction, quantifier equivalences, resolution, semantics, Skolemization, syntax, undecidability, unification

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propositional formulas are built from

- atoms $p, q, r, p_1, p_2, \ldots$

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▸ atoms	$p,\ q,\ r,\ p_1,\ p_2,\ \dots$		
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according to following Backus–Naur Form:

$$\varphi ::= p \mid \bot \mid \top \mid (\neg \varphi) \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi) \mid (\varphi \rightarrow \varphi)$$

Notational Conventions

- binding precedence $\neg \quad > \quad \wedge, \vee \quad > \quad \rightarrow$

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- $\rightarrow$, $\wedge$, $\vee$ are right−associative: $p \rightarrow q \rightarrow r$ denotes $p \rightarrow (q \rightarrow r)$

Example

formula $\neg(\neg(p \vee (q \wedge \neg p)) \rightarrow r)$

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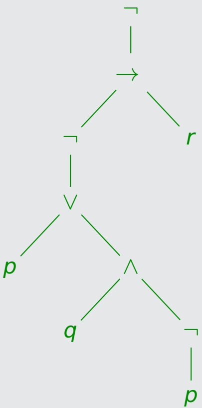
parse tree



Example

formula $\neg(\neg(p \vee (q \wedge \neg p)) \to r)$ $\neg\neg(p \vee q) \wedge (\neg p \to r)$

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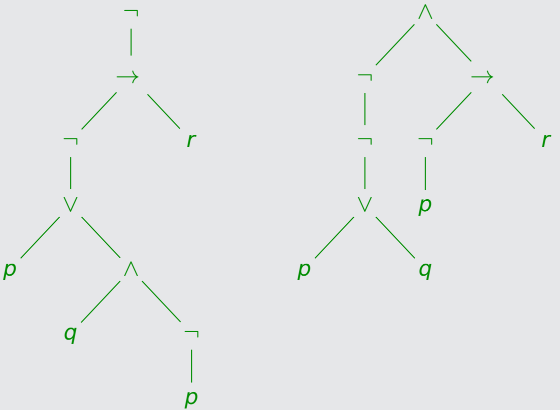

Example

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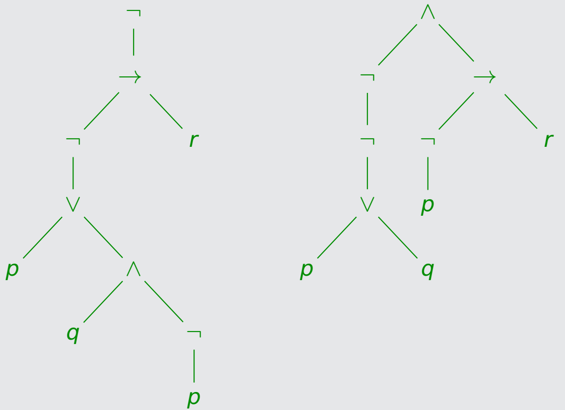
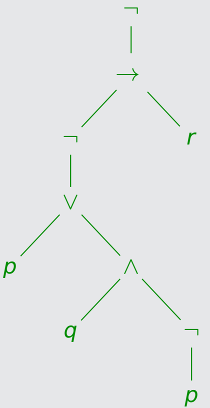
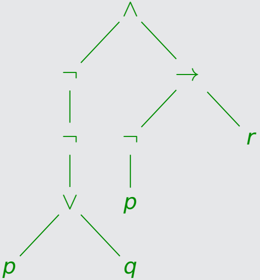


Example

formula	$\neg(\neg(p \vee (q \wedge \neg p)) \to r)$	$\neg\neg(p \vee q) \wedge (\neg p \to r)$	$\neg\neg p \vee q \wedge \neg p \to r$
parse tree			?

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Example

$v(p) = \mathsf{T}$ and $v(q) = \mathsf{F} \quad \Longrightarrow \quad \bar{v}(p \wedge \neg q \to \neg p) = \mathsf{F}$

Definition

truth tables

φ	$\neg\varphi$
T	F
F	T

φ	ψ	$\varphi \wedge \psi$	$\varphi \vee \psi$	$\varphi \rightarrow \psi$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

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F	F	F	F	T

truth tables for propositional formulas are constructed bottom−up

Example ❶

p	q	$(p \rightarrow \neg q) \rightarrow (q \vee \neg p)$
T	T	
T	F	
F	T	
F	F	

Example ❶

p	q	$\neg p$	$(p \to \neg q) \to (q \vee \neg p)$
T	T	F	
T	F	F	
F	T	T	
F	F	T	

Example ①

p	q	$\neg p$	$\neg q$	$(p \to \neg q) \to (q \vee \neg p)$
T	T	F	F	
T	F	F	T	
F	T	T	F	
F	F	T	T	

Example 1

p	q	$\neg p$	$\neg q$	$p \to \neg q$	$(p \to \neg q) \to (q \vee \neg p)$
T	T	F	F	F	
T	F	F	T	T	
F	T	T	F	T	
F	F	T	T	T	

Example ①

p	q	$\neg p$	$\neg q$	$p \rightarrow \neg q$	$q \vee \neg p$	$(p \rightarrow \neg q) \rightarrow (q \vee \neg p)$
T	T	F	F	F	T	
T	F	F	T	T	F	
F	T	T	F	T	T	
F	F	T	T	T	T	

Example ❶

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T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

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T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Example ❷

p	q	$(p \rightarrow \neg q) \rightarrow (q \vee \neg p)$
T	T	
T	F	
F	T	
F	F	

Example 1

p	q	$\neg p$	$\neg q$	$p \to \neg q$	$q \lor \neg p$	$(p \to \neg q) \to (q \lor \neg p)$
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Example 2

p	q	$(p \to \neg q) \to (q \lor \neg p)$
T	T	T
T	F	
F	T	T T
F	F	T

Example ❶

p	q	$\neg p$	$\neg q$	$p \to \neg q$	$q \vee \neg p$	$(p \to \neg q) \to (q \vee \neg p)$
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Example ❷

p	q	$(p \to \neg q)$	$\to$	$(q \vee \neg p)$
T	T		**T**	T
T	F			
F	T	T	**T**	T
F	F	T		

Example ①

p	q	$\neg p$	$\neg q$	$p \to \neg q$	$q \lor \neg p$	$(p \to \neg q) \to (q \lor \neg p)$
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Example ②

p	q	$(p \to$	$\neg q)$	$\to$	$(q \lor \neg p)$
T	T			T	T
T	F		T		
F	T	T		T	T
F	F	T			

Example ❶

p	q	$\neg p$	$\neg q$	$p \to \neg q$	$q \vee \neg p$	$(p \to \neg q) \to (q \vee \neg p)$
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Example ❷

p	q	$(p$	$\to$	$\neg q)$	$\to$	$(q$	$\vee$	$\neg p)$
T	T				T		T	
T	F		T	T				
F	T		T		T		T	
F	F		T					

Example ❶

p	q	$\neg p$	$\neg q$	$p \rightarrow \neg q$	$q \lor \neg p$	$(p \rightarrow \neg q) \rightarrow (q \lor \neg p)$
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Example ❷

p	q	$(p \rightarrow$	$\neg q)$	$\rightarrow$	$(q \lor$	$\neg p)$
T	T			T	T	
T	F	T	T			F
F	T	T		T	T	
F	F	T				T

Example ❶

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T	T	F	F	F	T	T
T	F	F	T	T	F	F
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F	F	T	T	T	T	T

Example ❷

p	q	$(p$	$\to$	$\neg q)$	$\to$	$(q$	$\vee$	$\neg p)$
T	T				**T**		T	
T	F		T	T			F	F
F	T		T		**T**		T	
F	F		T				T	T

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T	F	F	T	T	F	F
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Example ❷

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T	T				T		T	
T	F		T	T	F		F	F
F	T		T		T		T	
F	F		T		T		T	T

Definitions

- semantic entailment

$$\varphi_1, \varphi_2, \ldots, \varphi_n \vDash \psi$$

if $\bar{v}(\psi) = \mathsf{T}$ whenever $\bar{v}(\varphi_1) = \bar{v}(\varphi_2) = \cdots = \bar{v}(\varphi_n) = \mathsf{T}$ for every valuation v

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Examples ❶

p	q	$p \to q \vDash \neg p \vee q$
T	T	T
T	F	F
F	T	T
F	F	T

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Examples ❶

p	q	$p \to q$	$\vDash$	$\neg p \vee q$
T	T	T		T
T	F	F		
F	T	T		T
F	F	T		T

Definitions

- semantic entailment

$$\varphi_1, \varphi_2, \ldots, \varphi_n \vDash \psi$$

  if $\bar{v}(\psi) = \mathsf{T}$ whenever $\bar{v}(\varphi_1) = \bar{v}(\varphi_2) = \cdots = \bar{v}(\varphi_n) = \mathsf{T}$ for every valuation v

- tautology is formula φ such that $\vDash \varphi$

Examples ①

p	q	$p \to q$	$\vDash$	$\neg p \vee q$
T	T	T		T
T	F	F		
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F	T	T		T
F	F	T		T

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T	F	F
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F	F	T		T

p	q	$p \to q,$	$p \to \neg q$	$\vDash$	$\neg p$
T	T	T	F		
T	F	F			
F	T	T	T		T
F	F	T	T		T

Examples ❷

p	q	$p \to q \models q \to p$
T	T	T
T	F	F
F	T	T
F	F	T

Examples ❷

p	q	$p \to q$	$\vDash$	$q \to p$
T	T	T		T
T	F	F		
F	T	T		
F	F	T		

Examples ❷

p	q	$p \to q$	$\not\models$	$q \to p$
T	T	T		T
T	F	F		
F	T	T		F
F	F	T		

Examples ❷

p	q	$p \rightarrow q$	$\not\models$	$q \rightarrow p$
T	T	T		T
T	F	F		
F	T	T		F
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T	T	T		
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p	q	$p \to q$	$\not\models$	$q \to p$
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p	q	$p \rightarrow q$	$\not\models$	$q \rightarrow p$
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T	T	T	F		
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Examples ②

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T	T	T		T
T	F	F		
F	T	T		F
F	F	T		

p	q	$p \to q,$	$p \to \neg q$	$\not\models$	q
T	T	T	F		
T	F	F			
F	T	T	T		T
F	F	T	T		F

p	q	$p \to q,$	$p \wedge \neg q$	$\models$	$\bot$
T	T	T			
T	F	F			
F	T	T			
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p	q	$p \to q$	$\nvDash$	$q \to p$
T	T	T		T
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F	T	T		F
F	F	T		

p	q	$p \to q,$	$p \to \neg q$	$\nvDash$	q
T	T	T	F		
T	F	F			
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Question

$$\models (\neg p \wedge \neg q) \vee (s \wedge u) \vee (r \wedge w) \vee (\neg t \wedge \neg u) \vee (p \wedge r) \vee (q \wedge s) \vee (p \wedge t) \vee (q \wedge u) \vee (\neg r \wedge \neg s) \vee (t \wedge v) \vee (\neg v \wedge \neg w) \quad ?$$

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... truth table has $2^8 = 256$ rows ...

Question

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. . . truth table has $2^8 = 256$ rows . . .

	6		1		4		5	
		8	3		5	6		
2								1
8			4		7			6
		6				3		
7			9		1			4
5								2
		7	2		6	9		
	4		5		8		7	

. . . truth table has $2^{459} > 2^{4\times 100} = 16^{100} > 10^{100}$ rows . . .

Outline

Definitions

formula φ is

- valid if $\bar{v}(\varphi) = \mathsf{T}$ for every valuation v

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formula φ is valid $\iff$ $\neg\varphi$ is unsatisfiable

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formula φ is valid $\iff$ $\neg\varphi$ is unsatisfiable

Proof

$$\begin{aligned} \varphi \text{ is valid} &\iff \bar{v}(\varphi) = \mathsf{T} \quad \text{for every valuation } v \\ &\iff \bar{v}(\neg\varphi) = \mathsf{F} \quad \text{for every valuation } v \end{aligned}$$

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$\iff$ $\neg\varphi$ is not satisfiable

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&\iff \bar{v}(\neg\varphi) = \mathsf{T} \quad \text{for no valuation } v \\
&\iff \neg\varphi \text{ is not satisfiable} \iff \neg\varphi \text{ is unsatisfiable}
\end{aligned}
$$

Definition

formulas φ and ψ are semantically equivalent ($\varphi \equiv \psi$) if both $\varphi \vDash \psi$ and $\psi \vDash \varphi$

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$$\neg(\varphi \vee \psi) \equiv \neg\varphi \wedge \neg\psi \qquad \neg\neg\varphi \equiv \varphi \qquad \varphi \vee (\psi \wedge \chi) \equiv (\varphi \vee \psi) \wedge (\varphi \vee \chi)$$

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construct truth table of φ

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construct truth table of φ and inspect last column:

- φ is valid if and only if all entries are T

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$\neg(\varphi \vee \psi) \equiv \neg\varphi \wedge \neg\psi$ $\qquad$ $\neg\neg\varphi \equiv \varphi$ $\qquad$ $\varphi \vee (\psi \wedge \chi) \equiv (\varphi \vee \psi) \wedge (\varphi \vee \chi)$

Theorem

validity and satisfiability are decidable

Proof

construct truth table of φ and inspect last column:

- φ is valid if and only if all entries are T
- φ is satisfiable if and only if T entry exists

Outline

Particify with session ID **6893 6178**

Question

Given that one and only one answer is correct, which of the following is true ?

- **A** All of the below.
- **B** None of the below.
- **C** One of the above.
- **D** All of the above.
- **E** None of the above.
- **F** None of the above.

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- ✅ None of the above.
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validity of CNFs is efficiently decidable

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validity of CNFs is efficiently decidable:

CNF φ is valid $\iff$ every clause of φ contains complementary literals

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- literals ℓ_1 and ℓ_2 are complementary if $\ell_1 = \neg \ell_2$ or $\neg \ell_1 = \ell_2$

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validity of CNFs is efficiently decidable:

$$\text{CNF } \varphi \text{ is valid} \iff \text{every clause of } \varphi \text{ contains complementary literals}$$

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Examples

1. CNF

$$(p \vee q \vee \neg r) \wedge (\neg p \vee \neg r \vee p) \wedge (\neg q)$$

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$$(p \vee q \vee \neg r) \wedge (\neg p \vee \neg r \vee p) \wedge (\neg q)$$

clause

Examples

1. CNF

$$(p \lor q \lor \neg r) \land (\neg p \lor \neg r \lor p) \land (\neg q)$$

complementary literals

Examples

1. CNF

$$(p \vee q \vee \neg r) \wedge (\neg p \vee \neg r \vee p) \wedge (\neg q)$$

not valid

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not valid witness: $v(p) = v(q) = \mathrm{F}$ and $v(r) = \mathrm{T}$

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2. CNF

$$(p \vee q \vee \neg p) \wedge (\neg r \vee \neg p \vee r) \wedge (\neg q \vee q)$$

valid

Examples

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Special Cases

- $\bot$ represents empty clause (no literals)

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2. CNF

$$(p \vee q \vee \neg p) \wedge (\neg r \vee \neg p \vee r) \wedge (\neg q \vee q)$$

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Special Cases

- $\bot$ represents empty clause (no literals)
- $\top$ represents empty CNF (no clauses)

Theorem

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Procedure

① eliminate implications

$$\varphi \to \psi \quad \xrightarrow{①} \quad \neg\varphi \vee \psi$$

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Procedure

① eliminate implications

② push negations towards atoms and remove double negations

$$\varphi \rightarrow \psi \xrightarrow{①} \neg\varphi \vee \psi \qquad \neg\neg\varphi \xrightarrow{②} \varphi$$

$$\neg(\varphi \wedge \psi) \xrightarrow{②} \neg\varphi \vee \neg\psi$$

$$\neg(\varphi \vee \psi) \xrightarrow{②} \neg\varphi \wedge \neg\psi$$

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Remark

CNF ψ for formula φ might be exponentially larger

Example (CNFs are not unique)

$$\varphi = \neg(p \vee (q \wedge r))$$

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$$\varphi = \neg(p \vee (q \wedge r))$$

$$\xrightarrow{③} \neg((p \vee q) \wedge (p \vee r)) \xrightarrow{②} \neg(p \vee q) \vee \neg(p \vee r)$$

$$\xrightarrow{②} (\neg p \wedge \neg q) \vee \neg(p \vee r) \xrightarrow{②} (\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)$$

$$\xrightarrow{③} ((\neg p \wedge \neg q) \vee \neg p) \wedge ((\neg p \wedge \neg q) \vee \neg r)$$

$$\xrightarrow{③} ((\neg p \vee \neg p) \wedge (\neg q \vee \neg p)) \wedge ((\neg p \wedge \neg q) \vee \neg r)$$

$$\xrightarrow{③} ((\neg p \vee \neg p) \wedge (\neg q \vee \neg p)) \wedge ((\neg p \vee \neg r) \wedge (\neg q \vee \neg r))$$

Example (CNFs are not unique)

$$\varphi = \neg(p \vee (q \wedge r))$$

$$\xrightarrow{②} \neg p \wedge \neg(q \wedge r) \xrightarrow{②} \neg p \wedge (\neg q \vee \neg r)$$

$$\varphi = \neg(p \vee (q \wedge r))$$

$$\xrightarrow{③} \neg((p \vee q) \wedge (p \vee r)) \xrightarrow{②} \neg(p \vee q) \vee \neg(p \vee r)$$

$$\xrightarrow{②} (\neg p \wedge \neg q) \vee \neg(p \vee r) \xrightarrow{②} (\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)$$

$$\xrightarrow{③} ((\neg p \wedge \neg q) \vee \neg p) \wedge ((\neg p \wedge \neg q) \vee \neg r)$$

$$\xrightarrow{③} ((\neg p \vee \neg p) \wedge (\neg q \vee \neg p)) \wedge ((\neg p \wedge \neg q) \vee \neg r)$$

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$$\xrightarrow{②} (\neg p \wedge \neg q) \vee \neg(p \vee r) \xrightarrow{②} (\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)$$

$$\xrightarrow{③} ((\neg p \wedge \neg q) \vee \neg p) \wedge ((\neg p \wedge \neg q) \vee \neg r)$$

$$\xrightarrow{③} ((\neg p \vee \neg p) \wedge (\neg q \vee \neg p)) \wedge ((\neg p \wedge \neg q) \vee \neg r)$$

$$\xrightarrow{③} ((\neg p \vee \neg p) \wedge (\neg q \vee \neg p)) \wedge ((\neg p \vee \neg r) \wedge (\neg q \vee \neg r))$$

CNFs are not unique, even if rules ①, ②, ③ are applied in order

Procedure (extended)

⓪ simplify formulas with $\bot$ and $\top$

① eliminate implications

② push negations towards atoms and remove double negations

③ distribute disjunction over conjunction

Procedure (extended)

⓪ simplify formulas with $\bot$ and $\top$

① eliminate implications

② push negations towards atoms and remove double negations

③ distribute disjunction over conjunction

$\neg\bot \xrightarrow{⓪} \top$	$\bot \wedge \varphi \xrightarrow{⓪} \bot$	$\bot \vee \varphi \xrightarrow{⓪} \varphi$	$\bot \to \varphi \xrightarrow{⓪} \top$
$\neg\top \xrightarrow{⓪} \bot$	$\top \wedge \varphi \xrightarrow{⓪} \varphi$	$\top \vee \varphi \xrightarrow{⓪} \top$	$\top \to \varphi \xrightarrow{⓪} \varphi$
	$\varphi \wedge \bot \xrightarrow{⓪} \bot$	$\varphi \vee \bot \xrightarrow{⓪} \varphi$	$\varphi \to \bot \xrightarrow{⓪} \neg\varphi$
	$\varphi \wedge \top \xrightarrow{⓪} \varphi$	$\varphi \vee \top \xrightarrow{⓪} \top$	$\varphi \to \top \xrightarrow{⓪} \top$

Example

$$p \vee (q \wedge (\top \rightarrow (\neg p \vee \bot)) \rightarrow (\top \wedge \neg q))$$

Example

$$p \vee (q \wedge (\top \rightarrow (\neg p \vee \bot)) \rightarrow (\top \wedge \neg q))$$

$$\stackrel{⓪}{\longrightarrow} \quad p \vee (q \wedge (\top \rightarrow \neg p) \rightarrow (\top \wedge \neg q))$$

Example

$$p \vee (q \wedge (\top \to (\neg p \vee \bot)) \to (\top \wedge \neg q))$$

$$\xrightarrow{⓪} \quad p \vee (q \wedge (\top \to \neg p) \to (\top \wedge \neg q)) \quad \xrightarrow{⓪} \quad p \vee (q \wedge (\top \to \neg p) \to \neg q)$$

Example

$$p \vee (q \wedge (\top \to (\neg p \vee \bot)) \to (\top \wedge \neg q))$$

$$\xrightarrow{⓪} \; p \vee (q \wedge (\top \to \neg p) \to (\top \wedge \neg q)) \;\xrightarrow{⓪}\; p \vee (q \wedge (\top \to \neg p) \to \neg q)$$

$$\xrightarrow{⓪} \; p \vee (q \wedge \neg p \to \neg q)$$

Example

$$p \vee (q \wedge (\top \to (\neg p \vee \bot)) \to (\top \wedge \neg q))$$

$$\xrightarrow{⓪} p \vee (q \wedge (\top \to \neg p) \to (\top \wedge \neg q)) \xrightarrow{⓪} p \vee (q \wedge (\top \to \neg p) \to \neg q)$$

$$\xrightarrow{⓪} p \vee (q \wedge \neg p \to \neg q) \xrightarrow{①} p \vee (\neg(q \wedge \neg p) \vee \neg q)$$

Example

$$p \vee (q \wedge (\top \to (\neg p \vee \bot)) \to (\top \wedge \neg q))$$

$$\overset{⓪}{\longrightarrow} \; p \vee (q \wedge (\top \to \neg p) \to (\top \wedge \neg q)) \; \overset{⓪}{\longrightarrow} \; p \vee (q \wedge (\top \to \neg p) \to \neg q)$$

$$\overset{⓪}{\longrightarrow} \; p \vee (q \wedge \neg p \to \neg q) \; \overset{①}{\longrightarrow} \; p \vee (\neg(q \wedge \neg p) \vee \neg q)$$

$$\overset{②}{\longrightarrow} \; p \vee ((\neg q \vee \neg\neg p) \vee \neg q)$$

Example

$$p \vee (q \wedge (\top \to (\neg p \vee \bot)) \to (\top \wedge \neg q))$$

$$\xrightarrow{⓪} p \vee (q \wedge (\top \to \neg p) \to (\top \wedge \neg q)) \xrightarrow{⓪} p \vee (q \wedge (\top \to \neg p) \to \neg q)$$

$$\xrightarrow{⓪} p \vee (q \wedge \neg p \to \neg q) \xrightarrow{①} p \vee (\neg (q \wedge \neg p) \vee \neg q)$$

$$\xrightarrow{②} p \vee ((\neg q \vee \neg\neg p) \vee \neg q) \xrightarrow{②} p \vee ((\neg q \vee p) \vee \neg q)$$

Example (CNF from truth table)

$\varphi = \neg(p \vee (q \wedge r))$

p	q	r	φ
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	F

p	q	r	φ
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	T

Example (CNF from truth table)

$\varphi = \neg(p \vee (q \wedge r))$

p	q	r	φ
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	F

p	q	r	φ
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	T

$\varphi \equiv \neg((p \wedge q \wedge r) \qquad)$

Example (CNF from truth table)

$\varphi = \neg(p \vee (q \wedge r))$

p	q	r	φ
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	F

p	q	r	φ
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	T

$$\varphi \equiv \neg((p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \qquad)$$

Example (CNF from truth table)

$\varphi = \neg(p \vee (q \wedge r))$

p	q	r	φ
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	F

p	q	r	φ
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	T

$$\varphi \equiv \neg((p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \qquad)$$

Example (CNF from truth table)

$\varphi = \neg(p \vee (q \wedge r))$

p	q	r	φ
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	F

p	q	r	φ
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	T

$$\varphi \equiv \neg((p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \qquad)$$

Example (CNF from truth table)

$\varphi = \neg(p \vee (q \wedge r))$

p	q	r	φ
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	F

p	q	r	φ
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	T

$$\varphi \equiv \neg((p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge r))$$

Example (CNF from truth table)

$\varphi = \neg(p \vee (q \wedge r))$

p	q	r	φ
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	F

p	q	r	φ
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	T

$$\begin{aligned}\varphi &\equiv \neg\big((p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge r)\big)\\ &\equiv (\neg p \vee \neg q \vee \neg r) \wedge (\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee \neg r) \wedge (\neg p \vee q \vee r) \wedge (p \vee \neg q \vee \neg r)\end{aligned}$$

Theorem

for every formula φ there exists CNF ψ such that $\varphi \equiv \psi$

Procedure

① eliminate implications

② push negations towards atoms and remove double negations

③ distribute disjunction over conjunction

Theorem

for every formula φ there exists CNF ψ such that $\varphi \equiv \psi$

Deterministic Procedure

① eliminate implications

function IMPL_FREE(φ):
begin function
case φ is atom: **return** φ
φ is $\neg\varphi_1$: **return** $\neg$ IMPL_FREE(φ_1)
φ is $\varphi_1 \wedge \varphi_2$: **return** IMPL_FREE($\varphi_1$) $\wedge$ IMPL_FREE(φ_2)
φ is $\varphi_1 \vee \varphi_2$: **return** IMPL_FREE(φ_1) $\vee$ IMPL_FREE(φ_2)
φ is $\varphi_1 \to \varphi_2$: **return** $\neg$ IMPL_FREE(φ_1) $\vee$ IMPL_FREE(φ_2)
end case
end function

Theorem

for every formula φ there exists CNF ψ such that $\varphi \equiv \psi$

Deterministic Procedure

② push negations towards atoms and remove double negations

function NNF(φ):

begin function

case	φ is literal:	**return**	φ
	φ is $\neg\neg\varphi_1$:	**return**	NNF(φ_1)
	φ is $\varphi_1 \wedge \varphi_2$:	**return**	NNF(φ_1) $\wedge$ NNF(φ_2)
	φ is $\varphi_1 \vee \varphi_2$:	**return**	NNF(φ_1) $\vee$ NNF(φ_2)
	φ is $\neg(\varphi_1 \wedge \varphi_2)$:	**return**	NNF($\neg\varphi_1$) $\vee$ NNF($\neg\varphi_2$)
	φ is $\neg(\varphi_1 \vee \varphi_2)$:	**return**	NNF($\neg\varphi_1$) $\wedge$ NNF($\neg\varphi_2$)

end case

end function

Theorem

for every formula φ there exists CNF ψ such that $\varphi \equiv \psi$

Deterministic Procedure

② push negations towards atoms and remove double negations

function NNF(φ):

begin function

case	φ is literal:	**return**	φ
	φ is $\neg\neg\varphi_1$:	**return**	$\mathrm{NNF}(\varphi_1)$
	φ is $\varphi_1 \wedge \varphi_2$:	**return**	$\mathrm{NNF}(\varphi_1) \wedge \mathrm{NNF}(\varphi_2)$
	φ is $\varphi_1 \vee \varphi_2$:	**return**	$\mathrm{NNF}(\varphi_1) \vee \mathrm{NNF}(\varphi_2)$
	φ is $\neg(\varphi_1 \wedge \varphi_2)$:	**return**	$\mathrm{NNF}(\neg\varphi_1) \vee \mathrm{NNF}(\neg\varphi_2)$
	φ is $\neg(\varphi_1 \vee \varphi_2)$:	**return**	$\mathrm{NNF}(\neg\varphi_1) \wedge \mathrm{NNF}(\neg\varphi_2)$

end case

end function

Theorem

for every formula φ there exists CNF ψ such that $\varphi \equiv \psi$

Deterministic Procedure

③ distribute disjunction over conjunction

function $\mathsf{CNF}(\varphi)$:

begin function

case	φ is literal:	**return**	φ
	φ is $\varphi_1 \wedge \varphi_2$:	**return**	$\mathsf{CNF}(\varphi_1) \wedge \mathsf{CNF}(\varphi_2)$
	φ is $\varphi_1 \vee \varphi_2$:	**return**	$\mathsf{DISTR}(\mathsf{CNF}(\varphi_1), \mathsf{CNF}(\varphi_2))$

end case

end function

Theorem

for every formula φ there exists CNF ψ such that $\varphi \equiv \psi$

Deterministic Procedure

③ distribute disjunction over conjunction

function CNF(φ):

begin function

case	φ is literal:	**return**	φ
	φ is $\varphi_1 \wedge \varphi_2$:	**return**	$\mathrm{CNF}(\varphi_1) \wedge \mathrm{CNF}(\varphi_2)$
	φ is $\varphi_1 \vee \varphi_2$:	**return**	DISTR(CNF(φ_1), CNF(φ_2))

end case

end function

Theorem

for every formula φ there exists CNF ψ such that $\varphi \equiv \psi$

Deterministic Procedure

③ distribute disjunction over conjunction

function $\mathrm{DISTR}(\eta_1, \eta_2)$:

begin function

case	η_1 is $\eta_{11} \wedge \eta_{12}$:	**return**	$\mathrm{DISTR}(\eta_{11}, \eta_2) \wedge \mathrm{DISTR}(\eta_{12}, \eta_2)$
	η_2 is $\eta_{21} \wedge \eta_{22}$:	**return**	$\mathrm{DISTR}(\eta_1, \eta_{21}) \wedge \mathrm{DISTR}(\eta_1, \eta_{22})$
	otherwise:	**return**	$\eta_1 \vee \eta_2$

end case

end function

Theorem

for every formula φ there exists CNF ψ such that $\varphi \equiv \psi$

Deterministic Procedure

1. eliminate implications
2. push negations towards atoms and remove double negations
3. distribute disjunction over conjunction

Theorem

1. CNF(NNF(IMPL_FREE(φ))) is CNF

Theorem

for every formula φ there exists CNF ψ such that $\varphi \equiv \psi$

Deterministic Procedure

① eliminate implications

② push negations towards atoms and remove double negations

③ distribute disjunction over conjunction

Theorem

❶ $\mathsf{CNF}(\mathsf{NNF}(\mathsf{IMPL\_FREE}(\varphi)))$ is CNF

❷ $\mathsf{CNF}(\mathsf{NNF}(\mathsf{IMPL\_FREE}(\varphi))) \equiv \varphi$

Theorem

for every formula φ there exists CNF ψ such that $\varphi \equiv \psi$

Deterministic Procedure

① eliminate implications

② push negations towards atoms and remove double negations

③ distribute disjunction over conjunction

Theorem

1. $\mathsf{CNF}(\mathsf{NNF}(\mathsf{IMPL\_FREE}(\varphi)))$ is CNF
2. $\mathsf{CNF}(\mathsf{NNF}(\mathsf{IMPL\_FREE}(\varphi))) \equiv \varphi$
3. executing $\mathsf{CNF}(\mathsf{NNF}(\mathsf{IMPL\_FREE}(\varphi)))$ terminates

Outline

Huth and Ryan

- Section 1.1
- Section 1.3
- Sections 1.4.1 and 1.4.2
- Sections 1.5.1 and 1.5.2

Huth and Ryan

- Section 1.1
- Section 1.3
- Sections 1.4.1 and 1.4.2
- Sections 1.5.1 and 1.5.2

Differences (slides – book)

- role of $\bot$ and $\top$
- terminology concerning CNFs

Important Concepts

- atom
- bottom
- clause
- complementary literals
- conjunction
- conjunctive normal form
- disjunction
- disjunctive normal form
- implication
- literal
- negation
- top
- right−associativity
- satisfiability
- semantic entailment
- semantic equivalence
- tautology
- truth table
- truth values
- validity
- valuation

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homework for March 5 and 12